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Estimates of Technology and Convergence: Simulation Results

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Estimates of Technology and Convergence: Simulation Results.

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Abstract

Using a Solow-Swan model with a stochastic saving rate and stochastic productivity we analyse the distributions of parameter estimates that emerge under various choices of technology, and of the dimension of the panel on which cross-section regressions are based. There are distinct asymmetries that characterize these distributions. These asymmetries become more pronounced when the effects of a near-unit root in the productivity shock become magnified over a longer time horizon. Consequently, relying on traditional econometric transformations of these parameter estimates based on symmetric distributions, such as $t-$ratios, will be quite misleading if one tries to assess technology parameters and $\beta-$convergence.

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1 Introduction

In recent times researchers have made intensive use of cross-country evidence to assess alternative growth models. A central part of this research agenda concerns the empirical implications of different theories for the dynamic adjustment of per capita incomes. That these models converge to their country-specific steady state growth paths means that, in terms of the empirical growth literature, they satisfy conditional $\beta-$convergence[1]. Within the class of neoclassical growth models, however, the robustness of the conclusions of the empirical growth literature has come under scrutiny. Our approach in this paper is to revisit some of the early and influential empirical work which applies OLS to estimate technology parameters and convergence speed $-$ our contribution being to apply these techniques to artificial data generated from a stochastic Solow-Swan model in which both the saving rate and productivity are stochastic.

There is a well-known theoretical literature on stochastic growth models, although as far as we are aware there is no theoretical literature dealing with steady-state distributions of variables (or ratios of variables) in the Solow-Swan model with a stochastic saving rate[2].

The work most closely related to ours is that of Lee, Pesaran and Smith (1997). They examine properties of estimates of $\beta-$ convergence obtained using the common cross-section approach in which observations on each country in the panel are averaged over the time interval T, but the initial level of output is included in the OLS regression to capture dynamics. Each of the R countries in the sample shares a common Cobb-Douglas technology but is subject to individual stochastic processes for productivity and the labour supply. They derive an expression for the asymptotic R, small T bias in the estimate of $\beta-$convergence.

Our paper extends the work of Lee, Pesaran and Smith (1997) in several

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[1] There is a variety of terminology with respect to β-convergence. We follow that used by Durlauf and Quah (1999).

[2] See, for instance, Mirman (1972, 1973), Merton (1975), Binder and Pesaran (1999), Stachurski (2003).

ways. Unlike them, we rely on a Monte Carlo investigation of the underlying Solow-Swan growth model. We also extend their analysis by considering stochastic saving as well as stochastic productivity. Monte Carlo analysis allows us to consider the small R, small T case. This situation is perhaps more relevant in empirical work where the assumption of a common technology is most naturally applied to, say, the OECD countries in which case R is about 30. As we show later, this is much less than is required to justify reliance on asymptotic - R results. Finally, we report features of the distribution of the estimated capital share parameter as well as the measure of $\beta-$ convergence - it is the latter which is the primary focus of Lee, Pesaran and Smith (1997).

The rest of the paper is structured as follows. The next section specifies the stochastic closed economy Solow-Swan model with disturbances to the rate of productivity growth and the saving rate, and in the following section we discuss the distributions of the parameter estimates of the capital share and convergence parameters as they emerge from the synthetic data. Finally we conclude.

2 The Simulation Model

The discrete-time specification of the model is as follows. Output is given by a constant returns production function with Harrod-neutral technical progress and, in the case of Cobb-Douglas production, satisfying Inada conditions. The rate of capital accumulation is determined by saving at (stochastic) rate s, and (deterministic) depreciation at rate δ. Effective labour input E is made up of a stochastic productivity term A and deterministic stock of labour N. The initial labour force is normalised to unity. Basic equations of the model are as follows, with country subscripts (j) suppressed. The production technology is

$$Y_t = K_{t-1}^{\alpha}, E_{t-1}^{1-\alpha}, \tag{1}$$

while capital depreciates at rate δ

$$K_t = s_t Y_t + (1-\delta) K_{t-1}, \tag{2}$$

and the labour force grows at rate n,

$$E_t = A_t N_t,\ N_t = \exp(n't). \tag{3}$$

where $n' = \ln(1+n)$.

We distinguish between two different cases for the generation of the saving rate. In the first case, we assume a stochastic process, since in individual countries s appears to show positive serial correlation. We assume the process for s to be

$$s_t = s + \theta_t,\ \theta_t = \rho_s \theta_{t-1} + \epsilon_{s,t},\ 0 \leq \rho_s \leq 1 \tag{4}$$

where $\epsilon_{s,t}$ is $N(0, \sigma_s^2)$[3]. Since it is implicit that this is a single-commodity model, there is no reason to constrain the saving rate to be positive. In Mankiw, Romer and Weil (1992) the saving rate is proxied by the ratio of gross investment to GDP, a ratio that exhibits considerable short-term variation. In empirical data the persistence in the investment ratio appears to smaller than for productivity and this is reflected in the simulations by our assumption of a lower first-order autoregressive coefficient on the saving rate than productivity. However, the counterfactual assumption implicit in the Mankiw, Romer and Weil (1992) approach to estimation is that while saving rate can vary across countries, it is constant over the sample period.

In the second nonstochastic case, we incorporate this counterfactual assumption as an alternative way of generating s from that of equation (4), where now s is allowed to vary across countries, but is constant over time. To do that we assume that savings rates are uniformly distributed over the interval 0.15 to 0.35, and we hold them constant for each country throughout the analysis.

The productivity shock is

$$A_t = \exp(\mu' t + \varsigma_t),\ \varsigma_t = \rho_A \varsigma_{t-1} + \epsilon_{A,t},\ 0 \leq \rho_A \leq 1 \tag{5}$$

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[3] Binder and Pesaran (1999) provide sufficient (but not necessary) conditions for egodicity of per capita output. One of the conditions is that the distribution of shocks is truncated on the left to exclude large negative shocks. Rather than impose this condition we assume normality. In the simulations conducted for this paper there were no cases of shocks being 'too large' in the sense of inducing a negative level of output.

where, again $\mu' = \ln(1+\mu)$ is the mean growth rate, and $\epsilon'_{A,t}$ is $N(0, \sigma_A^2)$. Note that the literature assumes that the productivity shock to be typically very persistent. In the following simulations we will do the same.

The deterministic component of the above model is assumed to be the same for all countries in the sample, reflecting the usual assumption that all countries have access to a common technology. In the case analysed in this paper we assume, for simplicity, that commonality of deterministic components also implies that population growth rates are the same across countries. In the empirical growth literature, OECD countries are often considered to be homogeneous in terms of technology, and this motivates the initial choice of 30 as the number of countries used in the simulations. We also let this number increase to 120 and examine the effect that such an increase will have on the resulting distributions of the parameter estimates. Starting from the same initial point, each 'country' is simulated for H periods, and observations between $t = 70$ and $t = H$ are retained as data for estimation. The unrestricted 'Barro regression' is

$$\ln(Y_H/N_H) - \ln(Y_{70}/N_{70}) = \beta_0 + \beta_1 \ln s + \beta_2 \ln(Y_{70}/N_{70}) + \epsilon \tag{6}$$

where s is the sample-mean saving rate for country j. The interpretation of $\beta_1$and β_2 is usually motivated from the well-known result for the non-stochastic case, namely

$$\begin{aligned} & \ln(Y(t+H)/N(t+H)) - \ln(Y(t)/N(t)) && \text{(7a)} \\ = \quad & \gamma_1 + \gamma_2 \left(\frac{\alpha}{1-\alpha} \ln s - \frac{\alpha}{1-\alpha} \ln(n+\delta+\mu) - \ln(Y_j(t)/N_j(t)) \right) && \text{(7b)} \end{aligned}$$

where $\gamma_2 = 1 - \exp \beta H$, $\gamma_1 = (t + H - t \exp \beta H)\mu + \gamma_2 \ln A(0)$, and the speed of convergence is $\beta = -(1-\alpha)(\delta + n + \mu)$ [4]. Equation (6) is the basis for the estimation of convergence speed in Mankiw, Romer and Weil (1992), where the only stochastic element in the model is the initial condition $Y_{70/}N_{70}$ which is assumed to be independent of the cross-sectional disturbance term. In particular μ, s, δ and n are nonstochastic and, on the common-technology assumption, μ and δ are the same across countries. Following Mankiw, Romer and Weil (1992)

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[4] See Durlauf and Quah (1999), p.256.

in ignoring the variability implied by the inclusion of the sample mean μ in the expression for β, an estimate of the conditional convergence parameter β and technology parameter α can be obtained from (8) and (9) where

$$\beta_1 = \frac{\alpha}{1-\alpha}(1 - \exp(\beta[H - 70]) \; and \tag{8}$$

$$\beta_2 = \exp(\beta[H - 70]) - 1 \tag{9}$$

Although the objective of this paper is to revisit results obtained in the OLS framework used by Mankiw, Romer and Weil (1992) and others subsequently, we recognise that the use of OLS estimates $\widehat{\beta}_1$and $\widehat{\beta}_2$ to obtain estimates of α and β is problematic. There are several issues. The deterministic model of the previous section implies a restriction on estimation of β_1 and β_2, encapsulated in (8) and (9). Imposition of this restriction would also require that cross-sectional variation in μ'_j, as in (5) be taken into account[5]. Further, it follows from the setup of this section that measurement error is induced by taking sample means of the saving rate and the productivity growth rate. Finally, it is clear that the assumption (maintained by Mankiw, Romer and Weil (1992) and others) that the regressors are independent of the disturbance term in *not* valid in the present setup. Our simulation framework, which is intended to be a literal interpretation of the Solow-Swan model with common (stochastic) technology and stochastic saving, generates cross-sectional variation in the initial condition Y_{70}/N_{70} from a prior history of productivity and saving shocks. Since these shocks have persistence, sample-period values of s and μ from $t = 70$ to H are correlated with the initial condition and with the disturbance term.

3 Simulation Results

Our simulation results are based on parameter choices that are consistent with stylized facts. They are s_0 =0.25, $\alpha = 1/3$, $n = 0.02$, $\delta = 0.04$, $\mu = 0.02$,

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[5] With few exceptions (of which Lee, Pesaran and Smith (1997) is one), empirical work emphasises cross-sectional variation in sample population growth rather than in sample realisations of productivity. For simplicity we abstract from cross-sectional variation in population growth.

σ_s^2 =0.02, $\sigma_A^2 = 0.02$, ρ_s =0.5 and ρ_A =0.95. Tables 1 and 2 present the analysis for the stochastic and nonstochastic saving rate respectively. We define the period of analysis T as $H - 70$ from the previous section.

The first row of Table 1 gives the simulation results for a group of 30 homogeneous countries over a 35 period horizon ($R = 30$, $T = 35$) using 10,000 artificial data sets generated with the above parameter choices. This group for example may represent the OECD countries over the last 35 years. For each (R, T) combination we report the mean and the mean-squared error (MSE) and the mean absolute bias (MAB) of the estimated α and β parameters as well as the proportion of times that the estimate of the convergence parameter β was undefined, when the solutions to equations (8) and (9) generate imaginary values. The mean of the estimates is less than the true value of -0.06 for β, and the proportion of ill-defined βs is problematically high at about 7 percent. Also the estimate of α is not very close to the true value of 1/3 and its MSE is quite high. Things worsen considerably when T increases while R is fixed. In that case the effects of the near unit root in the productivity shock are magnified and the downward bias in parameter estimates increases. There is also a dramatic increase in the proportion of ill-defined (imaginary) values of β. In fact when $T = 100$ this proportion is nearly 50 percent and the distribution of the $\beta-$estimates is highly skewed to the right, irrespective of the number of countries R. For fixed values of T, on the other hand, as R increases the distribution of the estimates becomes more symmetric and tight and the MSE of the both $\widehat{\alpha}$ and $\widehat{\beta}$ decreases. We have also conducted experiments with values of ρ_s 0.8 and 0.2 and of ρ_A 0.8, 0.5 and 0.2 to check the sensitivity of the reported results[6]. The results for the β parameter are not sensitive to these choices, except that the estimates of α are more precise with lower values of ρ_A and higher values of ρ_s.

Hence we obtain a striking and at first hand disconcerting pattern of the shape of the distribution of the convergence parameter β and to a lesser extent

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[6] Results for different choices of ρ_A and ρ_s are available from the authors.

of the capital share parameter α^7. These tend to be quite skewed to the right and hence econometric results using standard statistics based on symmetry assumptions, such as t-ratios will not be valid to assess statistical significance. However, the problem of evaluating convergence goes deeper than that. Even if we were to use robust econometric methods such as Least Absolute Deviation regression we would still run into similar problems (see the lower panel of Table 1 for examples). Standard regression estimates which underlie $\beta-$convergence, represent average behavior over time and ignore what happens to the whole distribution, see Quah (1996, 1997). In that case using robust methods, see Koenker and Basset (1978), Koenker (1982) and Buchinsky (1994), would not be of help, since these methods concentrate on what happens to specific parts of the distribution such as the median or specific quantiles. Consequently when testing for convergence one may want to look at methods that emphasize the use of distribution dynamics based on transition matrices and their continuous counterpart, stochastic kernels as in Quah (1997) or nonparametric density estimates of the growth rate distribution over time as in Bianchi (1997).

The results of the nonstochastic saving rate are presented in Table 2. The pattern of ill-defined estimates of the convergence parameter β remains the same as in the case of the stochastic saving rate and the same issues that are discussed above remain. However, the estimates of the capital share parameter α are now very precise. The results for α in this case are similar to the case of a stochastic saving rate with higher values of ρ_s (not reported). In that case high persistence of that parameter would imply near constancy of the saving rate of each country around its trend, something that is captured exactly in the nonstochastic case. As in the case of Table 1, the pattern of results is the same whether one uses OLS or LAD methods.

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[7] Although most of the literature is based on Cobb-Douglas production we also analysed the effects of assuming that the production technology is Cobb-Douglas when in fact the elasticity of substitution is less than one. Small mis-specifications in the form of departures from Cobb-Douglas technology induce large changes in the estimate of α, which in this case would be mistakenly identified with the capital share parameter in a Cobb-Douglas production function. Results are available from the authors.

4 Conclusions

Using a Solow-Swan model for the case with a stochastic saving rate and stochastic productivity growth we analyze the distribution of parameter estimates that emerges under various parameter choices. The examination of the Monte Carlo distributions of the parameter estimates suggests that these parameter estimates cannot be used to assess $\beta-$convergence when combined with traditional econometric statistics that are based on symmetric distributions, such as $t-$ratios. There are distinct asymmetries that characterize these distributions which become more pronounced when the effects of a near-unit root in the productivity shock become more pronounced. For the setup analysed here, the use of robust methods is not likely to overcome the estimation problems induced by these asymmetries.

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Table 1

$\alpha = 1/3, \beta = -0.06, \rho_A = 0.95, \rho_S = 0.5, N = 10,000$

(R,T)	$Mean(\widehat{\alpha})$	$MSE(\widehat{\alpha})$	$MAB(\widehat{\alpha})$	$Mean(\widehat{\beta})$	$MSE(\widehat{\beta})$ (x10^{-4})	$MAB(\widehat{\beta})$ (x10^{-2})	Prop. ill-defined $\widehat{\beta}$
OLS estimates							
(30,35)	0.2698	4.773	0.3561	-0.0368	6.747	2.394	0.0668
(30,50)	0.3762	176.4	0.5630	-0.0283	6.809	2.939	0.2318
(30,100)	0.0581	647.2	1.352	-0.0118	10.78	4.145	0.4659
(60,35)	0.3324	0.0744	0.1446	-0.0384	4.625	1.913	0.0176
(60,50)	0.3252	5.438	0.1822	-0.0338	5.093	2.390	0.1278
(60,100)	0.2483	2.130	0.2723	-0.0142	8.609	3.910	0.4477
LAD estimates							
(30,35)	0.1421	88.57	0.7217	-0.0346	9.129	2.708	0.1076
(30,50)	0.2622	2196	1.641	-0.0251	0.888	3.197	0.2563
(30,100)	0.7601	817.8	1.727	-0.0107	12.17	4.253	0.4713

Table 2

$\alpha = 1/3, \beta = -0.06, \rho_A = 0.95, N = 10,000, s$ nonstochastic

(R,T)	$Mean(\widehat{\alpha})$	$MSE(\widehat{\alpha})$ $(x10^{-4})$	$MAB(\widehat{\alpha})$ $(x10^{-2})$	$Mean(\widehat{\beta})$	$MSE(\widehat{\beta})$ (x10^{-4})	$MAB(\widehat{\beta})$ (x10^{-2})	Prop. ill-defined $\widehat{\beta}$
				OLS estimates			
(30,35)	0.3353	9.499	2.173	-0.0361	6.982	2.440	0.0664
(30,50)	0.3341	6.079	1.861	-0.0278	6.812	2.930	0.2113
(30,100)	0.3334	4.348	1.583	-0.0112	10.81	4.131	0.4573
(60,35)	0.3349	3.447	1.453	-0.0378	4.891	1.966	0.0161
(60,50)	0.3342	2.502	1.243	-0.0336	5.198	2.399	0.1215
(60,100)	0.3334	1.838	1.068	-0.0141	8.747	3.922	0.4471
				LAD estimates			
(30,35)	0.3348	21.28	3.614	-0.0336	9.069	2.745	0.1117
(30,50)	0.3338	22.34	2.370	-0.0250	8.795	3.190	0.2490
(30,100)	0.3333	7.143	1.956	-0.0110	1.214	4.230	0.4641

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