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Abstract

This paper builds a structural VARMA (SVARMA) model for investigating Canadian monetary policy. Using the scalar component model (SCM) methodology proposed by Athanasopoulos and Vahid (2008a), we first identify a VARMA model and then construct a SVARMA for Canadian monetary policy. We include a SVAR model in our study for the purpose of comparison and we generate impulse responses along with 68% confidence bands for both models. Relative to the SVAR, the impulse responses generated by the SVARMA appear to be consistent with those predicted by various economic theoretical models and solve the economic puzzles found commonly in the empirical literature on monetary policy. The successful construction and implementation of the SVARMA model for Canadian monetary policy analysis, along with its promising impulse responses and superior out-of-sample forecasting performance of its reduced form compared to the VAR alternatives, indicates the suitability of this framework for small open economies.

Keywords: VARMA models, Identification, Impulse responses, Confidence band, Open economy, Transmission mechanism.
JEL classification numbers: C32, E52, F41

1 Introduction

Over the past three decades, extensive investigations into modelling and analyzing monetary policies have led to the conclusion that differences in model specifications and parameter estimates across models can lead to widely different policy recommendations. In addition, the potential loss from basing monetary policy on an invalid model can be substantial. Since the seminal paper by Sims (1980), the use of vector autoregression (VAR) and structural VAR (SVAR) models has been prevalent in the empirical literature on monetary policy analysis. For an extensive review, see Leeper et al. (1996) and Christiano et al. (1999). Despite the sound theoretical and empirical justifications for the superiority of vector autoregressive moving average (VARMA) models over VAR-type models for policy modelling, the use of the former is still in its infancy; more on this later. The main reason for this is the lack of methodological advances in establishing uniquely identified VARMA models. Recently, however, Athanasopoulos and Vahid (2008a) proposed a complete methodology for identifying and estimating canonical VARMA models by extending the work of Tiao and Tsay (1989). They established necessary and sufficient conditions for exactly identifying a canonical VARMA model so that all parameters can be efficiently identified and estimated simultaneously using full information maximum likelihood (FIML). Furthermore, Dufour and Pelletier (2002, 2008) illustrated that the VARMA representation is more appropriate for modelling monetary policy analysis than the VAR counterpart.

The main contribution of this paper is the building of a SVARMA model for Canadian monetary policy, and the conducting of an investigation to uncover the underlying effects of the Bank of Canada's monetary policy on the inflation rate and the output level in the economy, among other things. The latter is accomplished by generating reliable dynamic impulse response functions along with 68% confidence bands. We apply the methodology of Athanasopoulos and Vahid (2008a) to examine the advantages of using VARMA models for the monetary policy framework of Canada - a small open economy. The aims of the paper are to: (i) adapt this new methodology to build a structural VARMA (SVARMA) framework for Canadian monetary policy analysis; (ii) compare the performance of the SVARMA framework with its SVAR counterpart in terms of impulse responses; and (iii) discover whether the new framework can resolve the economic puzzles commonly found in the empirical monetary literature of small open economies.[1]

Monetary policy is widely implemented as a stabilization policy instrument for steering economies in the direction of achieving sustainable economic growth and price stability. The efficacy of monetary policy depends on the ability of policy makers to make an accurate assessment of the timing and effect of the policy on economic activities and prices. In 1991, jointly with the Canadian government, the Bank of Canada adopted an inflation targeting monetary policy framework, with the intention of keeping annual inflation close to two per cent and within the range of one to three per cent. To achieve this objective, the Bank of Canada uses the overnight interest rate as it's policy instrument (see Bhuiyan, 2012). In a low inflation environment, spending, saving, investments and output are expected to increase which, in turn, lead to steadily increasing living standards in Canada; see, for example, Ragan (2005).

Although VARs provide useful tools for evaluating the effect of monetary policy shocks, there are ample warnings in the literature of their limitations based on both theoretical and practical grounds. We shall discuss some of the justifications for the use of VARMA models over VARs provided in the recent literature. In studies of monetary policy, the dominant part of the analysis is based on the dynamics of impulse response functions of domestic variables to various monetary shocks; these impulse responses are derived using Wold's decomposition theorem. In a multivariate Wold representation, however, any covariance stationary time series can be transformed into an infinite order vector moving

[1] The main economic puzzles are referred to as: (i) the price puzzle (an unanticipated tightening of monetary policy, which is identified with innovations in interest rates, is associated with an increase in the price level rather than a decrease); (ii) the output puzzle (an unanticipated tightening of monetary policy, is associated with an increase in the output level rather than a decrease); and (iii) the exchange rates puzzle (an unanticipated increase in interest rates is associated with a depreciation of the country's exchange rate relative to the US rather than an appreciation).

average (VMA(∞)) process of its innovations. A finite order VARMA model provides a better approximation to the Wold representation than a finite order VAR, with the former producing more reliable impulse responses than the latter.

Several authors have put forward several convincing arguments in support of VARMA processes over VARs for modelling macroeconomic variables.[2] In addition, economic and financial time series are, invariably, constructed data involving, for example, seasonal adjustment, de-trending and temporal and contemporaneous aggregation. Such constructed time series would include moving average dynamics even if their constituents were generated by pure autoregressive processes. Further, a subset of a system of variables that was generated by a vector autoregression would also follow a VARMA process.[3]

To simplify the modelling and estimating a system of variables, applied researchers tend to approximate a VARMA process by a high-order VAR process. The use of VAR approximations requires models with extremely long lag lengths, much longer than those selected by typical information criteria such as the AIC or BIC, in order to describe a system adequately and to obtain reliable impulse responses.[4] However, in practice, the available sample sizes are inadequate to accommodate a sufficiently long lag structure, and thus lead to poor approximations of the real business cycle models (see, for example, Chari et al., 2007). On the other hand, Dufour and Pelletier (2002, 2008) illustrate that the impulse responses obtained from the more parsimonious VARMA representation are more precisely estimated than those obtained from their VAR counterparts, while Athanasopoulos and Vahid (2008b) show that VARMA models forecast macroeconomic variables more accurately than VARs. Moreover, via a simulation study, Athanasopoulos and Vahid (2008b) demonstrate that the forecast superiority comes from the presence of moving average components.

[2] See for example, Zellner and Palm (1974); Granger and Morris (1976); Wallis (1977); Maravall (1993); Dufour and Pelletier (2002); Lütkepohl (2005); Fry and Pagan (2005).

[3] Cooley and Dwyer (1998) claim that the basic real business cycle models follow VARMA processes. More recently, Fernández-Villaverde et al. (2005) demonstrated that linearized dynamic stochastic general equilibrium models in general imply a finite order VARMA structure.

[4] In a simulation study, Kapetanios et al. (2007) show that a sample size of 30,000 observations and a VAR of order 50 are required to sufficiently capture the dynamic effects of some of the economic shocks.

Despite the aforementioned justifications and recommendations to employ VARMA models rather than VARs, the use of the former is not prevalent in applied macroeconomics, mainly due to difficulties in identifying a unique VARMA representation and its estimation. A search for an identified VARMA model is far more challenging than a simple VAR-type model specification, and the lack of enthusiasm for the use of VARMA models is due to such difficulties.[5] In this paper, we build a SVARMA model for Canadian monetary policy analysis in two stages: (i) we identify a VARMA model by implementing the methodology of Athanasopoulos and Vahid (2008a); and (ii) we impose a recursive structure on the contemporaneous matrix of SVARMA in order to identify the orthogonal policy and non-policy shocks as opposed to a non-recursive contemporaneous SVAR structure applied by Cushman and Zha (1997); Kim and Roubini (2000) and Bhuiyan (2012) on the Canadian economy.[6] In light of the foregoing discussions on its suitability, we expect the recursive SVARMA model to produce reliable dynamic impulse responses that are consistent with economic theoretical models and stylized facts, compared with the widely used SVAR model.

In our empirical modelling of Canadian monetary policy, we use a similar set of seven variables as Kim and Roubini (2000) who, among others, have estimated a SVAR model for Canadian monetary policy. Of the seven variables, the world oil price index and the Federal funds rates represent the foreign variables, while the industrial production index, consumer price index, monetary aggregate M1, short-term interest rate and exchange rate represent the domestic variables. However, in contrast to Kim and Roubini (2000) who employed the market interest rate as the policy instrument, in this paper as in Bhuiyan (2012) we use the overnight rate target to identify the monetary policy reaction function. Studies by Kim and Roubini (2000) and Brischetto and Voss (1999) demonstrated that these seven variables are sufficient to describe the monetary policy framework of small open economies. In fact, they have provided evidence that these seven variables can capture the features of large

[5]See for example Hannan and Deistler (1988); Tiao and Tsay (1989); Reinsel (1997); Lütkepohl (2005).

[6]Bhuiyan and Lucas (2007) also used a recursive VAR model to asses the real and nominal effects of Canadian monetary policy shocks.

and more complex open economy models, such as that investigated by Cushman and Zha (1997). In a quest to solve the empirical puzzles (mentioned in footnote 1) which are found largely in the VAR framework, Kim and Roubini (2000) developed SVARs for modelling Canadian and other non-US G7 economies. However, their results for Canada indicate that though the price puzzle did not exist, contrary to expectations, a monetary tightening induced a brief increase in output instead of a fall. In this paper, the SVARMA-based empirical results for the extended period of study show that there do not exist any of the empirical puzzles and that a positive monetary shock reduced inflation, output and money demand in the Canadian economy and the confidence bands around SVARMA responses appear to be much more narrower. This indicates that the parsimonious SVARMA model provides more precise impulse response functions compared to the SVAR model.

In a most recent study, Bhuiyan (2012) has developed a Bayesian SVAR model for the Canadian economy and estimated the impacts of monetary policy shocks and the overnight target rate was used as the policy instrument.[7] In this Bayesian SVAR framework, the policy variables and other domestic and foreign variables were allowed to react contemporaneously. The study found that the Bank of Canada responds to any home and foreign variables that include information on future inflation while monetary policy affects the Canadian real economy through the market interest rate and exchange rate.

The paper is organized as follows: Section 2 discusses briefly the VARMA methodology proposed by Athanasopoulos and Vahid (2008a). Section 3 describes the variables used in the models and their time series properties. Section 4 explains the impulse response functions. Section 5 demonstrates in detail the identification of orthogonal shocks to monetary policy-related variables and the estimation of the SVARMA model for Canadian monetary policy, and reports and analyzes the empirical results. Section 6 concludes the paper.

[7] See Bhuiyan (2012) for the discussion on why overnight rate target would provide more precise measures of exogenous monetary policy shocks compared to market interest rate.

2 Identification of a VARMA model

For identifying and estimating a VARMA model we use the Athanasopoulos and Vahid (2008a) extension of the Tiao and Tsay (1989) scalar component model (SCM) methodology. The aim of identifying scalar components is to examine whether there are any simplifying embedded structures underlying a VARMA(p, q) process.

For a given K dimensional VARMA(p, q) process

$$\boldsymbol{y}_t = \boldsymbol{\Phi}_1 \boldsymbol{y}_{t-1} + \ldots + \boldsymbol{\Phi}_p \boldsymbol{y}_{t-p} + \boldsymbol{v}_t - \boldsymbol{\Theta}_1 \boldsymbol{v}_{t-1} - \ldots - \boldsymbol{\Theta}_q \boldsymbol{v}_{t-q}, \tag{1}$$

a non-zero linear combination $z_t = \boldsymbol{\alpha}' \boldsymbol{y}_t$ follows a SCM(p_1, q_1) if $\boldsymbol{\alpha}$ satisfies the following properties:

$$\begin{aligned} \boldsymbol{\alpha}' \boldsymbol{\Phi}_{p_1} &\neq \mathbf{0}' \text{ where } 0 \leq p_1 \leq p, \\ \boldsymbol{\alpha}' \boldsymbol{\Phi}_l &= \mathbf{0}' \text{ for } l = p_1 + 1, \ldots, p, \\ \boldsymbol{\alpha}' \boldsymbol{\Theta}_{q_1} &\neq \mathbf{0}' \text{ where } 0 \leq q_1 \leq q, \\ \boldsymbol{\alpha}' \boldsymbol{\Theta}_l &= \mathbf{0}' \text{ for } l = q_1 + 1, \ldots, q. \end{aligned}$$

The scalar random variable z_t depends only on lags 1 to p_1 of all variables and lags 1 to q_1 of all innovations in the system. The determination of embedded scalar component models is achieved through a series of canonical correlation tests.

Denote the estimated squared canonical correlations between $\boldsymbol{Y}_{m,t} \equiv \left(\boldsymbol{y}'_t, \ldots, \boldsymbol{y}'_{t-m}\right)'$ and $\boldsymbol{Y}_{h,t-1-j} \equiv \left(\boldsymbol{y}'_{t-1-j}, \ldots, \boldsymbol{y}'_{t-1-j-h}\right)'$ by $\widehat{\lambda}_1 < \widehat{\lambda}_2 < \ldots < \widehat{\lambda}_K$. As suggested by Tiao and Tsay (1989), the test statistic for at least s SCM(p_i, q_i), i.e., s insignificant canonical correlations, against the alternative of less than s scalar components is

$$C(s) = -(n - h - j) \sum_{i=1}^{s} \ln \left\{ 1 - \frac{\widehat{\lambda}_i}{d_i} \right\} \overset{a}{\sim} \chi^2_{s \times \{(h-m)K + s\}} \tag{2}$$

where d_i is a correction factor that accounts for the fact that the canonical variates could be moving averages of order j, and is calculated as follows:

$$d_i = 1 + 2 \sum_{v=1}^{j} \widehat{\rho}_v \left(\widehat{\boldsymbol{r}}'_i \boldsymbol{Y}_{m,t}\right) \widehat{\rho}_v \left(\widehat{\boldsymbol{g}}'_i \boldsymbol{Y}_{h,t-1-j}\right), \tag{3}$$

where $\widehat{\rho}_v\,(.)$ is the v^{th} order autocorrelation of its argument and $\widehat{\boldsymbol{r}}_i'\boldsymbol{Y}_{m,t}$ and $\widehat{\boldsymbol{g}}_i'\boldsymbol{Y}_{h,t-1-j}$ are the canonical variates corresponding to the i^{th} canonical correlation between $\boldsymbol{Y}_{m,t}$ and $\boldsymbol{Y}_{h,t-1-j}$. Let $\boldsymbol{\Gamma}(m,h,j) = E(\boldsymbol{Y}_{h,t-1-j}\boldsymbol{Y}_{m,t}^{'})$. This is a sub-matrix of the Hankel matrix of the autocovariance matrices of $\boldsymbol{y}_t$. Note that zero canonical correlations imply and are implied by $\boldsymbol{\Gamma}(m,h,j)$ having a zero eigenvalue.

In what follows, we provide a brief description of the complete VARMA methodology based on scalar components. For further details, refer to Athanasopoulos and Vahid (2008a) and Tiao and Tsay (1989).

Stage I: Identifying the scalar components

First, by strategically choosing $\boldsymbol{Y}_{m,t}$ and $\boldsymbol{Y}_{h,t-1-j}$, we identify the overall tentative order of the VARMA(p,q) by searching for $s+K$ components of order SCM(p,q), given that we have found s SCM$(p-\kappa, q-\mu)$ for $\{\kappa,\mu\} = \{0,1\}$ or $\{1,0\}$ or $\{1,1\}$. The process of exploring the various possibilities of underlying simplifying structures in the form of SCMs is a hierarchical one. Hence, the identification process begins by searching for K SCMs of the most parsimonious possibility, i.e. SCM$(0,0)$ (which is a white noise process), by testing for the rank of $\boldsymbol{\Gamma}(0,0,0) = E(\boldsymbol{Y}_{0,t-1}\boldsymbol{Y}_{0,t}^{'})$, where $\boldsymbol{Y}_{m,t} = \boldsymbol{Y}_{0,t}$ and $\boldsymbol{Y}_{h,t-1-j} = \boldsymbol{Y}_{0,t-1}$. If we do not find K linearly independent white noise scalar processes, we set $m = h$, and by incrementing m and j we search for the next set of K linearly independent scalar components. First, we search for first order "moving average" components by testing for the rank of $\boldsymbol{\Gamma}(0,0,1) = E(\boldsymbol{Y}_{0,t-2}\boldsymbol{Y}_{0,t}^{'})$, and then we search for the first order "autoregressive" components by testing for the rank of $\boldsymbol{\Gamma}(1,1,0) = E(\boldsymbol{Y}_{1,t-1}\boldsymbol{Y}_{1,t}^{'})$, and then $\boldsymbol{\Gamma}(1,1,1) = E(\boldsymbol{Y}_{1,t-2}\boldsymbol{Y}_{1,t}^{'})$ for SCM$(1,1)$, and so on.

Conditional on the overall tentative order (p,q), we then repeat the search process, but this time search for individual components. So, starting again from the most parsimonious SCM(0,0), we sequentially search for K linearly independent vectors $(\boldsymbol{\alpha}_1,\ldots,\boldsymbol{\alpha}_K)$ for $m = 0,\ldots,p$, $j = 0,\ldots,q$ and $h = m + (q-j)$. As for a tentative order of (p,q), each series is serially uncorrelated after lag q.

The test results from first identifying the overall tentative order and then the individual SCMs are tabulated in what are referred to as *Criterion* and *Root* tables. Reading from the Criterion table allows us to identify the overall tentative order of the model, while reading from the Root table allows us to identify the individual orders of the scalar components. Since an $SCM(m, j)$ nests all scalar components of order $(\leq m, \leq j)$, for every one $SCM(p_1 < p, q_1 < q)$ there will be $s = \min\{m-p_1+1, j-q_1+1\}$ zero canonical correlations at position $(m \geq p_1, j \geq q_1)$. Therefore, for every increment above s, a new $SCM(m, j)$ is found. We demonstrate the reading of these tables in Section 5. For a complete exposition of how to read from these tables and recognize the patterns of zeros, as well as for further details on the sequence of testing, see Athanasopoulos and Vahid (2008a).

Suppose that we have identified K linearly independent scalar components characterized by the transformation matrix $\boldsymbol{A}_0 = (\boldsymbol{\alpha}_1, \ldots, \boldsymbol{\alpha}_K)'$. If we rotate the system in (1) by $\boldsymbol{A}_0$, we obtain

$$\boldsymbol{A}_0\boldsymbol{y}_t = \boldsymbol{A}_1\boldsymbol{y}_{t-1} + \ldots + \boldsymbol{A}_p\boldsymbol{y}_{t-p} + \boldsymbol{\eta}_t - \boldsymbol{M}_1\boldsymbol{\eta}_{t-1} - \ldots - \boldsymbol{M}_q\boldsymbol{\eta}_{t-q}, \tag{4}$$

where $\boldsymbol{A}_i = \boldsymbol{A}_0\boldsymbol{\Phi}_i$, $\boldsymbol{\eta}_t = \boldsymbol{A}_0\boldsymbol{v}_t$ and $\boldsymbol{M}_i = \boldsymbol{A}_0\boldsymbol{\Theta}_i\boldsymbol{A}_0^{-1}$ for $\boldsymbol{i} > \boldsymbol{0}$. This rotated model incorporates whole rows of zero restrictions in the AR and MA parameter matrices on the RHS, as each row represents one identified SCM(p_i, q_i). However, we should note that obtaining the orders of SCMs does not necessarily lead to a uniquely identified system. For example, if two scalar components were identified such that $z_{r,t} = SCM\,(p_r, q_r)$ and $z_{s,t} = SCM\,(p_s, q_s)$, where $p_r > p_s$ and $q_r > q_s$, the system will not be identified. To obtain an identified system, we need to set $\min\{p_r - p_s, q_r - q_s\}$, i.e. set either the autoregressive or moving average parameters to be zero. This process is known as the "general rule of elimination", and in order to identify a canonical VARMA model as defined by Athanasopoulos and Vahid (2008a), we set the moving average parameters to zero.

Stage II: Imposing identification restrictions on matrix $\boldsymbol{A}_0$

Athanasopoulos and Vahid (2008a) recognized that some of the parameters in $\boldsymbol{A}_0$ are redundant and can be eliminated. This stage mainly outlines this process, and a brief description of the rules of placing restrictions on the redundant parameters is as follows:

1. Given that each row of the transformation matrix $\boldsymbol{A}_0$ can be multiplied by a constant without changing the structure of the model, one parameter in each row can be normalized to one. However, there is a danger of normalizing the wrong parameter, i.e. a zero parameter might be normalized to one. To overcome this problem, we add tests of predictability using subsets of variables. Starting from the SCM with the smallest order (the SCM with minimum $p+q$), exclude one variable, say the K^{th} variable, and test whether a SCM of the same order can be found using the $K-1$ variables alone. If the test is rejected, the coefficient of the K^{th} variable is then normalised to one, and the corresponding coefficients in all other SCMs that nest this one are set to zero. If the test concludes that the SCM can be formed using the first $K-1$ variables only, the coefficient of the K^{th} variable in this SCM is zero, and should not be normalised to one. It is worth noting that if the order of this SCM is uniquely minimal, then this extra zero restriction adds to the restrictions discovered before. Continue testing by leaving out variables $K-1$ and testing whether the SCM could be formed from the first $K-2$ variables only, and so on.

2. Any linear combination of a SCM(p_1, q_1) and a SCM(p_2, q_2) is a SCM$(\max\{p_1, p_2\}, \max\{q_1, q_2\})$. The row of matrix $\boldsymbol{A}_0$ corresponding to the SCM(p_1, q_1) is not identified if there are two embedded scalar components with weakly nested orders, i.e., $p_1 \geq p_2$ and $q_1 \geq q_2$. In this case arbitrary multiples of SCM(p_2, q_2) can be added to the SCM(p_1, q_1) without changing the structure. To achieve identification, if the parameter in the i^{th} column of the row of $\boldsymbol{A}_0$ corresponding to the SCM(p_2, q_2) is normalized to one, the parameter in the same position in the row of $\boldsymbol{A}_0$ corresponding to SCM(p_1, q_1) should be restricted to zero. A detailed explanation of this issue, together with an example, can be found in Athanasopoulos and Vahid (2008a).

Stage III: Estimating the uniquely identified system

Finally, in the third stage, the identified model is estimated using FIML. As in Hannan and Rissanen (1982), a long VAR was used to obtain the initial values of the parameters.

3 Variables and their time series properties

As was mentioned in the introduction, in this study we use a similar set of seven variables as Kim and Roubini (2000) for modelling Canadian monetary policy; these variables are listed in Table 1. The variables OPI and R_U represent the foreign block. The OPI is included to account for inflation expectations, mainly to capture the non-policy-induced changes in inflationary pressure to which the central bank may react when setting monetary policy. Hence, it is essential to include OPI in the monetary model to account for forward-looking monetary policy (see Brischetto and Voss, 1999). It is also common in the monetary literature of small open economies to use the US Federal fund rates as a proxy for foreign monetary policy (see, for example, Cushman and Zha, 1997; Kim and Roubini, 2000; Dungey and Pagan, 2000). Since Canada is an open economy and has relatively open capital markets, it is also reasonable to assume that domestic interest rates are related to US interest rates. The remaining five variables are the standard set of variables used in the monetary literature to represent open economy monetary business cycle models (see, for example, Sims, 1992). YP and INF are taken as the target variables of monetary policy, known as non-policy variables, while $M1$ and R_C represent money market and policy variables respectively and ER is the information market variable.

The data are collected from the International Financial Statistics (IFS), covering January 1974 to December 2007, excluding the period of the global financial crisis. The variables are seasonally adjusted and in logarithms, except for inflation and interest rates which are expressed in percentages. The results of unit root tests - Augmented Dickey Fuller and Philips-Perron - of all variables over the whole sample show that the variables are I(1) and I(0) in first-differences. In addition, Johansen's co-integration test also pro-

Table 1: Variables included in the Canadian Monetary Policy Models

Variable	Description	Abbrevation
Foreign		
Oil Price	World Oil Price Index, logs	OPI
US Interest Rate	Federal Funds Rate, Per cent	R_U
Canada		
Output	Industrial Production (SA), Logs	YP
Inflation Rate	Consumer Price Index (% change per annum)	INF
Money	Monetary Aggregate M1 (SA), Logs	$M1$
Interest Rate	Overnight Rate Target, Per cent	R_C
Exchange Rate	Exchange Rate (USA/CAN), Logs	ER

Sources: International Financial Statistics

vides evidence of long run relationships among the variables. Given that the variables are non-stationary and cointegrated, VAR or VARMA models with variables in first differences will lead to loss of information in the long run relationships.[8] Since the objective of this study is to assess the interrelationships among the variables and to correctly identify the effects of monetary shocks, all variables are detrended with the exception of the inflation rate, which is expressed as a percentage change per annum in monthly CPI.[9]

4 Impulse response functions

Impulse response functions are estimated to assess the persistence and dynamic effects of various macroeconomic shocks on policy and non-policy related variables. It is also apparent that economically interpretable shocks are obtained to assess these responses and these issues are briefly discussed below.

The effects of monetary policy shocks are analysed from impulse response functions which are derived from pure moving average representations of models. For a VARMA(p, q)

[8]The choice between a VAR (unrestricted VAR) and a VECM (restricted VAR) depends on the economic interpretation of the impulse response functions from the two specifications (see Ramaswamy and Sloke, 1997, for details). The impulse response functions generated from VECM models tend to imply that the impact of monetary shocks is permanent, while the unrestricted VAR/VARMA allows the data series to decide whether the effects of the monetary shocks are permanent or temporary. It is also common in the monetary literature to estimate the unrestricted VAR model (see, for example, Sims, 1992; Cushman and Zha, 1997; Bernanke and Mihov, 1998; Kim and Roubini, 2000).

[9]We are thankful to Professor Adrian Pagan for suggesting that in order to improve the modelling framework, the variables in levels are replaced with de-trended variables.

process

$$\mathbf{A}(L)\mathbf{y}_t = \mathbf{M}(L)\boldsymbol{\upsilon}_t \tag{5}$$

the impulse responses can be obtained from

$$\mathbf{y}_t = \boldsymbol{\Xi}(L)\boldsymbol{\upsilon}_t = \boldsymbol{\upsilon}_t + \sum_{i=1}^{\infty} \boldsymbol{\Xi}_i \boldsymbol{\upsilon}_{t-i}, \tag{6}$$

where $\boldsymbol{\Xi}_i = \mathbf{M}_i + \sum_{j=1}^{i} \mathbf{A}_j \boldsymbol{\Xi}_{i-j}$, $\boldsymbol{\Xi}_0 = \mathbf{I}_k$ and $\boldsymbol{\upsilon}_t$ is a white noise process with $E(\boldsymbol{\upsilon}_t) = 0$ and $E(\boldsymbol{\upsilon}_t \boldsymbol{\upsilon}_t') = \boldsymbol{\Sigma}_{\boldsymbol{\upsilon}}$. Similarly, we obtain the impulse responses from orthogonal shocks for a reduced form VAR(p) model

$$\boldsymbol{\Phi}(L)\mathbf{y}_t = \mathbf{e}_t \tag{7}$$

with a pure VMA representation $\mathbf{y}_t = \boldsymbol{\Phi}^*(L)\mathbf{e}_t = \mathbf{e}_t + \sum_{i=1}^{\infty} \boldsymbol{\Phi}_i^* \mathbf{e}_{t-i}$ where $\boldsymbol{\Phi}_i^* = \sum_{j=1}^{i} \boldsymbol{\Phi}_j \boldsymbol{\Phi}_{i-j}^*$ and $\boldsymbol{\Phi}_0^* = \mathbf{I}_K$.

In order to directly attribute the responses of variables to economically interpretable shocks, we need to transform the exogenous shocks in equation (6) to a new set of orthogonal shocks. A traditional and convenient method is to use the Choleski decomposition, as first applied by Sims (1980). A major criticism of the Choleski decomposition approach is that the assumed Wold ordering of the variables is considered atheoretical. In contrast, SVARMA and SVAR models use economic theory to identify the contemporaneous relationships between variables (see, for example, Bernanke, 1986; Sims, 1986; Blanchard and Watson, 1986). The relationship between the reduced form VARMA disturbances ($\boldsymbol{\upsilon}_t$) and the orthogonal shocks $\mathbf{v}_t$ is

$$\mathbf{B}_0 \boldsymbol{\upsilon}_t = \mathbf{v}_t, \tag{8}$$

where $\mathbf{B}_0$ is an invertible square matrix, $E(\mathbf{v}_t) = 0$, $E(\mathbf{v}_t \mathbf{v}_t') = \boldsymbol{\Sigma}_{\mathbf{v}}$ and $\boldsymbol{\Sigma}_{\mathbf{v}}$ is a diagonal matrix. $\mathbf{B}_0$ is normalized across the main diagonal, so that each equation in the system has a designated dependent variable. The innovations of the structural model are related to the reduced form innovations by $\boldsymbol{\Sigma}_{\boldsymbol{\upsilon}} = \mathbf{B}_0^{-1} \boldsymbol{\Sigma}_{\mathbf{v}} (\mathbf{B}_0^{-1})'$. The impulse responses from the SVARMA are obtained from

$$\mathbf{y}_t = \mathbf{B}_0^{-1} \mathbf{v}_t + \sum_{i=1}^{\infty} \boldsymbol{\Xi}_i \mathbf{B}_0^{-1} \mathbf{v}_{t-i}. \tag{9}$$

while the impulse responses from the SVAR are obtained as follows

$$\mathbf{y}_t = \mathbf{B}_0^{-1}\boldsymbol{\epsilon}_t + \sum_{i=1}^{\infty} \boldsymbol{\Phi}_i^* \mathbf{B}_0^{-1}\boldsymbol{\epsilon}_{t-i}, \tag{10}$$

where $\boldsymbol{\epsilon}_t = \mathbf{B}_0\mathbf{e}_t$. For both the SVARMA and SVAR models, a recursive identification structure on the contemporaneous matrix $\mathbf{B}_0$ is imposed and the structure is as follows

$$\mathbf{B}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ b_{21}^0 & 1 & 0 & 0 & 0 & 0 & 0 \\ b_{31}^0 & 0 & 1 & 0 & 0 & 0 & 0 \\ b_{41}^0 & 0 & b_{43}^0 & 1 & 0 & 0 & 0 \\ 0 & 0 & b_{53}^0 & b_{54}^0 & 1 & 0 & 0 \\ b_{61}^0 & b_{62}^0 & b_{63}^0 & b_{64}^0 & 0 & 1 & 0 \\ b_{71}^0 & b_{72}^0 & b_{73}^0 & b_{74}^0 & b_{75}^0 & b_{76}^0 & 1 \end{bmatrix} \tag{11}$$

with the variables ordered as in Table 1. The above contemporaneous structure is used to estimate the orthogonal shocks for Canada where the sizes of these shocks actually represent the one standard deviation of the corresponding orthogonal errors obtained from the SVARMA and SVAR models.

The two foreign variables are identified recursively, with the assumption that the OPI is contemporaneously exogenous to all other variables in the model, while the R_U is assumed to be contemporaneously affected by OPI. YP is influenced contemporaneously only by OPI, while INF is affected by both OPI and YP. The $M1$ equation which represents the demand for real money balances is contemporaneously dependent on YP and INF. The domestic monetary policy equation is assumed to be the reaction function of the Bank of Canada which sets the interest rate after observing the current OPI, R_U, YP and INF, reflecting an open economy Taylor rule. Finally, ER is seen as an information market variable that reacts quickly to all relevant economic disturbances and hence is contemporaneously affected by all the variables in the SVARMA and SVAR systems. With an exception of the monetary policy equation, the rest of the restrictions are similar to Kim and Roubini (2000). The identification restrictions on the monetary policy equation differs from that imposed by Kim and Roubini (2000) who assumes that central banks react immediately to OPI, $M1$ and ER but does not react immediately to the R_U, YP

and INF. The justification of Kim and Roubini (2000) for this assumption is that within a month a central bank is more concerned about the impact of global and foreign exchange rate shocks on the economy than the impact of target variables and foreign monetary shocks.[10] However, in our extended data set we found such restrictions did not provide plausible results where a contractionary monetary policy causes an immediate fall rather then a rise in interest rate. On the other hand, we found the open economy Taylor rule restrictions appear to provide theoretically consistent results which are discussed in detail in Section 5.3.[11]

Apart from the restrictions imposed on the contemporaneous structure, no restrictions are imposed on the lag structures of the SVAR model. On the other hand, due to the identification issues discussed in Section 2, further restrictions are imposed on the SVARMA model in order to identify a unique VARMA process.

5 Empirical results

In this section, we apply the VARMA methodology outlined in Section 2 to the Canadian monetary model of seven variables. The impulse responses generated from the identified SVARMA and SVAR models are then used to assess the effects of various monetary shocks.[12]

5.1 Identifying SVARMA and SVAR models

In Stage 1, we identify the overall order of the VARMA process and the orders of embedded SCMs in the data for Canada. In Panel A of Table 2 we report the results of all the canonical correlations test statistics divided by their χ^2 critical values and this table is known as the "Criterion Table". If the entry in the $(m, j)^{th}$ cell is less than one, this shows that there are seven SCMs of order (m, j) or lower in this system.

[10] Bhuiyan (2012) also imposed similar restrictions as Kim and Roubini (2000) but with an additional assumption that the Bank of Canada also responds immediately to R_U.

[11] We thank an anonymous referee for making this point clear.

[12] All computations are carried out using Gauss code, which is available from the authors upon request.

From Panel A in Table 2 we infer that the overall order of the system is VARMA$(2, 1)$. Conditional on this overall order, the canonical correlation tests are employed to identify the individual orders of embedded SCMs. The number of insignificant canonical correlations identified are tabulated in Panel B of Table 2 which is referred to as the "Root Table". The root table shows the test results from identifying the individual orders of the SCMs conditional on the overall order being VARMA(2,1). Since Bank of Canada implemented inflation targeting from 1991 onwards, as robustness check we carried out sub-sample analysis for the period January 1991 to December 2007 and the identified model is also VARMA$(2, 1)$.

We first identify two SCM(1,0)s and one SCM(0,1). As it is possible for an SCM(0,1) to be observationally equivalent to an SCM(1,0), which leads to an identification problem (see Tiao and Tsay, 1989, page 161), we proceed with only the autoregressive components. Next, we find five SCM(2,0)s and five SCM(1,1)s. However, every SCM(m, j) nests all scalar components of order $(\leq m, \leq j)$. For each individual SCM$(p_1 < p, q_1 < q)$, there will be $\xi = \min\{m - p_1 + 1, j - q_1 + 1\}$ zero canonical correlations at position $(m \geq p_1, j \geq q_1)$. Therefore, a new SCM(m, j) is found for every increment above ξ, and hence only three of the five SCMs found are new. We proceed with three SCM(1,1)s. Finally, we also find 2 new SCMs of order (2,1). The identified VARMA$(2, 1)$ consists of two SCM$(1, 0)$s, three SCM$(1, 1)$s and two SCM$(2, 1)$s.

Table 2: Stage I of the identification process of a VARMA model for the Canadian Monetary System

PANEL A: Criterion Table

m \ j	0	1	2	3	4
0	137.58[a]	15.42	8.19	5, 45	4.07
1	3.55	1.26	1.22	1.21	0.93
2	1.26	**0.97**	1.08	1.08	0.89
3	1.26	1.08	0.98	0.95	0.92
4	1.19	1.07	0.97	1.02	0.91

PANEL B: Root Table

m \ j	0	1	2	3	4
0	0	0	0	0	0
1	**2**	**5**	6	6	7
2	**6**	**10**	12	12	13
3	6	12	17	19	19
4	6	12	19	23	26

[a] *The statistics are normalized by the corresponding 5% χ^2 critical values*

Implementing Stage II of the identification process described in Section 2 has led to additional zero restrictions on the matrix containing the contemporaneous relationships between the variables, and the canonical SCM representation of the identified VARMA$(2,1)$ of the Canadian monetary model is given above, where $\boldsymbol{y}_t = (OPI_t, R_{U,t}, YP_t, INF_t, M1_t, R_{C,t}, ER_t)'$. Among the variables, INF_t and $M1_t$ are found to be loading as SCM$(1,0)$, while OPI_t, $R_{U,t}$ and $R_{C,t}$ were loading as SCM$(1,1)$ and YP_t and ER_t were loading on as SCM$(2,1)$. We also ensured that the individual tests described in Section 2 do not contradict the normalization of the diagonal parameters of the contemporaneous matrix to one.

$$
\begin{aligned}
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & \alpha_{23} & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & \alpha_{56} & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix} \boldsymbol{y}_t = \boldsymbol{c} + &
\begin{bmatrix}
\psi^{(1)}_{11} & \psi^{(1)}_{12} & \psi^{(1)}_{13} & \psi^{(1)}_{14} & \psi^{(1)}_{15} & \psi^{(1)}_{16} & \psi^{(1)}_{17} \\
\psi^{(1)}_{21} & \psi^{(1)}_{22} & \psi^{(1)}_{23} & \psi^{(1)}_{24} & \psi^{(1)}_{25} & \psi^{(1)}_{26} & \psi^{(1)}_{27} \\
\psi^{(1)}_{31} & \psi^{(1)}_{32} & \psi^{(1)}_{33} & \psi^{(1)}_{34} & \psi^{(1)}_{35} & \psi^{(1)}_{36} & \psi^{(1)}_{37} \\
\psi^{(1)}_{41} & \psi^{(1)}_{42} & \psi^{(1)}_{43} & \psi^{(1)}_{44} & \psi^{(1)}_{45} & \psi^{(1)}_{46} & \psi^{(1)}_{47} \\
\psi^{(1)}_{51} & \psi^{(1)}_{52} & \psi^{(1)}_{53} & \psi^{(1)}_{54} & \psi^{(1)}_{55} & \psi^{(1)}_{56} & \psi^{(1)}_{57} \\
\psi^{(1)}_{61} & \psi^{(1)}_{62} & \psi^{(1)}_{63} & \psi^{(1)}_{64} & \psi^{(1)}_{65} & \psi^{(1)}_{66} & \psi^{(1)}_{67} \\
\psi^{(1)}_{71} & \psi^{(1)}_{72} & \psi^{(1)}_{73} & \psi^{(1)}_{74} & \psi^{(1)}_{75} & \psi^{(1)}_{76} & \psi^{(1)}_{77}
\end{bmatrix} \boldsymbol{y}_{t-1} \\
+ & \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\psi^{(2)}_{31} & \psi^{(2)}_{32} & \psi^{(2)}_{33} & \psi^{(2)}_{34} & \psi^{(2)}_{35} & \psi^{(2)}_{36} & \psi^{(2)}_{37} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\psi^{(2)}_{71} & \psi^{(2)}_{72} & \psi^{(2)}_{73} & \psi^{(2)}_{74} & \psi^{(2)}_{75} & \psi^{(2)}_{76} & \psi^{(2)}_{77}
\end{bmatrix} \boldsymbol{y}_{t-2} \\
+\boldsymbol{u}_t - & \begin{bmatrix}
\mu^{(1)}_{11} & \mu^{(1)}_{12} & \mu^{(1)}_{13} & \mu^{(1)}_{14} & \mu^{(1)}_{15} & \mu^{(1)}_{16} & \mu^{(1)}_{17} \\
\mu^{(1)}_{21} & \mu^{(1)}_{22} & \mu^{(1)}_{23} & \mu^{(1)}_{24} & \mu^{(1)}_{25} & \mu^{(1)}_{26} & \mu^{(1)}_{27} \\
\mu^{(1)}_{31} & \mu^{(1)}_{32} & \mu^{(1)}_{33} & 0 & 0 & \mu^{(1)}_{36} & \mu^{(1)}_{37} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mu^{(1)}_{61} & \mu^{(1)}_{62} & \mu^{(1)}_{63} & \mu^{(1)}_{64} & \mu^{(1)}_{65} & \mu^{(1)}_{66} & \mu^{(1)}_{67} \\
\mu^{(1)}_{71} & \mu^{(1)}_{72} & \mu^{(1)}_{73} & 0 & 0 & \mu^{(1)}_{76} & \mu^{(1)}_{77}
\end{bmatrix} \boldsymbol{u}_{t-1}.
\end{aligned}
$$

For the SVAR model, we found that twelve lags are necessary to capture all of the dynamics in the data. The lag length specification tests suggest that either one (Schwartz Bayesian Information Criterion, Hannan Quinn information criterion) or between three to four (Akaike Information Criteria, Likelihood Ratio test) lags should be included. Including few lags may not be sufficient to capture the lag dynamics. The Ljung-Box and LM tests for serial autocorrelation in the residuals show that at least twelve lags are required to capture the dynamics in the data.[13]

To obtain the orthogonal shocks, we use the recursive identification structure described in Section 4. Five additional restrictions were imposed on Canada and the over-identifying restrictions were not rejected at the 1% significance level, thus suggesting that the identified model specifications are appropriate.[14]

5.2 An out-of-sample forecast evaluation of VARMA versus VAR Models

Before proceeding with the structural analysis we evaluate the out-of-sample forecasting performance of the identified VARMA(2,1) model. As our main focus in this paper is monetary policy analysis and not forecasting, the VAR(12) is selected for capturing all the dynamics in the data based on statistical inference (as we have discussed in the previous section). In order to perform a robust out-of-sample forecast evaluation of the VARMA(2,1) we also include as alternatives VAR(4) and VAR(1) models. These are the models selected respectively by the AIC and the SBIC. In Table 3 we report the model selection criteria for all the models. The results show that if we were to choose a model based purely on model selection criteria both the AIC and the SBIC would select the VARMA model over any of the VAR alternatives.

We split our data in an in-sample period with 288 observations, covering January 1974 to December 1997, and an out-of-sample period with 120 observations, covering January 1998 to December 2007. We re-estimate all models in the in-sample period and forecast 1 to 12-steps-ahead. We then role all models forward (without re-estimating) and generate

[13] This result is not surprising as Kim and Roubini (2000) included six lags with twelve seasonal dummies while Bhuiyan (2012) included eight lags and Dufour and Pelletier (2008) included twelve lags to their respective SVAR models.

[14] The contemporaneous matrix $\mathbf{B}_0$ requires $((7^2 - 7)/2 = 21)$ restrictions for exact identification while in (9) there are 26 restrictions imposed, leading to over-identification.

Table 3: Model selection criteria for the estimated VARMA and VAR models.

	AIC	BIC
VARMA(2,1)	-27.86	-26.88
VAR(1)	-24.88	-24.39
VAR(4)	-25.14	-23.21
VAR(12)	-24.90	-19.13

1 to 12-steps-ahead forecasts until the end of the out-of-sample period. This generates 120 1-step-ahead forecasts, 119 2-steps ahead forecasts up to 109 12-steps-ahead forecasts, which are used for forecast evaluation.

In Table (4) we present the percentage gains (losses for negative entries) in RMSFE (Root Mean Squared Forecast Error) from forecasting with the VARMA(2,1) model compared to the alternative VARs. We present the results for the two key economic indicators for Canada YP and INF, as well as for all the seven variables together. The results show that the VARMA(2,1) model forecasts the macroeconomic variables considered in the multivariate system more accurately than the VAR counterparts. This is reflected by the large gains in RMSFE from the VARMA model when considering all variables as shown in the last row of each panel. The times that one of the VAR alternatives was more accurate were very few. In these cases, as is the one with the INF variable forecasts generated by the VAR(4), the loss from using the VARMA(2,1) instead of the VAR alternative were very small in comparison to the gains. These findings are consistent with those in previous studies such as Athanasopoulos and Vahid (2008b) and Dufour and Pelletier (2008) which also evaluate the forecasting accuracy of VARMA models versus VARs.

5.3 Responses of policy and non-policy variables to various shocks

The dynamic impulse response functions of domestic variables to various independent shocks are generated from the SVARMA and SVAR models and are revealed in Figures 1 to 6. The behavior of these responses over a period of 48 months is analyzed and discussed in this section. The sizes of the shocks are measured by one-standard deviation of the

Table 4: Out-of-sample percentage gains in RMSFE from forecasting h-steps-ahead with a VARMA instead of the VAR alternatives. Negative entries correspond to a percentage loss.

	h 1	2	4	6	8	12	Average
			VARMA(2,1) v VAR(1)				
YP	9.26	189.21	85.47	45.60	26.47	12.78	53.73
INF	4.17	217.35	144.74	100.36	70.96	37.58	93.12
All variables	8.65	1275.41	454.23	231.38	139.40	69.83	306.71
			VARMA(2,1) v VAR(4)				
YP	24.99	25.87	37.29	39.09	39.62	39.02	36.08
INF	6.26	2.80	-0.74	-2.42	-2.60	-4.81	-1.85
All variables	26.85	24.77	26.03	24.54	24.50	26.93	25.38
			VARMA(2,1) v VAR(12)				
YP	95.57	99.86	103.92	96.39	94.07	92.75	97.01
INF	18.95	15.41	20.05	22.32	14.56	-4.42	13.12
All variables	132.99	144.86	143.51	125.40	107.18	104.69	122.06

orthogonal errors of the respective models and are presented in Table 5 below. The sizes of the orthogonal shocks in the SVARMA and SVAR models appear to be somewhat similar. 68% confidence bands for the impulse functions are computed via bootstrapping 10000 samples, using the bootstrap-after-bootstrap method of Kilian (1998).

Table 5: Magnitude of One Standard Deviation Shocks from the SVAR and SVARMA Models

Model	OPI	R_U	YP	INF	$M1$	R_C	ER
SVAR	0.164	0.183	0.173	0.115	0.162	0.155	0.103
SVARMA	0.155	0.229	0.182	0.124	0.203	0.182	0.082

5.3.1 Impulse responses to an oil price shock

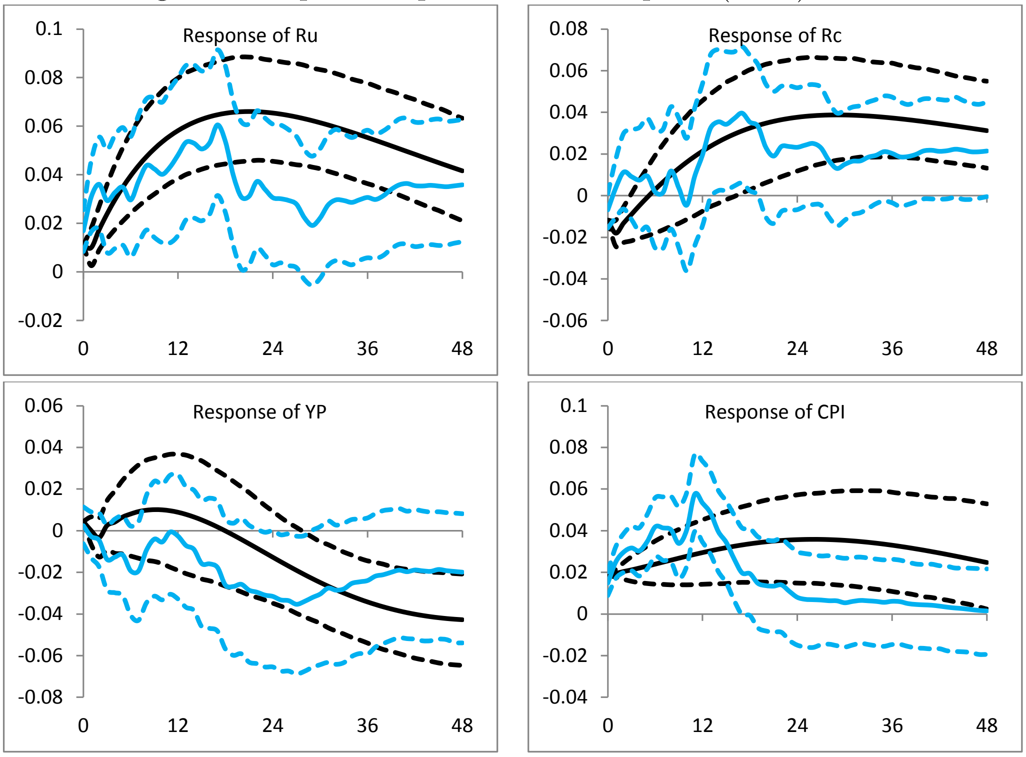
The responses of the Canadian variables to an oil price shock are shown in Figure 1. A positive OPI shock is expected to induce inflationary pressure on the economy. As expected, both the US and Canadian monetary policies responded to higher oil prices, resulting in increasing policy rates, R_U and R_C. Both of these rates continue to rise for about 18 months and peaked at around 6% and 3.5%, respectively. Since OPI is based on the world market price, the rise in oil price leads to an inflationary pressure in the economy. To combat the rise in INF and the demand for money, the Bank of Canada has responded by increasing the R_C.

Considering that Canada is a net oil exporter, although not significant a positive OPI shock led to a positive movement in YP within the first year, followed by a negative response, which becomes significant after two years. This outcome is not surprising as Canada, unlike many oil producers, does not heavily subsidize fuel. As observed in Figure 1, the rise in R_C is higher than the rise in INF, leading to a positive rise in real interest rate. Consequently, these changes could lead to an immediate appreciation of the Canadian dollar, ER. Both a rise in R_C and an appreciation of ER reduce the size of the positive impact of an oil shock on Canadian output. After a year, output falls as a result of a fall in consumption, investment and non-oil exports. It is evident that these expected movements of variables, R_U, R_C, YP and INF, to a positive oil price shock are well captured by impulse responses generated by the SVARMA model.

5.3.2 Impulse responses to a US monetary policy shock

A positive R_U shock, which is defined as an unanticipated monetary contraction, leads to a rise in the US interest rate, resulting in excess demand for US currency in the foreign exchange market. The US dollar appreciates, while the Canadian dollar depreciates. To lean against the exchange rate depreciation, the Bank of Canada increases its policy rate R_C and consequently $M1$ and YP fall. As observed in Figure 2, the rise in both R_U and R_C peaked at around three months and then declined and reached a steady state after two years, while $M1$ and YP continue to fall for eighteen months. For about thirty months, the R_U response to an US monetary shock is larger than the R_C responses. This interest rate differential appear to be causing the Canadian exchange to have persistent negative response. Since the Canadian output (YP) is also responding negatively to a positive R_U

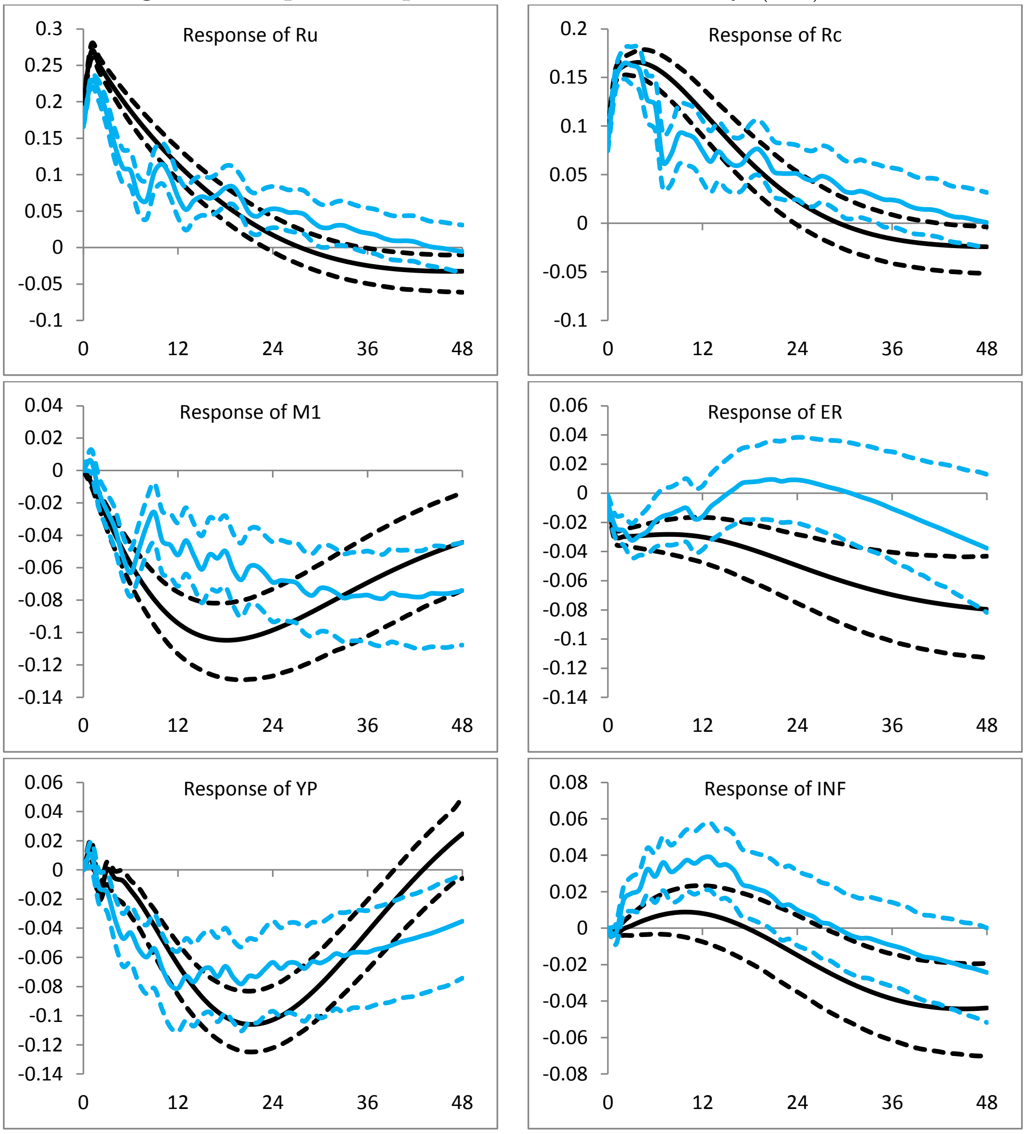
Figure 1: Impulse responses to an oil price (OPI) shock



Notes: SVARMA and SVAR impulse responses are shown as unbroken black and blue lines respectively with 68% confidence bands (obtained from 10000 bootstrap replications) shown as dashed lines.

shock, which could further contribute to the persistent negative effects on the exchange rate. The depreciating Canadian dollar could also explain the immediate rise in INF in response to the US monetary shock and the subsequent fall after a year.

Figure 2: Impulse responses to a US monetary (R_U) shock



Notes: SVARMA and SVAR impulse responses are shown as unbroken black and blue lines respectively with 68% confidence bands (obtained from 10000 bootstrap replications) shown as dashed lines.

5.3.3 Impulse responses to a domestic monetary policy shock

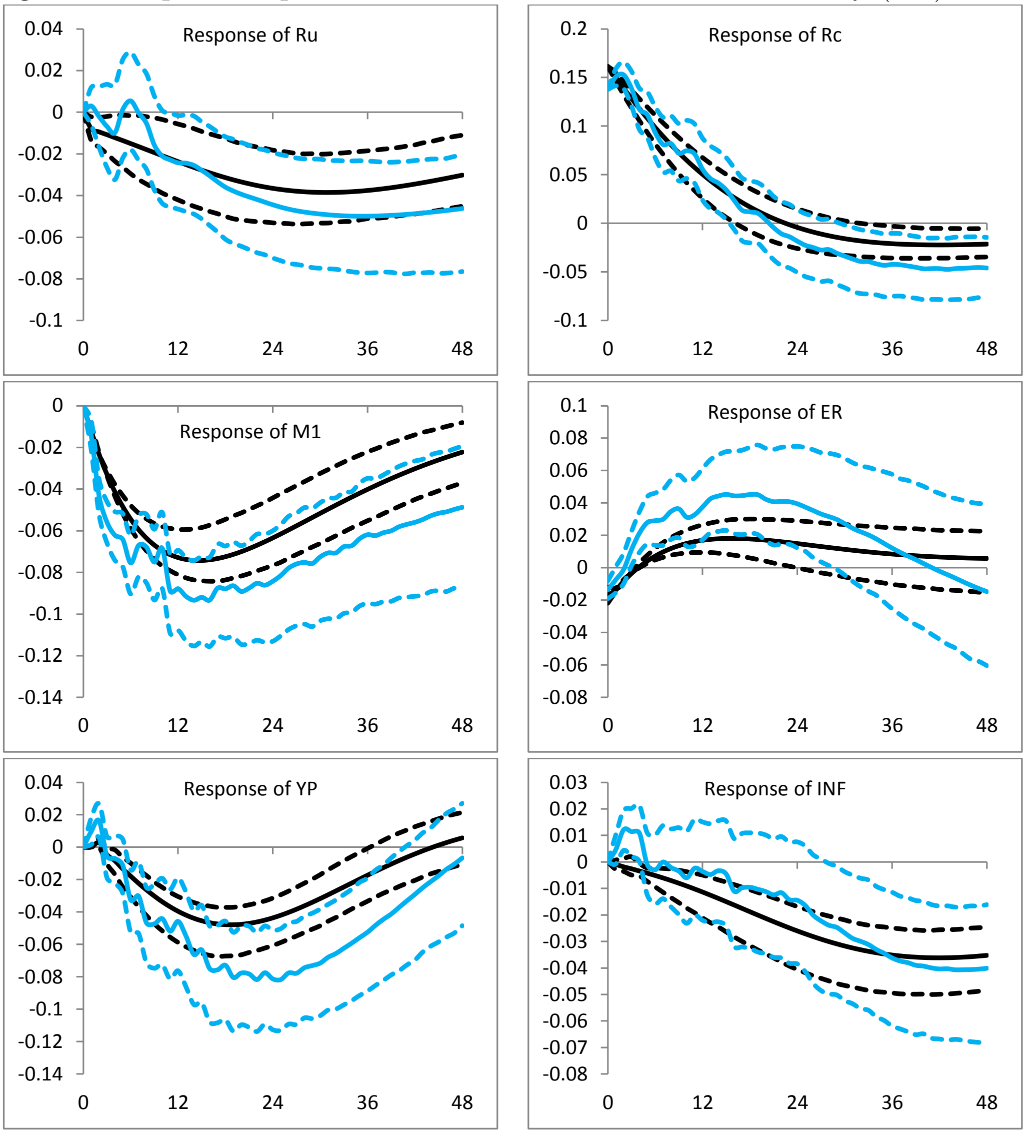
Canada is a small open economy, and hence its monetary policy changes are not expected to affect the US interest rate - a proxy for the world interest rate. However, the impulse responses generated by both SVARMA and SVAR models with 68% confidence bands plotted in Figure 3 indicate significant but negative response of R_U to a positive R_C shock. We believe that the observed negative movement in R_U is not a response to the R_C shock, but maybe a response to the US Federal Reserve's independent monetary policy measures taken around this period.

According to Kim and Roubini (2000), if the identified monetary shock is indeed orthogonal, in the sense that it is not a systematic response to any shock, a tighter monetary policy stance would lead to a rise in R_C and a fall in $M1$, and subsequently these results will be reversed due to persistent deflationary pressures in the economy. It is worth noting that, as observed in Figure 3, the R_C increases on impact following a contractionary monetary shock and then falls, reaching subsequently after two years to a new lower steady state, while $M1$ continues to decline for the first 18 months, followed by a rise thereafter.

An immediate fall in INF response to a contractionary monetary policy shock is only observed for SVARMA response, where it declines smoothly, becoming persistent over the entire 48 months's horizon. As for the SVAR model, a price puzzle exist for the first six months and a significant fall is only observed after thirty months.

A rise in R_C followed by a fall in INF leads to both a rise in real interest rate and an appreciation of the nominal exchange rate. That is, a positive interest differential in favour of Canadian financial assets is associated with a persistent appreciation of the Canadian dollar. This result is consistent with that of Eichenbaum and Evans (1995) and Grilli and Roubini (1996). In response to a positive R_C shock, YP declines, indicating that Canadian money is non-neutral in the short run. As indicated by both the SVAR and SVARMA models, the negative YP responses to an R_C shock may be due to an increase in the real cost of borrowing and the appreciation of the currency. Overall, the absence of price, output and exchange rate puzzles highlights the adequacy of the recursive SVARMA model for identifying an appropriate monetary policy shock and producing impulse responses, which are consistent with economic theoretical model predictions.

Figure 3: Impulse responses of variables to a domestic monetary (R_C) shock

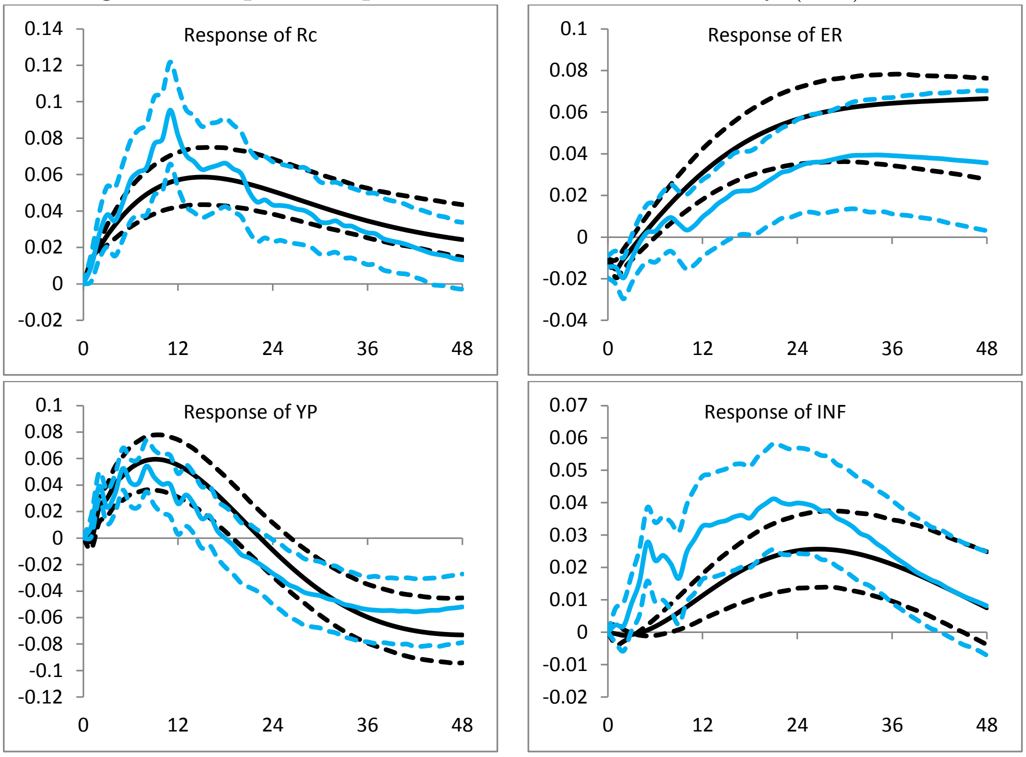


Notes: SVARMA and SVAR impulse responses are shown as unbroken black and blue lines respectively with 68% confidence bands (obtained from 10000 bootstrap replications) shown as dashed lines.

5.3.4 Impulse responses to a money shock

A positive $M1$ shock is expected to trigger an increase in demand for money and this is followed by a rise in YP and INF. The rise in $M1$, YP and INF are followed by a rise in R_C and eventually a rise in ER. As observed in Figure 4, these outcomes are clearly observed via both SVAR and SVARMA models.

Figure 4: Impulse responses of variables to a money ($M1$) shock

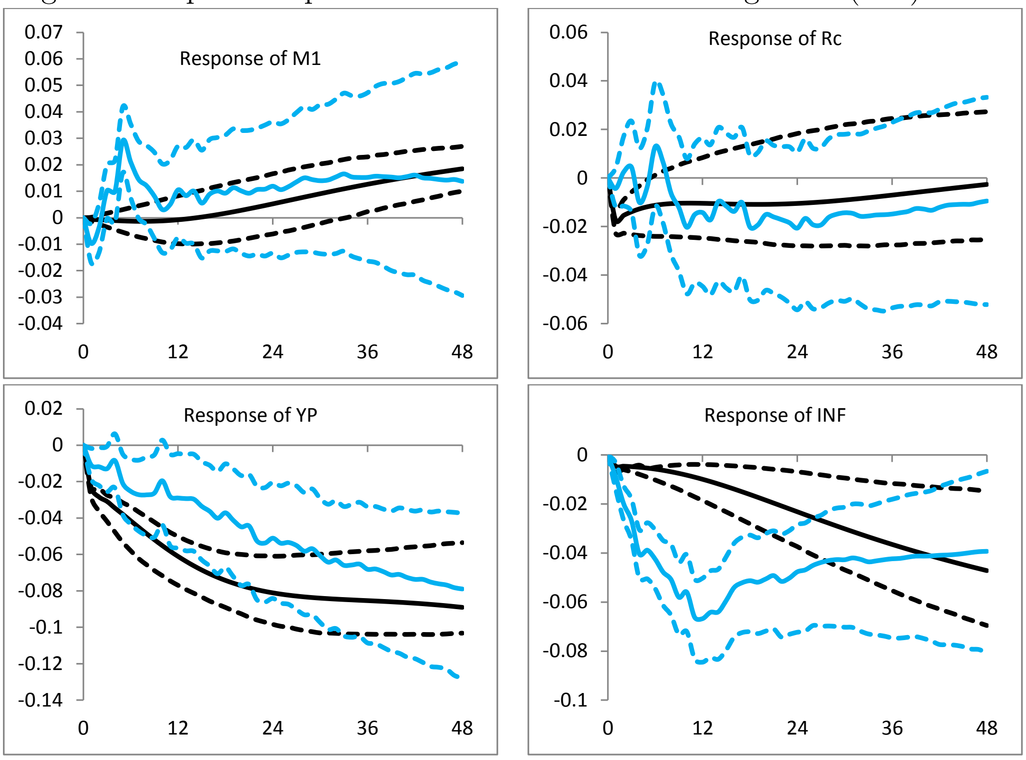


Notes: SVARMA and SVAR impulse responses are shown as unbroken black and blue lines respectively with 68% confidence bands (obtained from 10000 bootstrap replications) shown as dashed lines.

5.3.5 Impulse responses to an exchange rate shock

R_C is expected to decline in response to a positive ER shock, where an unanticipated appreciation of currency should prompt policy makers to lean against currency appreciation. Referring to Figure 5, as anticipated, a fall in the SVARMA R_C responses is immediate, lasting for about 6 months, before returning to a steady state. The currency appreciation is expected to have two opposing effects on YP. On the one hand, it decreases net exports as they become more expensive than the imports. On the other hand, it reduces the cost of production through lower prices of imported intermediate goods. These combined effects transpiring through the demand and supply channels would determine the net influence of an ER shock on YP. It is noted that for both the SVAR and SVARMA, YP responses decrease for the first few years. The reason for these models capturing the contractionary output maybe due to a persistent currency appreciation followed by a transitory decline in interest rate have led to a contractionary effect on output. INF is expected to respond negatively to an ER shock due to lower import prices and production costs. This outcome is observed in both the SVAR and SVARMA responses.

Figure 5: Impulse responses of variables to an exchange rate (ER) shock



Notes: SVARMA and SVAR impulse responses are shown as unbroken black and blue lines respectively with 68% confidence bands (obtained from 10000 bootstrap replications) shown as dashed lines.

5.3.6 Impulse responses to a an output shock and a price shock

The R_C response increases to a positive YP shock. This outcomes is consistent with the contractionary policy measure usually undertaken by central banks against expanding economies. The expansion in YP also induces inflationary pressure in the economy causing an increase an in INF. An unexpected increase in output would cause an immediate increase in the demand for money $M1$ and then followed by a fall due to the rise in the interest rate. Figure 6 shows that all the above expected directions of responses are observed in the SVARMA model. However, a larger than expected increase in YP leads to an increase in R_C, resulting in an ER appreciation. Such movements of responses are clearly evident in both the models.

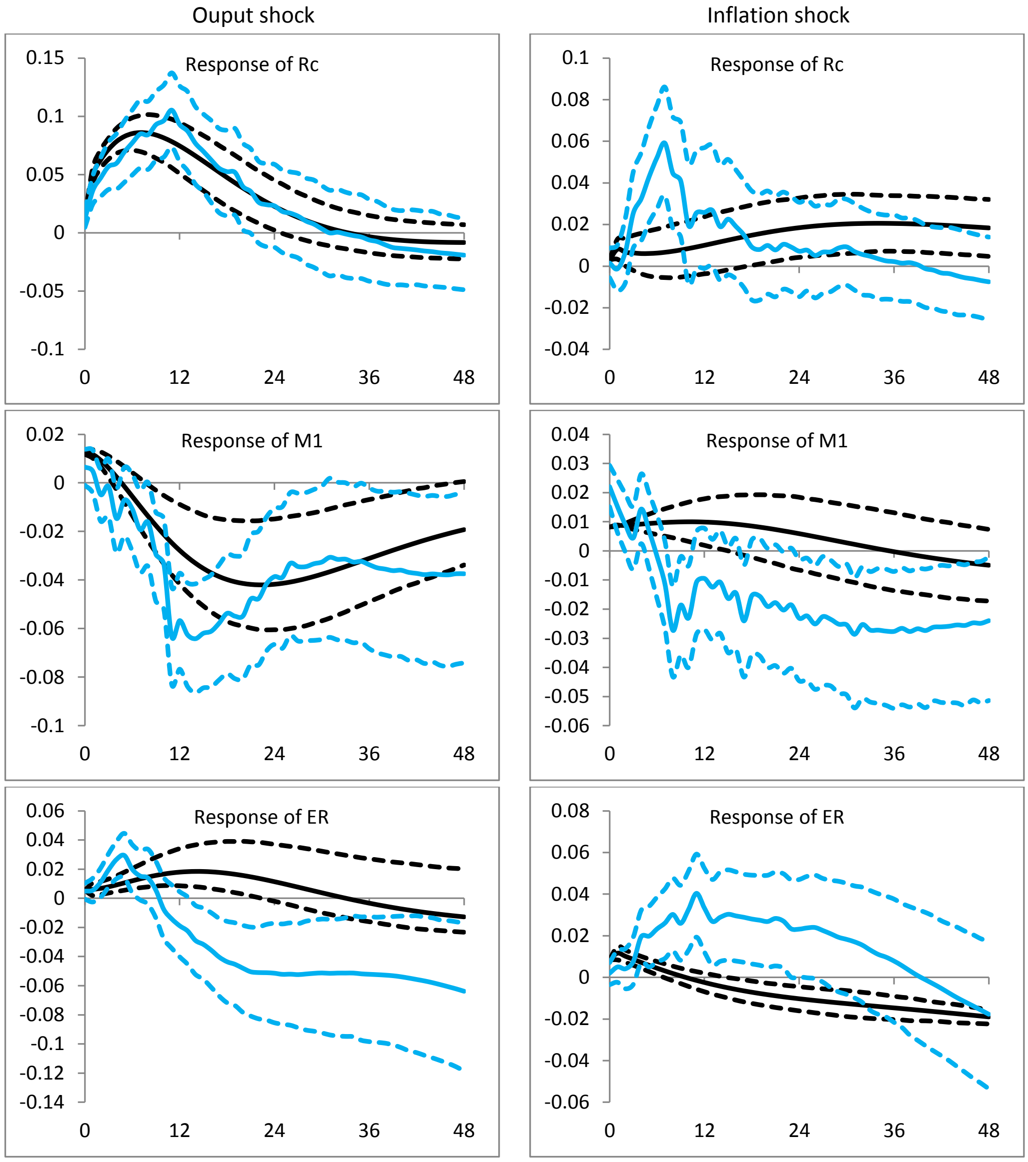
A positive shock to INF can be regarded as unanticipated inflationary pressure on the economy. As a consequence, R_C is expected to rise and demand for money $M1$ is also expected to rise. Figure 6 shows the SVARMA R_C response to a INF shock is gradual and positive but less pronounced, while the corresponding SVAR response increases sharply and then declines gradually. A comparison of both models' responses shows that the SVAR model tends to overstate the dynamic response of monetary policy to an inflationary shock. Although unanticipated inflationary pressure resulting in depreciation of ER is expected, a rise in R_C can offset this negative effect. This outcome is captured neatly only by the SVARMA model, while this response is positive and large in the SVAR model.

6 Conclusion

This paper builds a structural VARMA (SVARMA) model for investigating Canadian monetary policy in two stages. Firstly, using the scalar component model (SCM) developed by Athanasopoulos and Vahid (2008a), this paper identifies a VARMA model. Secondly, imposing a recursive identifying structure on its contemporaneous matrix, it establishes the identification conditions for a SVARMA model for Canadian monetary policy. Although the VARMA model has long been known as the preferred model for monetary policy analysis, the traditional VAR and SVAR models have been widely used, mostly due to the difficulties associated with the identification and estimation of the former. The SVAR model is included in this study for comparison purposes. To our knowledge, this is the first paper to successfully construct a canonical SVARMA model for Canadian monetary policy analysis. All computations are carried out using Gauss code, which is available from the authors upon request.

The results of our investigation are very promising. The impulse response functions

Figure 6: Impulse responses of $M1$, R_C and ER to an output (YP) shock and to an inflation (INF) shock



Notes: SVARMA and SVAR impulse responses are shown as unbroken black and blue lines respectively with 68% confidence bands (obtained from 10000 bootstrap replications) shown as dashed lines.

and the 68% confidence bands generated by the new SVARMA framework appear to have resolved the anomalies commonly found in the empirical monetary literature on small open economies. These anomalies include price, output and exchange rate puzzles. The presence of moving average components combined with an appropriate identification restrictions on the contemporaneous structure of the SVARMA model appear to have resolved these economic anomalies. By contrast, the impulse responses and the 68% confidence bands generated by the SVAR reveal the existence of the price puzzle. Overall, with an exception of a monetary shock, the SVAR and SVARMA models generated qualitatively similar impulse-response functions for other identified shocks. However, the widths of the confidence bands for the impulse responses generated by the SVARMA model are narrower than those generated by the SVAR. As highlighted by Dufour and Pelletier (2008) the MA operator in the SVARMA model allows the reduction of the required AR order so we can get more efficient estimates. This has led to generating both more precise impulse-response functions and more accurate out-of-sample forecasts compared to the VAR. We recommend that the SVARMA methodology be adapted to analyze the monetary policy of other open economies as the potential gain from basing monetary policy on an adequate model is immense, as shown by our investigation.

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