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QUALITY VERSUS QUANTITY IN VERTICALLY DIFFERENTIATED PRODUCTS UNDER NON-LINEAR PRICING *

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ABSTRACT

Quality is defined as being skewed when the marginal rate of substitution (MRS) between quantity and quality differs from the marginal rate of transformation (MRT). This definition is used to assess the balance of quality and quantity in each variety of good produced by a monopolist using non-linear pricing, where each variety can be differentiated using both quantity and quality. A variety's decisive customers face a binding self-selection or participation constraint. Skewing of a variety's quality occurs when there is a difference between its decisive customer(s) MRS and that of (i) its non-decisive customers (ii) the decisive customers of 'adjacent varieties'. Some important special cases are identified and analysed.

JEL Classification: L11

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* This paper utilises and extends some of the analysis of Sibly (2006). I would like to that Jong-Hee Hahn and Ann Marsden for comments on an earlier draft. All errors remain my responsibility.

QUALITY VERSUS QUANTITY IN VERTICALLY DIFFERENTIATED PRODUCTS UNDER NON-LINEAR PRICING

One often hears people make statements such as "I am a person who prefers quality over quantity". The implications of such comments are, of course, that such people recognize there is a trade-off between these attributes. Unlike this popular saying, people do not have a preference for *either* quantity *or* quality but, rather, have a rate of substitution between the two. Such preferences will be apparent in the individuals' demand, and therefore market equilibrium. When customers differ in their preferences of quality, it is in the interest of the firm to vertically differentiate its product (Mussa and Rosen, 1978). When customers differ in their preferences for quantity and quantity it will be in the interests of firms to differentiate its products which so that bundles differ in both quantity and quality. For example, suppliers of instant coffee usually provide their high quality variety in relatively small jars, whereas the low quality variety brand is supplied in larger jars. In contrast, some products that could be readily vertically differentiated, for example rice, are only supplied in varying quantities but only in one quality.

The aim of this paper is to provide an economic interpretation of the equilibrium in these, and analogous, examples. A definition of "skewed quality" is used to assess the balance of quality and quantity in each of the (equilibrium) bundles supplied by the firm. Quality is said to be skewed if the marginal rate of substitution (MRS) between quantity and quality differs from the marginal rate of transformation (MRT). This method of analysis differs from traditional practice in that it looks at the balance between quantity and quality rather than looking at the impact of marginal increments in quantity and quality individually. For example, an equilibrium variety (jar) of coffee would have skewed quality if, for a given level of cost, a substitution of quantity for quality results in a higher consumer benefit (and hence social surplus). This paper provides an analysis that allows identification of the skewness of quality of varieties in a market with a wide class of benefit and cost functions,

To place the contribution of this paper in context it is useful to review the existing literature. The pioneering papers of Spence (1975) and Sheshinski (1976) modelled the quality of a single variety supplied by a monopolist who utilizes linear pricing. These papers implicitly incorporated a trade-off between quantity and quality in a general way. These papers showed (following Swan, 1970) that there was no unambiguous relationship between the efficient and profit maximizing quality level. With Mussa and Rosen (1978) interest turned to the role of quality in differentiating products. Mussa and Rosen (1978), and the overwhelming bulk of the subsequent literature adopted the unit demand model. In these models each customer has a unit demand, and increases in quality linearly increase the willingness to pay. As is extremely well known, Mussa and Rosen show that all but the highest quality good is supplied at a sub-optimal quality. The customers purchasing the low quality good receive no consumer surplus, however all other customers do. These results rely critically on Mussa and Rosen's use of the unit demand model and of the ordering of utility in that model. In subsequent work this approach provides great analytical convenience, and it has often proved possible to provide unambiguous results that relate quality to its efficient level. However the unit demand model does not allow substitution possibilities between quantity and quality. It thus abstracts from one of the fundamental features of markets with endogenous quality. In addition, the unit demand model blurs the distinction between linear and nonlinear pricing.

Maskin and Riley (1984) consider how a monopolist can use nonlinear pricing to conduct price discrimination. In equilibrium the monopolist bundles output: low valuation customers purchase bundles with an inefficiently low quantity, whereas the highest valuation customer purchases a bundle with an efficient quantity. Maskin and Riley note that their model can be recast as the problem of vertical differentiation studied by Mussa and Rosen (1978). In doing so they adopt, as did Mussa and Rosen, the unit demand model. The results of this exercise parallel both their analysis of nonlinear pricing and Mussa and Rosen's results. Specifically they show, assuming a restriction on consumer preferences known as the "single crossing property", that the quality level supplied to low valuation customers is below the efficient level, whereas the quality level supplied to the highest valuation customer is efficient.

The preferences assumed by Mussa and Rosen (1978) and Maskin and Riley (1984) are ones that can be ordered by a single parameter (characteristic). These analyses are thus referred to as uni-dimensional screening problem. There has been interest in extending these early results to cases involving more general preferences, particularly to preferences allowing substitution possibilities between 'instruments' of a firm's output. This leads to a 'multi-dimensional screening problem', in which the aim is to identify the profit maximizing non-linear pricing schedule given the assumed distribution of preferences and cost function. Unfortunately, multi-dimensional screening problems, particularly in case of non-linear pricing, turn out to be analytically challenging. Rochet and Choné (1998) provide a useful survey of this small, and technical, literature. They note that the literature on multidimensional screening can be delineated by the number of instruments available to the firm relative to the number of characteristics of customers. Laffont, Maskin and Rochet (1987) consider the case in which the firm has one instruments but faces customers with two characteristics. On the other hand, the firm may have multiple instruments but face customers who have only one characteristic (Mathews and Moore, 1987). Finally the both the firm and customers may both have multiple characteristics (Armstrong, 1996, Rochet and Choné, 1998 and Sibley, Srinagesh, 1997, Wilson, 1993). Armstrong (1996) shows, in the context of a multi-product monopolist, that there are some (low valuation) customer types who will not be supplied by the monopolist.[1] Rochet and Choné (1998) consider a monopolist who supplies one unit to each customer, but each unit is differentiated by multiple quality characteristics.[2] They use a generalization of the approach of Mussa and Rosen (1978) to show that bunching (i.e. selling the same good to customers with differing characteristics) will commonly occur in multidimensional settings.

The purpose of this paper is not to provide a new solution to multidimensional screening problems, but to provide an economic interpretation of market equilibria in which the firm can differentiate bundles using both quantity and quality. (Hence the firm has two instruments.) The requirement to satisfy the self-selection constraints causes the equilibrium bundle of each variety to differ from the efficient one. The difference between each variety's equilibrium bundle and its efficient one can be decomposed into an unskewed and a skewed component. Intuitively the unskewed component representing the distorting of the 'desirability' of the bundle (while maintaining the optimal balance of quality and quantity), the skewed component represents the distorting of the balance of quality and quantity. It is shown that in equilibrium the skewness of variety i depends on (i) the weighted differences in the marginal rates of substitution of the customer types purchasing variety i and (ii) the difference between the variety i's customers marginal rate of substitution and the weighted marginal rate of substitution of other variety's customers who would switch to variety i with a marginal lowering of its fee.

When all customer types have a common marginal rate of substitution, no skewing of quality and quantity occurs. In this event, self-selection is contingent on the 'desirability' of

[1] The firm considered in this paper is one that bundles quantity and quality. This problem is formally identical to the one facing a monopolist that utilizes non-linear pricing to sell bundles consisting of two goods.

[2] In Rochet and Choné's model the unit could be interpreted a single 'bundle' and one of the quality dimensions could be interpreted as quantity.

the bundle, which can be ordered in one dimension. When, further, cost takes an iso-elastic functional form, the monopolist's problem can be transformed into, what is effectively, a one-dimensional problem. If additionally, in some case in which customer types' preferences can be represented by an iso-elastic benefit function, either quantity or quality can be common across all bundles. These cases represent a link between the one-dimensional analyses of Mussa and Rosen and Maskin and Riley, and the multidimensional analyses discussed above and in this paper.

Section 1 states the formal definition of skewed quality and discusses its implications. It also includes descriptions of equilibrium, including the distinction between bunched and unbunched equilibrium Section 2 states the monopolist's optimization problem. Sections 3 considers the cases in which preferences are such that no variety exhibits skewed quality. Section 4 considers the skewing of quality that occurs in the relatively straightforward cases of unbunched equilibria. Section 5 considers the skewing that occurs in all categories of equilibrium that can occur when there are three customer types. Section 6 the skewing of quality that occurs when there are two varieties produces, and many customer types purchasing both varieties. Section 7 concludes the paper.

1. Equilibrium and Skewed Quality

A monopolist produces n vertically differentiated varieties of good. Each variety i is a bundle with quantity X^i of quality Z^i. The firm offers a set of schedules $\Omega^i \equiv \langle X^i,Z^i,T^i\rangle$, i=1,..n, in which the bundle $\langle X^i,Z^i\rangle$ for fee T^i. The equilibrium (profit maximizing) schedule for variety i is denoted $\overset{*}{\Omega}{}^i \equiv \langle \overset{*}{X}{}^i,\overset{*}{Z}{}^i,\overset{*}{T}{}^i \rangle$.

A monopoly has a number of customer types that differ in preferences. For convenience the types are ordered by two indexes, i and φ with $\varphi=1,..n_i$. This allocation of indexes is chosen in such a way that in equilibrium customers types iφ purchase variety i. The total number of customer types is thus $\sum_{i=1}^{n} n_i$. There are $N^{i\varphi}$ type iφ customers. Each variety has $N^i = \sum_{\varphi=1}^{n_i} N^{i\varphi}$ customers in equilibrium, and the total number of customers is $N= \sum_{\varphi=1}^{n} N^i$. Note that much of the literature discussed in the introduction assumes a continuum of customer types. However, while this assumption aids in the derivation of solutions for a large number of customer types, the assumption of a finite number of customer types is more appropriate for the economic interpretation presented in this paper.

The consumer benefit (utility) type iφ customers receive from consuming variety i is $V^{i\varphi}(X^i,Z^i)$. The surplus from production of all varieties is therefore:

$$S(X^i,Z^i,X^{-i},Z^{-i}) = \sum_{i=1}^{n}[N^iV^i(X^i,Z^i)-C(N^iX^i,Z^i)] \qquad (1)$$

where $V^i(X^i,Z^i) = \sum_{\varphi=1}^{n_i}(N^{i\varphi}/N^i)V^{i\varphi}(X^i,Z^i)$ is the average consumer benefit of the customers of purchasing variety i and $C(N^iX^i,Z^i)$ is the total cost of producing variety i. It is assumed that cost is non-decreasing in quantity and quality, $C_1(N^iX^i,Z^i)>0$, $C_2(N^iX^i,Z^i) >0$, $C_{11}(N^iX^i,Z^i)\geq 0$ and $C_{22}(N^iX^i,Z^i)\geq 0$. In contrast to much of the literature there may be a fixed cost of producing each variety, so $C(0,Z^i)\geq 0$. The following definition is useful to discuss the importance of this latter assumption. Following Mussa and Rosen (1978) and Rochet and Choné (1998):

<u>Definition 1</u>: An equilibrium schedule, $\overset{*}{\Omega}{}^i$, is unbunched if $n_i=1$ and bunched if $n_i>1$. An equilibrium $\{\overset{*}{\Omega}{}^i$, i=1,…,n} is unbunched if $n_i=1$ for all i =1,..n, and bunched if $n_i>1$ for at least one i.

The literature usually restricts consideration to those cases in which bunching would not occur under first-degree price discrimination (in which firms can costlessly identify the type of a particular customer). In such cases each customer type is provided with a unique bundle (with the fee equal to their consumer benefit). This outcome usually requires the adoption of the constraint returns to scale cost function. In such instances, bunching emerges as the firm's optimal response to the need to satisfy the self-selection constraints. However bunching may occur for cost as well as screening reasons. In particular the firm may choose to adopt bunched schedules when there are fixed costs in offering additional varieties. The analysis of this paper allows for bunching for either cost or screening reasons.

The following definition introduces the method that is used in this paper to evaluate equilibrium:

<u>Definition 2</u>: In an equilibrium $\{\overset{*}{\Omega}{}^i$, i=1,…,n}, the quality of variety i is said to be downwardly (un-,upwardly) skewed if:

$$\frac{V_1^i(\overset{*}{X}{}^i, \overset{*}{Z}{}^i)}{V_2^i(\overset{*}{X}{}^i, \overset{*}{Z}{}^i)} < (=, >) \frac{N^i C_1(N^i\overset{*}{X}{}^i, \overset{*}{Z}{}^i)}{C_2(N^i\overset{*}{X}{}^i, \overset{*}{Z}{}^i)} \qquad (2)$$

where the LHS is the marginal rate of substitution (MRS) and the RHS is the marginal rate of transformation (MRT) of variety i.

The motivation for this definition is the observation that the unskewed combination of quantity and quality maximizes the surplus for a given level of resources devoted to production. Specifically the unskewed bundles of quantity and quality, denoted $\{\tilde{X}^i,\tilde{Z}^i\}$ satisfies:

$$\{\tilde{X}^i(\bar{C}),\tilde{Z}^i\bar{C}))\} = \underset{X^i\, Z^i}{\operatorname{argmax}}\ S(X^i,Z^i,X^{-i},Z^{-i}) \text{ subject to } C(N^iX^i,Z^i) \leq \bar{C} \qquad (3)$$

where $\bar{C}$ is a given level of cost. The unskewed combination has optimal balance of quantity and quality. A variety has downwardly skewed quality if, by substituting quality for quantity, a higher surplus is obtainable for a given level of resources devoted to production of that variety. This is depicted graphically in figure 1 for variety i. Quality is unskewed at the point $\tilde{\Pi}^i$ where the indifference curve $\tilde{V}^i$ is tangent to $\overset{*}{C}{}^i$, the iso-cost curve. That is, quality is unskewed when the quantity-quality combination is $\{\tilde{X}^i, \tilde{Z}^i\}$. Note that at the point $\overset{*}{\Pi}{}^i$ quality is downwardly skewed, as the MRS<MRT.

The following companion to definition 2 describes quantity distortion.

<u>Definition 3</u>: In a market outcome $\{\overset{*}{X}{}^i,\overset{*}{Z}{}^i\}$, the quantity of variety i is said to be downwardly (un-,upwardly) distorted if:

$$\overset{*}{X}{}^i < (=, >) \hat{X}^i \qquad (4)$$

and the quality of variety i is said to be downwardly (un-,upwardly) distorted if:

$$\overset{*}{Z}{}^i < (=, >) \hat{Z}^i \qquad (5)$$

where $\hat{X}^i$ is the efficient level of quantity and $\hat{Z}^i$ is the efficient level of quality of variety i, defined by:

$$V^i_1(\hat{X}^i, \hat{Z}^i) = C_1(N^i\hat{X}^i, \hat{Z}^i) \text{ and } V^i_2(\hat{X}^i, \hat{Z}^i) = C_2(N^i\hat{X}^i, \hat{Z}^i) \qquad (6)$$

Definition 3 is the widely adopted as the definition of quality distortion, including in the seminal work of Spence (1975) and Sheshinski (1976). This condition has proved a difficult analytical tool to use, (see, for example, Spence and Sheshinski). In addition, the interpretation of the 'efficient level of quantity and quality of a variety' is ambiguous when bunching occurs for screening purposes. In this case Pareto efficiency requires 'unbunching' of the variety, and offering each customer type their own specific variety. In contrast definition 3 describes the efficient bundle on the assumption that unbunching does not occur, and thus abstracts from issues associated with unbunching. Note also that definition 3 does not capture the idea of there being an optimal trade-off quantity and quality at different production levels

Note that the implicit function theorem can be used to rewrite (3) to describe the unskewed bundles by the function $Z^i = \tilde{z}^i(X^i)$. The function $\tilde{z}^i$ is called the contract curve for variety i, as it represents the locus of points of tangency between the indifference curves and the iso-cost curves. Quality is skewed when the equilibrium bundle does not lie on the contract curve. In particular, the difference between the efficient and efficient and the equilibrium bundle, $(\overset{*}{X}{}^i-\hat{X}, \overset{*}{Z}{}^i-\hat{Z}^i)$, can be decomposed into two components: (i) the unskewed component is $(\tilde{X}^i-\hat{X}^i, \tilde{Z}^i-\hat{Z}^i)$ and (ii) the skewed component is $(\overset{*}{X}{}^i-\tilde{X}^i, \overset{*}{Z}{}^i-\tilde{Z}^i)$.

2. The Monopolist’s Optimisation Problem

The consumer surplus of type iφ customers from the schedule <X,Z,T> is:

$$U^{i\varphi}(X,Z,T) = \begin{cases} V^{i\varphi}(X,Z) - T & \text{if } V^{i\varphi}(X,Z) \geq T \\ 0 & \text{if } V^{i\varphi}(X,Z) < T \end{cases} \qquad (7)$$

where it is assumed that customers who do not purchase a bundle receive zero benefit. The firm knows the distribution of customer types, but cannot identify specific individuals as belonging to a customer type.

To ensure that type i customers purchase variety i the schedules must satisfy the selection constraints:

$$V^{i\varphi}(X^i,Z^i) - T^i \geq V^{i\varphi}(X^j,Z^j) - T^j \qquad \text{for all } \varphi=1,..n_i \text{ and } j\neq i \qquad (8)$$

Types iφ only purchase variety i if, in addition to the self selection constraints holding, the following participation constraint also holds:

$$V^{i\varphi}(X^i,Z^i) \geq T^i \qquad (9)$$

If these constraints are satisfied, firm profit from type i customers is

$$\Pi^i = N^iT^i - C(N^iX^i,Z^i) \qquad (10)$$

The firm chooses the number of varieties, n, and the schedule for each variety for $<X^i,Z^i,T^i>$ i=1,..,n, to maximize total profits, Π, i.e:

$$\max_{X^i Z^i T^i n} \Pi = \max_{X^i Z^i T^i n} \sum_{i=1}^{n} \Pi^i \text{ subject to (8) and (9).} \qquad (11)$$

The following proposition follows from the optimization problem (11).

Proposition 1: The quality of variety i is downwardly (un-,upwardly) skewed if:

$$\sum_{\varphi=1}^{n_i-1}\left[\sum_{\kappa=\varphi+1}^{n_i}\omega^{i\varphi\kappa}\, s^{i\kappa}(\overset{*}{X}{}^i,\overset{*}{Z}{}^i)\left(\frac{V_1^{i\varphi}(\overset{*}{X}{}^i,\overset{*}{Z}{}^i)}{V_2^{i\varphi}(\overset{*}{X}{}^i,\overset{*}{Z}{}^i)}-\frac{V_1^{i\kappa}(\overset{*}{X}{}^i,\overset{*}{Z}{}^i)}{V_2^{i\kappa}(\overset{*}{X}{}^i,\overset{*}{Z}{}^i)}\right)\right]V_2^{i\varphi}(\overset{*}{X}{}^i,\overset{*}{Z}{}^i)) < \;(=, >)$$

$$\left[\frac{V_1^i(\overset{*}{X}{}^i,\overset{*}{Z}{}^i)}{V_2^i(\overset{*}{X}{}^i,\overset{*}{Z}{}^i)}-\frac{\sum_{j\neq i}\sum_{\varphi=1}^{n_j}\lambda^{j\varphi i}V_1^{j\varphi}(\overset{*}{X}{}^i,\overset{*}{Z}{}^i)}{\sum_{j\neq i}\sum_{\varphi=1}^{n_j}\lambda^{j\varphi i}V_2^{j\varphi}(\overset{*}{X}{}^i,\overset{*}{Z}{}^i)}\right]\sum_{j\neq i}\sum_{\varphi=1}^{n_j}\lambda^{j\varphi i}V_2^{j\varphi}(\overset{*}{X}{}^i,\overset{*}{Z}{}^i) \qquad (12)$$

where:

$$\omega^{i\varphi\kappa} = [\mu^{i\kappa} + \sum_{j\neq i}\lambda^{i\kappa j}](N^{i\varphi}/N^{i\kappa}) \; -\mu^{i\varphi} - \sum_{j\neq i}\lambda^{i\varphi j} \qquad (13)$$

and

$$s^{i\kappa}(\overset{*}{X}{}^i,\overset{*}{Z}{}^i) = \frac{N^{i\kappa}V_2^{i\kappa}(\overset{*}{X}{}^i,\overset{*}{Z}{}^i)}{\sum_{\varphi=1}^{n_i}N^{i\varphi}\,V_2^{i\varphi}(\overset{*}{X}{}^i,\overset{*}{Z}{}^i)} \qquad (14)$$

and $\mu^{i\varphi}\geq 0$ and $\lambda^{i\varphi j}\geq 0$, $\varphi=1,..,n_i$, $j\neq i$, are Lagrange multipliers.

Proof: The Lagrangian for the optimization problem (11) is:

$$L=\sum_{i=1}^{n}\left[N^iT^i-C(N^iX^i,Z^i)+\sum_{\varphi=1}^{n_i}\mu^{i\varphi}(V^{i\varphi}(X^i,Z^i)-T^i)+\sum_{j\neq i}\sum_{\varphi=1}^{n_i}\lambda^{i\varphi j}\{V^{i\varphi}(X^i,Z^i)-V^{i\varphi}(X^j,Z^j)-T^i+T^j\}\right] \qquad (15)$$

where $\mu^{i\varphi}\geq 0$ and $\lambda^{i\varphi j}\geq 0$, $\varphi=1,..,n_i$, and $j\neq i$, are the Langrange multipliers. The first order conditions for profit maximization are:

$$\partial L/\partial T^i = N^i - \sum_{\varphi=1}^{n_i}\mu^{i\varphi} + \sum_{j\neq i}\left\{\sum_{\varphi=1}^{n_j}\lambda^{j\varphi i} - \sum_{\varphi=1}^{n_i}\lambda^{i\varphi j}\right\} = 0 \qquad (16)$$

$$\partial L/\partial X^i = -\,N^iC_1(N^iX^i,Z^i) + \sum_{\varphi=1}^{n_i}\mu^{i\varphi}V_1^{i\varphi}(X^i,Z^i)$$

$$+ \sum_{j\neq i} \left\{ \sum_{\varphi=1}^{n_i} \lambda^{i\varphi j} V_1^{i\varphi}(X^i,Z^i) - \sum_{\varphi=1}^{n_j} \lambda^{j\varphi i} V_1^{j\varphi}(X^i,Z^i) \right\} = 0 \qquad (17)$$

$$\partial L/\partial Z^i = - C_2(N^iX^i,Z^i) + \sum_{\varphi=1}^{n_i} \mu^{i\varphi} V_2^{i\varphi}(X^i,Z^i)$$

$$+ \sum_{j\neq i} \left\{ \sum_{\varphi=1}^{n_i} \lambda^{i\varphi j} V_2^{i\varphi}(X^i,Z^i) - \sum_{\varphi=1}^{n_j} \lambda^{j\varphi i} V_2^{j\varphi}(X^i,Z^i) \right\} = 0 \qquad (18)$$

Dividing (17) by (18) gives:

$$\frac{N^iC_1(N^iX^i,Z^i)}{C_2(N^iX^i,Z^i)} = \frac{\sum_{\varphi=1}^{n_i} \mu^{i\varphi} V_1^{i\varphi}(X^i,Z^i) + \sum_{j\neq i} \left\{ \sum_{\varphi=1}^{n_i} \lambda^{i\varphi j} V_1^{i\varphi}(X^i,Z^i) - \sum_{\varphi=1}^{n_j} \lambda^{j\varphi i} V_1^{j\varphi}(X^i,Z^i) \right\}}{\sum_{\varphi=1}^{n_i} \mu^{i\varphi} V_2^{i\varphi}(X^i,Z^i) + \sum_{j\neq i} \left\{ \sum_{\varphi=1}^{n_i} \lambda^{i\varphi j} V_2^{i\varphi}(X^i,Z^i) - \sum_{\varphi=1}^{n_j} \lambda^{j\varphi i} V_2^{j\varphi}(X^i,Z^i) \right\}} \qquad (19)$$

Thus the quality of variety i is downwardly (un-,upwardly) skewed with respect to customer type ib if:

$$\frac{V_1^{i}(X^i,Z^i)}{V_2^{i}(X^i,Z^i)} < (=, >) \frac{\sum_{\varphi=1}^{n_i} \mu^{i\varphi} V_1^{i\varphi}(X^i,Z^i) + \sum_{j\neq i} \left\{ \sum_{\varphi=1}^{n_i} \lambda^{i\varphi j} V_1^{i\varphi}(X^i,Z^i) - \sum_{\varphi=1}^{n_j} \lambda^{j\varphi i} V_1^{j\varphi}(X^i,Z^i) \right\}}{\sum_{\varphi=1}^{n_i} \mu^{i\varphi} V_2^{i\varphi}(X^i,Z^i) + \sum_{j\neq i} \left\{ \sum_{\varphi=1}^{n_i} \lambda^{i\varphi j} V_2^{i\varphi}(X^i,Z^i) - \sum_{\varphi=1}^{n_j} \lambda^{j\varphi i} V_2^{j\varphi}(X^i,Z^i) \right\}} \qquad (20)$$

Cross multiplying yields (12). ||

The Lagrange multipliers in (12) are non-negative, and can be interpreted as weights. Equation (12) can be interpreted as a condition involving differences in the MRS between customer types. To further interpret proposition 1 the following definition, which categories equilibrium schedules, is useful.

<u>Definition 4</u>: (a) An equilibrium schedule i is said to be downwardly binding ($<_b$) on another equilibrium schedule j, $\overset{*}{\Omega}{}^i <_b \overset{*}{\Omega}{}^j$, for i≠j, if there is a type j customer, js, for which:

$$V^{js}(\overset{*}{X}^j,\overset{*}{Z}^j) - \overset{*}{T}^j = V^{js}(\overset{*}{X}^i, \overset{*}{Z}^i) - \overset{*}{T}^i \tag{21}$$

and:

$$V^{i\varphi}(X^i,Z^i) - T^i > V^{i\varphi}(X^j,Z^j) - T^j \tag{22}$$

for all type i customers. The two varieties i and j are said to be adjacent if $\overset{*}{\Omega}^i <_b \overset{*}{\Omega}^j$. (b) Schedule j is singly binding if it is downwardly binding on exactly one variety. (c) An equilibrium schedule is non-binding if it does not downwardly bind another equilibrium schedule. (d) In a bunched variety, a customer type for whom the participation or self-selection constraint is binding is called a 'decisive' type, and other customer types that purchase the variety are 'non-decisive' types.

Definition 4 limits the types of equilibrium schedules under consideration. This proves useful in interpreting (12). Furthermore it is assumed below that there is at most one decisive customer purchasing each variety.[3] Let i1 be the decisive customer who purchases variety i (so that s=1 in (20)). To use definition 4 to interpret proposition 1, consider two singly binding schedules: schedule i-1 is the schedule that downwardly binds schedule i and schedule i is the schedule that downwardly binds schedule i+1. Suppose that variety i has only two customer types (φ=1,2) and customer type i1 is decisive. Similarly customer type i+1 1 is the decisive customer purchasing variety i+1. Then, by proposition 1, the quality of variety i is downwardly (un-,upwardly) skewed if:

$$\frac{N^{i2}\lambda^{i1i-1}V_2^{i1}(\overset{*}{X}^i, \overset{*}{Z}^i)V_2^{i2}(\overset{*}{X}^i, \overset{*}{Z}^i)}{\sum_{\varphi=1}^{n_i} N^{i\varphi}\, V_2^{i\varphi}(\overset{*}{X}^i, \overset{*}{Z}^i)}\left(\frac{V_1^{i1}(\overset{*}{X}^i, \overset{*}{Z}^i)}{V_2^{i1}(\overset{*}{X}^i, \overset{*}{Z}^i)} - \frac{V_1^{i2}(\overset{*}{X}^i, \overset{*}{Z}^i)}{V_2^{i2}(\overset{*}{X}^i, \overset{*}{Z}^i)}\right)$$

$$+ \lambda^{i+1\,1\,i}\, V_2^{i+1\,1}(\overset{*}{X}^i, \overset{*}{Z}^i)\left[\frac{V_1^{i}(\overset{*}{X}^i, \overset{*}{Z}^i)}{V_2^{i}(\overset{*}{X}^i, \overset{*}{Z}^i)} - \frac{V_1^{i+1\,1}(\overset{*}{X}^i, \overset{*}{Z}^i)}{V_2^{i+1\,1}(\overset{*}{X}^i, \overset{*}{Z}^i)}\right] > (=, <)\ 0 \tag{23}$$

Equation (23) is the weighted sum of two differences in MRSs. The first expression of (23) represents the difference between the MRS of the decisive and non-decisive customer types that purchase variety i. The second term represents the difference between the MRS of type i

[3] The interpretation below is largely unaffected by assuming more than one decisive customer per variety. However assuming more than one decisive customer per variety adds somewhat to the exposition required.

customers and the marginal rate of substitution of customer type i+1 1. Equation (23) indicates that the quality of variety i is downwardly skewed if (i) the decisive type i1 customers have a greater MRS than the non-decisive type i2 customers and (ii) type i customers have a greater MRS than the type i+1 1 customers. Proposition 1 can be understood as an extension of this reasoning. Equation (12) states that variety i is downwardly skewed if (i) the weighted difference between the MRS of decisive and non decisive types which purchase variety i is positive and (ii) if the MRS of type i customers is greater than the weighted marginal rate of substitution of the decisive customers who purchase varieties that are downwardly bound by variety i.

The Lagrange multipliers, $\lambda^{i\varphi j}$, can be thought of as a weight, measuring the relative profitability of relaxing the constraint (8). Specifically it can be interpreted as the hypothetical increase in profits that would occur if the fee to type $i\varphi$ customers alone could be marginally increased without violating the self-selection constraints. Proposition 1 does not indicate how the weights, $\lambda^{i\varphi j}$ are determined mathematically. This is considered below by discussing different classes of equilibrium.

3. Unskewed Equilibria

This section considers unskewed equilibria. In an unskewed equilibrium the firm does not distort the balance of quality and quantity of any variety to satisfy self-selection. Proposition 1 indicates that there is no skewing in equilibrium when all customers have a common MRS. This occurs when consumer benefit can be written as:

$$V^{i\varphi}(X^i,Z^i) = v^{i\varphi}(y(X^i,Z^i)) \qquad (24)$$

where the function y(,) is common to all customer types. With preference (24) all customer types have a common MRS. Preferences of the type (24) yield vertical differentiation *in bundles*. That is, all customer types agree on the ordering of bundles. Specifically all customers agree that bundle (X^2,Z^2) is preferred to bundle (X^1,Z^1) if $y(X^2,Z^2)>y(X^1,Z^1)$. If the MRS of customer types differs then there is a degree of horizontal differentiation *in bundles*.

This is illustrated in figure 2. Customer type 1, (with the black indifference curves $\overline{v}^{1A}$ and $\overline{v}^{1B}$), prefers bundle (X^A,Z^A) to bundle (X^B,Z^B) whereas type 2 customers (with the grey indifference curves $\bar{v}^{2A}$ and $\bar{v}^{2B}$) prefers bundle (X^B,Z^B) to bundle (X^A,Z^A). Thus there is disagreement between customer types as to the desirability of some bundles, i.e. there is potentially horizontal differentiation in bundles, when MRSs differ across customer types.

When preferences take the form (24) the equilibrium schedules of variety i lies on the contract curve:

$$\frac{y_1(X^i,Z^i)}{y_2(X^i,Z^i)} = \frac{N^i C_1(N^i X^i,Z^i)}{C_2(N^i X^i,Z^i)} \qquad (25)$$

Note that the contract curve (25) may differ from variety to variety because of differences in customer numbers. However suppose also that variable cost is iso-elastic in quantity, $C(X,Z) = F + X^{\chi}\psi(Z)$, where $F\geq 0$ is the fixed cost, and $\chi>1$, and $\psi'(Z)>0$, Then all varieties lie on the common contract curve:

$$\frac{Xy_1(X,Z)}{Zy_2(X,Z)} = \frac{\chi\psi(Z)}{Z\psi'(Z)} \qquad (26)$$

The LHS represents the elasticity of (all types) indifference curve between quality and quality, and the RHS represents the elasticity of the iso-cost curve.

Consider the case in which desirability of bundles is iso-elastic in quantity, i.e.:

$$y(X,Z) = [X.\Gamma(Z)]^{\omega} \qquad (27)$$

where $0<\omega<1$ and $\Gamma'(Z)>0$ and $\Gamma''<0$. In this case in which the elasticity of the indifference curve is independent of X. Then the contract curve is horizontal at quality level $\hat{Z}$, where:

$$\chi\varepsilon_{\Gamma}(\hat{Z}) = \varepsilon_{\psi}(\hat{Z}) \qquad (28)$$

where $\varepsilon_{\Gamma}(Z)\equiv Z\Gamma'(Z)/\Gamma'(Z)$ and $\varepsilon_{\psi}(Z)\equiv Z\psi'(Z)\psi(Z)$. $\hat{Z}$ is the efficient level of quality. Thus Swan Invariance holds (Swan, 1970) holds in this example. As quality is independent of quantity (and variety), the firm can be thought of as only choosing the quantity of each variety (bundle) given the quality level (28). Thus Maskin and Riley’s one instrument (quantity) model can be interpreted as being generated by the preferences (27). Quality is fixed, and quantity in the bundle (and the fee) is used to screen customers.

The contract curve is vertical if the desirability of bundles is iso-elastic in quality in the following way:

$$y(X,Z) = [(X-\underline{X}).Z^{\gamma}]^{\varpi} \qquad (29)$$

and cost is $C(X,Z) = F + \xi[XZ^{\phi}]^{\chi}$ where ξ, ϖ, ϕ, and χ are positive parameters. The contract curves for all variety is $\hat{X} = (\gamma - \phi)\underline{X}/\phi$, where $\phi<\gamma$. $\hat{X}$ is the efficient quantity.

If $\hat{X}^i$ is interpreted as "one unit" then this is the case that corresponds with the unit demand model introduced by Mussa and Rosen. In this case it is quality that is used to screen customers.

More generally, when consumer benefit is given by (24), the two-instrument problem can be transformed into a one (screening) instrument problem by defining the instrument:

$$Y \equiv y(X,Z) \qquad (30)$$

Y can unambiguously be interpreted as the 'desirability' of the bundle. In this case the firm's problem is transformed to:

$$\max N^iT^i - f(Y^i,Z^i,N^i) \qquad (31)$$

subject to the self-selection constrains:

$$v^{i\varphi}(Y^i) - T^i \geq v^{i\varphi}(Y^j) - T^j \qquad \text{for all } \varphi=1,..n_i \text{ and } j\neq i \qquad (32)$$

and the participation constraints:

$$v^{i\varphi}(Y^i) \geq T^i \qquad \text{for all } \varphi=1,..n_i \text{ and } j\neq i \qquad (33)$$

where:

$$f(Y^i,Z^i,N^i) \equiv C(N^iX(Y^i,Z^i),Z^i) \qquad (34)$$

In this transformation, the firm can be thought of as choosing the desirability of the bundle (Y^i) according to the process analysed by Maskin and Riley (1984). The level of quality of each variety is chosen to minimize cost of producing the desirability of the bundle, i.e. $f_2(Y^i,Z^i,N^i)=0$ or, equivalently, (25). Use the implicit function theorem to define the cost minimising quality as a function of desirability $Z^i=\underline{z}(Y,N^i)$. The next proposition follows from these observations:

> Proposition 2: (a) Suppose $f_{21}(Y^i,Z^i,N^i) = 0$ for all $Y^i \geq 0$ $Z^i \geq 0$. Then the quality of variety i is undistorted. (b) Suppose $f_{21}(Y^i,Z^i,N^i) < 0$ for all $Y^i \geq 0$ $Z^i \geq 0$. Then if Y^i is downwardly (un-, upwardly) distorted the quality of variety i is downwardly (un-, upwardly) distorted. (c) Suppose $f_{21}(Y^i,Z^i,N^i) > 0$ for all $Y^i \geq 0$ $Z^i \geq 0$. Then if Y^i is downwardly (un-, upwardly) distorted the quality of variety i is upwardly (un-, downwardly) distorted.

When $f_{21}(Y^i,Z^i,N^i) < 0$ an increase in Y^i increases the cost minimizing Z^i. If the firm downwardly distorts Y^i to satisfy self-selection, it will also lower Z^i to minimize cost.

Figure 3 illustrates proposition 2 for two customer types. In drawing this diagram it is assumed, following Maskin and Riley, that $v^i(Y^i)$ satisfies the (one dimensional) single crossing property. That is, $v^2(Y)>v^1(Y)$ and $v^{2\prime}(Y)>v^{1\prime}(Y)$ for all relevant Y. Assume also that bunching does not occur (in this case it would only occur for cost reasons). The common cost-minimizing curve for both customer types shown is shown as the curve $\underline{z}(Y)$ in figure 3(b). The firm chooses to produce bundles that lie on the contract curve. Then firm profit from type i customers, when these customers are guaranteed consumer surplus $\bar{U}^i$, and the firm supplies quality, $\underline{z}(Y)$, is given by:

$$\Pi^i(Y,\bar{U}^i) = v^i(Y) - \bar{U}^i \qquad (35)$$

Firm profit for each type, $\Pi^i(Y,\bar{U}^i)$, is plotted in figure 3(a). This diagram mirrors the approach used by Maskin and Riley in their figure 1. Maskin and Riley interpret the curves $\Pi^i(Y,\bar{U}^i)$ as the customer type i's indifference curves across schedules (rather than bundles), i.e. they show lines of constant consumer surplus as a function of profits (or the fee) and quantity. If the firm could identify customer type (first degree price discrimination), it would be profit maximizing to offer the schedule <$\hat{Y}^i$ $\hat{Z}^i$ $\hat{T}^i$>, where $\hat{T}^i$ is equal to the consumer benefit of type i. Under these schedules consumers gain utility $\bar{U}^i_0$, which is the level of utility at which type i consumers are indifferent between purchasing the bundle <$\hat{Y}^i$ $\hat{Z}^i$> or not. However, where the firm cannot identify customer type, type 2 customers would have an incentive to switch to type 1's bundle.

Maskin and Riley show that, in order to optimally satisfy self selection, the firm must reduce the quantity to type 1 customer from $\hat{Y}^1$ to $\overset{*}{Y}{}^1$ and increase the level of utility to type 2 customers from $\bar{U}^2_0$ to $\bar{U}^2_1$ (by reducing the fee below the level of benefit), where $\bar{U}^2_1$ is defined by $\Pi^2(Y,\bar{U}^2_1)=\Pi^1(Y,\bar{U}^1_0)$. Specifically the quantity level $\overset{*}{Y}{}^1$ occurs at the point where the slope of the curve $\Pi^1(Y,\bar{U}^1_0)$ multiplied by the number of type 1 customers equals the slope of the curve $\Pi^2(Y,\bar{U}^2_0)$ - $\Pi^2(Y,\bar{U}^2_1)$ multiplied by the number of type 2 customers.

The cost-minimizing curve is shown as upward sloping in figure 3(b). In this case quality is downwardly distorted as self-selection causes the equilibrium level of desirability, $\overset{*}{Y}{}^1$, to be less than the efficient level $\hat{Y}^1$. Had the cost-minimizing curve been downward sloping (horizontal) quality would have been upwardly distorted (undistorted). Note also that the analysis using figure 3(b) requires that the cost function is such that the contract curve can be written as (26). When the cost function does not have this form inspection of (25) shows that the contract curve of each variety will depend on the number of customers purchasing that variety. Further note that cases in which preferences, $v^i(Y^i)$, do not satisfy the (one dimensional) single crossing property can also be analysed using an appropriately modified variant of figure 3.

4. Unbunched Equilibria

This section analyses equilibrium in which bunching does not occur. When bunching does not occur the LHS of (12) is zero. It is shown below that this enables a straightforward interpretation of the impact of skewing.

4.1 Singly Binding Equilibrium

In a singly binding equilibrium each schedule is either singly binding or non-binding. The following proposition identifies the manner in which quality is skewed in this class of equilibria:

<u>Proposition 3</u>: Consider an unbunched, singly binding equilibrium. Then (i) the non-binding schedules offer the efficient bundle of quantity and quality (and thus quality is unskewed) and (ii) if schedule i is downwardly binding on schedule j, then variety i is downwardly (un-, upwardly) skewed if:

$$\frac{V^j_1(\overset{*}{X}{}^i,\overset{*}{Z}{}^i)}{V^j_2(\overset{*}{X}{}^i,\overset{*}{Z}{}^i)} < (=, >) \ \frac{V^i_1(\overset{*}{X}{}^i,\overset{*}{Z}{}^i)}{V^i_2(\overset{*}{X}{}^i,\overset{*}{Z}{}^i)} \qquad (36)$$

Furthermore if the equilibrium schedule i does not have another equilibrium schedule downwardly binding it has a fee that equals consumer benefit.

Proof: The equilibrium is unbunched. so n_i=1 for all i. Schedule i is assumed only to downwardly binding on schedule j, hence $\lambda^{j1i}>0$ and $\lambda^{m1i}=0$ for $m\neq j$. Schedule j is either (i) downwardly binding on only one other schedule, say k, so that $\lambda^{i1k}>0$ and $\lambda^{i1m}=0$ for $m\neq j$ or (ii) is non-binding, so that $\lambda^{i1m}=0$ for all m. The result then follows from Proposition 1. ||

To discuss the intuition and implications of Proposition 3 it is useful to consider different examples of singly binding equilibrium. To this end, define two schedules $\overset{*}{\Omega}{}^i$ and $\overset{*}{\Omega}{}^j$ as being linked if there are a set of equilibrium schedules

$\{\underline{\overset{*}{\Omega}}{}^k : \underline{\overset{*}{\Omega}}{}^k <_b \underline{\overset{*}{\Omega}}{}^{k+1}$ for k=1,..,τ-1 and τ<n-2$\}$ such that

$$\overset{*}{\Omega}{}^i <_b \underline{\overset{*}{\Omega}}{}^1 <_b \ldots <_b \underline{\overset{*}{\Omega}}{}^k <_b \ldots\ <_b \underline{\overset{*}{\Omega}}{}^\tau <_b \overset{*}{\Omega}{}^j$$

A linked set of schedules consists of those schedules that linked to every other member of the set. Treat a schedule that is not linked to another as a special case of a linked set. Then each singly binding equilibrium has ν linked sets, where $1 \leq \nu \leq n$. Suppose the linked set $\iota \in [1,..,\nu]$ has $n_\iota \leq n$ schedules. Equilibrium schedules in the linked set $\iota \in [1,..,\nu]$ (and, because discussion is restricted to unbunched equilibrium, customer types), $\overset{*}{\Omega}{}^{\iota\,\varphi}$, can be ordered using the index φ so that $\overset{*}{\Omega}{}^{\iota\,\varphi} <_b \overset{*}{\Omega}{}^{\iota\,\varphi+1}$ for all $\varphi=1,..,n_\iota$. Note that the equilibrium schedule $\overset{*}{\Omega}{}^{\iota\,n_\iota}$ does not downwardly bind another equilibrium schedule. By Proposition 3 each of these schedules offers the efficient quantity and quality. The equilibrium schedule $\overset{*}{\Omega}{}^{1}_{\iota}$ does not have another equilibrium schedule downwardly binding it, and hence by Proposition 3 the fee is equal to the consumer benefit.

4.1.1 Uniform Ordering

The following restriction on preferences, which can be thought of as a straightforward generalization of the one dimensional single crossing property of Maskin and Riley, generates only one linked set of schedules.

Definition 5: Preferences exhibit uniform ordering if for all i=2,..n: (i) $V^i(X,Z) > V^{i-1}(X,Z)$, (ii) $V^i_1(X,Z) > V^{i-1}_1(X,Z)$ and, (iii) $V^i_2(X,Z) > V^{i-1}_2(X,Z)$.

This definition of uniform ordering is adapted from the definition of Sibley and Srinagesh (1997). Sibley and Srinagesh (p.699) view uniform ordering as "unappealing" because it so restrictive on allowable preferences. However it ensures that preferences can be ranked as in an analogous way to the single instrument case studied by Maskin and Riley. This allows the ordering of equilibrium schedules to be related to preferences.

It is shown in the appendix that, under uniform ordering, only the downward-adjacent incentive compatibility constraints are binding, and that only type 1's participation constraint is binding. (This is the same result as the single instrument case studied by Maskin and Riley.) Hence:

Corollary 1: Consider an unbunched equilibrium in which preferences satisfy under uniform ordering. Variety i is downwardly (un-, upwardly) skewed if:

$$\frac{V^{i+1}_1(\overset{*}{X}{}^i, \overset{*}{Z}{}^i)}{V^{i+1}_2(\overset{*}{X}{}^i, \overset{*}{Z}{}^i)} < (=, >) \frac{V^{i}_1(\overset{*}{X}{}^i, \overset{*}{Z}{}^i)}{V^{i}_2(\overset{*}{X}{}^i, \overset{*}{Z}{}^i)} \qquad (37)$$

for i=1,…,n-1. Schedule $\overset{*}{\Omega}^n$ offers the efficient levels of quantity and quality for variety n (which thus has unskewed quality). Furthermore the schedule $\overset{*}{\Omega}^1$ has a fee that equals consumer benefit.

Note that the requirement of uniform ordering does not impose a ranking in the magnitudes of types' MRS. Thus uniform ordering does not impose a relationship between a variety's skewness and its customers' ranking.

To explore the intuition of this result further consider the following example:

<u>Example 1 (Iso-elastic example)</u>: Consider the case in which preferences are given by $V^i(X,Z) = A^i[(X-\underline{X})(Z-\underline{Z})^{\gamma_i}]^{\omega_i}$ where A^i, $\underline{X}$, $\underline{Z}$, ω_i and γ_i, are positive parameters, and $X \geq \underline{X}$ and $Z \geq \underline{Z}$. If, in addition, cost is given by the Cobb-Douglas form $C(X,Z) = \xi[XZ^{\phi}]^{\chi}$ where ξ, ϖ, ϕ, and χ are positive parameters, the contract curve of variety i is:

$$\tilde{Z}^i = \frac{\phi \underline{Z} \tilde{X}^i}{\gamma_i \underline{X} + (\phi-\gamma_i)\, \tilde{X}^i} \qquad (38)$$

If $\underline{X} = 0$ and $\underline{Z} > 0$ (with $\phi > \gamma_i$) the contract curves for variety i is horizontal at quality level $Z^i = \hat{Z}^i = \phi \underline{Z} / (\phi-\gamma_i)$.

Assume that (i) $A^i < A^{i+1}$, (ii) $\gamma_i A^i < \gamma_{i+1} A^{i+1}$, and (iii) $\omega_i = \omega$ for all i, in such a way that uniform ordering holds for relevant values of X and Z. In this case, by corollary 1, variety i is downwardly (un-,upwardly) skewed if $\gamma_i <$ (=, >) γ_{i+1}. In this case $Z^i <$ (=, >) $\hat{Z}^i$ if $\gamma_i <$ (=, >) γ_{i+1}. Thus differences in the MRS between type i and i+1 (i.e. differences between γ_i and γ_{i+1}) determine the relationship between quality and its efficient level. In particular, write $Z^i = \tilde{Z}^i - \delta Z^i$. Ignoring 2nd order terms it is shown in the appendix that:

$$\delta Z^i \approx \frac{\phi A^{i+1} R^{i+1} (\gamma^{i+1}-\gamma^i) \left(\frac{\gamma^i \underline{Z}}{\phi-\gamma^i}\right)^{\omega(\gamma^{i+1}-\gamma^i)}}{\gamma^i(\phi-\gamma^i) A^i R^i + A^{i+1} R^{i+1} [\gamma^i(\phi-\gamma^{i+1}) + \gamma^{i+1}(\gamma^i-\gamma^{i+1})] \left(\frac{\gamma^i \underline{Z}}{\phi-\gamma^i}\right)^{\omega(\gamma^{i+1}-\gamma^i)}} \quad (39)$$

where $R^i = \sum_{j=i}^{n} N^j$. The quality of variety i is both upwardly distorted and upwardly skewed if $\gamma_{i+1} > \gamma_i$.

This representation of example 1 can be used to illustrate the intuitive reason that quality is skewed when the MRS differs across types. Assume there are two customer types, with type 1 customers having a lower marginal rate of substitution than type 2 customers, that is $\gamma_1 > \gamma_2$. In this case, Corollary 1 indicates that quality is downwardly skewed. The contract curve associated with each customer type shown in figure 4(b). The contract curve for type 1 and 2 customers is horizontal at quality levels $\hat{Z}^1$ and $\hat{Z}^2$. Suppose, for the moment, that the firm is constrained to produce bundles that lie on the contract curve. Then firm profit from type i customers, when these customers are guaranteed consumer surplus $\bar{U}^i$, and the firm supplies quality, $\hat{Z}^j$, (j=1,2) is given by:

$$\Pi^i(X, \hat{Z}^j, \bar{U}^i) = V^i(X, \hat{Z}^j) - \bar{U}^i \tag{40}$$

Firm profit for type i customers, when production is on variety j's contract curve, $\Pi^i(X,\hat{Z}^j,\bar{U}^i)$, is plotted in figure 4(a). Figure 4(a) is the Maskin and Riley diagram which has been augmented so that the iso-profit curves, $\Pi^i(X,\hat{Z}^j,\bar{U}^i)$, incorporate differing quality levels, $Z^j=\hat{Z}^j$, on the different contract curves. If the firm could identify customer type (first degree price discrimination), it would be profit maximizing to offer the schedule $<\hat{X}^i\ \hat{Z}^i\ \hat{T}^i>$, where $\hat{T}^i$ is equal to the consumer benefit of type i. Under these schedules consumers gain utility $\bar{U}^i_0$, which is the level of utility at which type i consumers are indifferent between purchasing the bundle $<\hat{X}^i\ \hat{Z}^i>$ or not. However, where the firm cannot identify customer type, type 2 customers would have an incentive to switch to type 1's bundle.

In order to optimally satisfy self selection, the firm must reduce the quantity to type 1 customer from $\hat{X}^1$ to $\tilde{X}^1$ and increase the level of utility to type 2 customers from $\bar{U}^2_0$ to $\bar{U}^2_1$ (by reducing the fee below the level of benefit), where $\bar{U}^2_1$ is defined by $\Pi^2(X,\hat{Z}^1,\bar{U}^2_1)=\Pi^1(X,\hat{Z}^1$

$,\bar{U}^1_0)$. Specifically the quantity level $\tilde{X}^1$ occurs at the point where the slope of the curve $\Pi^1(X, \hat{Z}^1,\bar{U}^1_0)$ multiplied by the number of type 1 customers equals the slope of the curve $\Pi^2(X,\hat{Z}^2,\bar{U}^2_0) - \Pi^2(X,\hat{Z}^2,\bar{U}^2_1)$ multiplied by the number of type 2 customers. In this analysis so for the firm has, by the assumption, been constrained to produce unskewed quality, quality is $\hat{Z}^1$. However observe that if the firm is not constrained to produce unskewed quality, it can increase profits by substituting quantity for quality in bundle 1. By doing so, type 1 customers continue to satisfy the participation constraint, and T^2 can be increased by $\tilde{V}^2 - \bar{V}^2$ while type 2 customers still satisfy self-selection. Because the marginal valuations may change with movements along the indifference curves, the bundle $<\bar{X}^i\ \bar{Z}^i>$ might not be the equilibrium one, and it may be possible to increase profit by further fine turning the bundle directed at type 1 customers. Nonetheless the analysis in figure 1 shows how the difference between the efficient and profit-maximizing bundle directed at type 1 customers can be decomposed into a skewed and unskewed component.

Example 1 can also be analysed on the assumption that $\underline{Z}=0$ and $\underline{X}>0$ (with $\phi<\gamma_i$). Under these assumptions the contract curves for each variety are vertical at $X^i = \hat{X}^i= (\gamma_i - \phi)\underline{X}/\phi$. In this case variations of quantity from the efficient level can be analysed as the effect of skewing.

Following the analysis of Maskin and Riley it is assumed above that the firm supplies all customer types. However Armstrong (1996) points out, in the context of a multiproduct monopolist, that it may not be profit maximizing for the firm to supply the low valuation customers. In particular, suppose the valuation of, say, type 1 customers is very much lower than all other customers. If the firm is to supply these customers it must set a relatively low fee to all other customer to satisfy incentive compatibility. In this case it may be optimal for the firm to forgo the (small) profits available from supplying type 1 customers in order to raise the fee to all other customers. In terms of figure 4, the weight slope of the curve $\Pi^1(X, \hat{Z}^1,\bar{U}^1_0)$ is less than the weighted slope of the curve $\Pi^2(X,\hat{Z}^2,\bar{U}^2_0) - \Pi^2(X,\hat{Z}^2,\bar{U}^2_1)$ for all X>0. It is straightforward to show that the condition (37) identifies quality skewness in those varieties that are actually supplied. Note, however, when the firm drops the production of one or more varieties the optimal bundle for type i changes. In this event it is possible that the quality skewness of those varieties produced also changes.

Note that the analysis used in figure 4 can also be readily used for arbitrary cost and utility functions. (For example, an analysis using example 1 with $\underline{X}>0$ and $\underline{Z}>0$ is easily derived from the above analysis.) Thus the approach can provide an intuitive interpretation of any equilibrium. In particular, the unskewed component can be interpreted as being determined by criteria that mirrors the single instrument case of Mussa and Rosen (1978) and Maskin and Riley (1984). The unskewed component can be thought of as the result of allowing firms to trade off quantity and quality in order to satisfy the self-selection constraints, and this is a natural consequence of the extension of the analysis to a two-instrument setting.

In the above analysis it is assumed that there are no cost spillovers in the production of different varieties. As pointed out by Kim and Kim (1996) many production processes, for example automobiles, are characterized by cost spillovers between varieties. For example, undertaking production of a small car (the low quality variety) will lower the cost of a mid-sized car (a higher quality variety). Definition 2 allows for the presence of cost spillovers. It is readily shown that Corollary 1 holds in the presence of cost spillover provided their presence does not alter which of the self-selection and participation constraints are binding. However this may not be the case. For instance, it is possible that the presence of a spillover might substantially lower the marginal cost of producing variety 1 relative to other varieties. It may be optimal for the firm to seek to offer type 1 customers a bundle with higher quantity and quality than that of type 2 customers. In this case the incentive compatibility constraints would be upwardly binding and thus Corollary 1 would not hold.

4.1.2 Diverse Ordering

Mussa and Rosen and Maskin and Riley's results are also often interpreted as implying that quality distortion is ubiquitous when consumers are free from available bundles. However this aspect of Maskin and Riley's results depends crucially on the assumption of the single crossing condition holding. The above results similarly depend on uniform ordering. This claim can be verified by considering the following alternative distribution of preferences:

Definition 6: Order consumers such that, i<j if $C(\hat{X}^i,\hat{Z}^i)< C(\hat{X}^{i+1},\hat{Z}^{i+1})$. Diverse ordering occurs if:

$$V^i(\hat{X}^i, \hat{Z}^i) > V^j(\hat{X}^i, \hat{Z}^i) \text{ for all } i \neq j. \qquad (41)$$

Under diverse ordering the distribution of consumer utility is such that self-selection holds if $T^i = V^i(\hat{X}^i, \hat{Z}^i)$. Thus $<\hat{X}^i \hat{Z}^i \hat{T}^i>$ is the incentive compatible profit maximising schedule. The firm produces the efficient quantity and quality is unskewed and undistorted.

Figure 5 is used to illustrate this outcome. Figure 5, like figure 4, is an augmented Maskin-Riley diagram. For simplicity, attention is restricted to two customer types, types L and H, in figure 5. Type H customer preferences are represented by the indifference curves labelled $\Pi^L(.)$, where $\Pi^L(X,\hat{Z}^L,\bar{U}^L_0)$ is the indifference curve along which the participation constraint is binding. Three types of preferences of type H customers are shown. A consumer preference satisfying uniform ordering (as discussed above), represented by the indifference curve $\Pi^H_U(X, \hat{Z}^H,\bar{U}^H_0)$, is included in figure 5 for comparison purposes. In contrast, the indifference curve $\Pi^H_D(X, \hat{Z}^H,\bar{U}^H_0)$ is an example of preferences which, when combined with those of type L's preferences, exhibits diverse ordering. If the firm offers schedules $<\hat{X}^i \hat{Z}^i \hat{T}^i>$, i=L,H, type i's bundle is represented by the point M^i. It is apparent from figure 5 that type i would receive lower utility if they switched to the alternative bundle. Thus the schedule $<\hat{X}^i \hat{Z}^i \hat{T}^i>$, which includes the efficient bundle, is incentive compatible.

4.1.3 Inverse Ordering

Using a unit demand model Donnenfeld and White (1988) and Srinagesh and Bradburd (1989) (DWSB) argue that, in contrast to the findings of Mussa and Rosen and Maskin and Riley, the high quality level may be upwardly distorted. Noting the direct parallel between quality choice with unit demand and quantity choice, the DWSB argument can be illustrated in figure 5. An example of the case considered by DWSB is represented by preferences $\Pi^H_T(X, \hat{Z}^H,\bar{U}^H_0)$. Observe that these preferences do not satisfy the single crossing condition – indeed as shown the indifference curves of different types may cross twice. DWSB do not provide a characterization of preferences under which their analysis holds.

Rather they note that the slope of customer type L's demand curve must be steeper than that of customer type H's demand curve.[4] This is the case for output levels above $\hat{X}^L$.

In this case the schedule <$\hat{X}^i$ $\hat{Z}^i$ $\hat{T}^i$> is inconsistent with incentive compatibility: type L customers (represented by the point M^L) receive a higher utility by consuming type H's bundle (represented by the point $\hat{N}^H$). In order to satisfy incentive compatibility it is necessary to lower the fee to type L customers so they receive utility $\bar{U}_1^L$ (at point N^L). It is also necessary to increase the quantity available to type H customers from $\hat{X}^H$ to $\tilde{X}^H$, so that they move along their participation constraint from the point $\hat{N}^H$ to N^H. Thus the quantity level in the unskewed component of the 'high quantity' (type H) consumer is above the efficient level. This is the movement identified by the analysis of DWSB.

However, as with the uniform ordering case, the point N^H only represents equilibrium for type H customers if quality is unskewed, that is if both consumer types have the same MRS. Figure 5 shows the case in which type L consumers have a greater MRS than type H consumers then, by corollary 1, quality is upwardly skewed. Starting from quality and quality levels, ($\tilde{X}^H$,$\tilde{Z}^H$), quality can be substituted for quantity along type H's indifference curve $\tilde{V}^H$. This lowers the utility provided by variety H to type L customers, thus allowing the firm to increase the fee to type L customers. This skewing of quality (in this instance) causes quantity to move in the opposite direction to the effect on quantity identified by the analysis of DWSB (i.e. the skewed and unskewed components work in opposite directions).

4.2 Downwardly Bound Equilibrium Schedules of Order β

Not all combinations of preferences and cost functions generate singly binding equilibrium. This section considers some equilibrium schedules that are not singly binding. The following definition is useful:

<u>Definition 7</u>: An equilibrium schedule j is downwardly bound of order β if exactly β equilibrium schedules are downwardly binding on it.

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[4] DWSB adopt the unit demand model, thus consider the choice of quality assuming quantity is fixed. In their papers they refer to the customers' trade-off between price and quality as the marginal rate of substitution. However this trade-off is better described as the demand for quality.

The following proposition considers an equilibrium in which one schedule (schedule k) is downwardly bound by a number of other schedules.

Proposition 4: Consider an unbunched equilibrium in which all schedules are singly binding or non-binding except schedule k, which is downwardly bound of order β. Then if schedule i is downwardly binding on schedule k, then variety i is downwardly (un-, upwardly) skewed if:

$$\frac{V_1^k(\overset{*}{X}{}^i,\overset{*}{Z}{}^i)}{V_2^k(\overset{*}{X}{}^i,\overset{*}{Z}{}^i)} < (=, >) \frac{V_1^i(\overset{*}{X}{}^i,\overset{*}{Z}{}^i)}{V_2^i(\overset{*}{X}{}^i,\overset{*}{Z}{}^i)} \qquad (42)$$

Proof: This follows directly from Proposition 1, by noting that, under the assumptions of proposition 4, that $\lambda^{j1i}=0$ for all $j\neq k$ and $\lambda^{k1i}>0$ ||

Proposition 4 is illustrated in figure 6. In that diagram it is assumed there are three customer types (H, M, L). Type H customer have the highest willingness to pay (and hence profit). However the efficient quantity of variety M is greater than that of variety H, and the efficient quantity of variety H is greater than that of variety M. The MRS of type L is greater than the MRS of type M customers, and the MRS of type M customers is greater than that of type H customers. The schedule directed to type H customers (M^H) is downwardly bound by the schedules directed to type L customers (M^L) and directed to type M customers (M^M). Varieties L and M customers are adjusted to ensure that type H does not switch to either of them. In particular, the attractiveness to type H customers of the bundle directed at type L customers can be reduced by downwardly skewing quality. Similarly the attractiveness to type H customers of the bundle directed at type M customers can be reduced by upwardly skewing quality.

Consider now the case in which one schedule downwardly binds a number of others. This case is most easily analysed by considering the example described in the following proposition.

Proposition 5: Consider an unbunched equilibrium in which n=k+1, with $\overset{*}{\Omega}{}^1 <_b \overset{*}{\Omega}{}^k$ with $\overset{*}{\Omega}{}^k$ non-binding, for k=2,..,n. Then (i) the equilibrium schedules $\overset{*}{\Omega}{}^k$ offer the efficient bundle of quantity and quality (and thus quality is unskewed) and (ii) the quality of variety 1 is downwardly (un-,upwardly) skewed if:

$$\frac{\sum_{k=2}^{n} N^k V_1^k(X^k,Z^k)}{\sum_{k=2}^{n} N^k V_2^k(X^k,Z^k)} < (=,>) \frac{V_1^1(X^i,Z^i)}{V_2^1(X^i,Z^i)} \tag{43}$$

Furthermore the equilibrium schedule $\overset{*}{\Omega}{}^1$ has a fee that equals consumer benefit. Proof: Under the assumption of proposition 5 and $\lambda^{i1k}=0$, $\mu^{k1}=0$ for $k=2,..,n$. The FOC (16) thus gives $\lambda^{k1i} = N^i$. The schedule $\overset{*}{\Omega}{}^1$ has a fee that equals consumer benefit for reason identical to the equivalent part of Proposition 3. ||

Proposition 5 is illustrated in figure 7. In that diagram it is assumed there are three customer types (H, M, L). The MRS of type L is greater than the MRS of type M customers, and the MRS of type M customers is greater than that of type H customers. The schedule directed to type M customers (M^M) and the schedule directed to type H customers (M^H) are both downwardly bound by the schedules directed to type L customers. The schedule directed to type M customers and the schedule directed to type H customers do not downwardly bind another schedule, hence their bundles consist of the efficient quantity and quality. The bundle directed to type L customers is adjusted to ensure that neither type M nor type H customers switch. In figure 7(a) the unskewed component is adjusted along type L's participation constraint $\pi^L(X, \hat{Z}^L, U_0^L)$ to minimize the loss to the firm from satisfying the self-selection constraints. X is below $\hat{X}^L$ if the loss of profit from type M customers is greater than that from type H customers. Further the attractiveness to type M customers and type H customers of the bundle directed at type L customers can be reduced by skewing quality. However, given the assumed ordering of MRS in figure 7, the direction of skewing required to deter type M customers is the opposite of type H customers. In particular upward skewing is required to deter type M customers and downward skewing is required to deter type H customers. The equilibrium direction of skewing will depend on the weighted MRS of the two customer types, as indicated by (43).

5. Three Customer Types

In this section the skewness of quality is described in each of the possible categories of equilibrium when there are three customer types. The equilibrium are categorised according to the number of varieties produced. Ideally equilibrium for an arbitrary number of customer types could be categorised. However this is a notoriously difficult task. Indeed, as is seen below, even with only three customer types (and thus a maximum of three varieties) there are many qualitatively different types of equilibrium. The number of qualitatively different equilibrium increases exponentially as the number of customer types increases. However the case in which there are three customer types can serve as a guide to the cases involving more than three customer types.

5.1 Three Varieties: Unbunched Equilibria

If three varieties are produced the equilibria are by necessity unbunched. The results of section 4 can be used to categorise these equilibrium. There are 5 categories of unbunched equilibria with varieties produced:

1. All 3 varieties are linked in a singly binding equilibrium: The skewing of the quality of each variety in this case is described by corollary 1.
2. One variety downwardly bound of order two: The skewing of the quality of each variety in this case is described by proposition 4.
3. Two varieties downwardly bound by the third: The skewing of the quality of each variety in this case is described by proposition 5.
4. Two varieties linked (in a singly binding equilibrium) and the third unlinked: The skewing of the quality of each variety in this case is described by corollary 1. In particular the non-binding schedules do not exhibit skewed quality.
5. All three varieties unlinked: The skewing of the quality of each variety in this case is described by corollary 1. All varieties have unskewed quality

5.2 Two Varieties

There are three categories of equilibrium in which the firm produces two varieties.

5.2.1 High Quality Variety a Bunched Equilibria

Let varieties 1 downwardly bind variety 2. Customer type 21 is decisive then by (23) variety 2 is downwardly (un-,upwardly) skewed if:

$$\frac{V_1^{21}(\overset{*}{X}{}^2, \overset{*}{Z}{}^2)}{V_2^{21}(\overset{*}{X}{}^2, \overset{*}{Z}{}^2)} > \ (=, <) \ \frac{V_1^{22}(\overset{*}{X}{}^2, \overset{*}{Z}{}^2)}{V_2^{22}(\overset{*}{X}{}^2, \overset{*}{Z}{}^2)} \qquad (44)$$

Variety 1 is downwardly (un-,upwardly) skewed if:

$$\frac{V_1^{1}(\overset{*}{X}{}^1, \overset{*}{Z}{}^1)}{V_2^{1}(\overset{*}{X}{}^1, \overset{*}{Z}{}^1)} > \ (=, <) \ \frac{V_1^{21}(\overset{*}{X}{}^1, \overset{*}{Z}{}^1)}{V_2^{21}(\overset{*}{X}{}^1, \overset{*}{Z}{}^1)} \qquad (45)$$

5.2.2 Low Quality Variety a Bunched Equilibria

Let varieties 1 downwardly bind variety 2. The participation constraint for type 11 customers is binding. By (23) variety 2 is unskewed. Variety 1 is downwardly (un-,upwardly) skewed if:

$$\frac{N^{12} N V_2^{11}(\overset{*}{X}{}^1,\overset{*}{Z}{}^1) V_2^{12}(\overset{*}{X}{}^1,\overset{*}{Z}{}^1)}{\sum_{\varphi=1}^{2} N^{1\varphi} V_2^{1\varphi}(\overset{*}{X}{}^1,\overset{*}{Z}{}^1)} \left(\frac{V_1^{11}(\overset{*}{X}{}^1,\overset{*}{Z}{}^1)}{V_2^{11}(\overset{*}{X}{}^1,\overset{*}{Z}{}^1)} - \frac{V_1^{12}(\overset{*}{X}{}^1,\overset{*}{Z}{}^1)}{V_2^{12}(\overset{*}{X}{}^1,\overset{*}{Z}{}^1)} \right)$$

$$+ N^2 V_2^2(\overset{*}{X}{}^1,\overset{*}{Z}{}^1) \left[\frac{V_1^{1}(\overset{*}{X}{}^1,\overset{*}{Z}{}^1)}{V_2^{1}(\overset{*}{X}{}^1,\overset{*}{Z}{}^1)} - \frac{V_1^{2}(\overset{*}{X}{}^1,\overset{*}{Z}{}^1)}{V_2^{2}(\overset{*}{X}{}^1,\overset{*}{Z}{}^1)} \right] > \ (=, <) \ 0 \qquad (46)$$

Customer type 12 may be non-decisive because they have low customer numbers. The first expression in (46) is close to zero if N^{12} is close to zero. In this event Variety 1 is downwardly (un-,upwardly) skewed if:

$$\frac{V_1^{1}(\overset{*}{X}{}^1,\overset{*}{Z}{}^1)}{V_2^{1}(\overset{*}{X}{}^1,\overset{*}{Z}{}^1)} > \ (=, <) \ \frac{V_1^{2}(\overset{*}{X}{}^1,\overset{*}{Z}{}^1)}{V_2^{2}(\overset{*}{X}{}^1,\overset{*}{Z}{}^1)} \qquad (47)$$

More generally, because of their relatively low numbers, non-decisive customers have a lower weighting in the calculation of the average marginal benefit of the customers of purchasing variety i than the decisive customer. In this case the average marginal benefit of the customers of purchasing variety i will be approximately equal to the marginal benefit of decisive customers. Consequently the MRS of the average customer type that purchases variety i would approximately equal the MRS of the decisive customer type. In such cases proposition 3 can also be used to analyse the skewness of varieties in bunched, singly binding equilibrium.

5.2.3 Unbunched Varieties

The firm may choose not to produce a variety that a 'low' willingness to pay type would purchase. The firm may do this for either screening purposes (for the reasons noted by Armstrong, 1996) or for cost reasons. In this case the firm may produce a variety for each of the remaining two customer types. The skewing of the quality of each variety in this case is described by corollary 1.

5.4 One Variety

There are three ways in which the firm would produce only one variety. The one variety is sold to (i) one customer type, with the remaining two customer types not purchasing (ii) two customer types with the remaining customer type not purchasing or (iii) all three customer types. By (23) the variety does not exhibit skewed quality in all three cases.

6. Two Varieties with Bunching of Many Types

Rochet and Choné (1998 p. 786) argue that their model implies that 'bunching is present in the solution of most multidimensional screening problems'. Rochet and Choné are referring to the bunching that occurs because of screening. In addition bunching is likely to occur because of the fixed cost of varieties. As is readily observed, the number of varieties is much less than the (presumably large number) of customer types. It is thus useful to consider the case in which there are two varieties that are each purchased by a large number of customer types.

Let varieties 1 downwardly bind variety 2. Customer type 21 is the decisive type that purchases variety 2. The participation constraint for type 11 customers is binding. By proposition 1 variety 2 is downwardly (un-,upwardly) skewed if:

$$\frac{V_1^2(\overset{*}{X}{}^2, \overset{*}{Z}{}^2)}{V_2^2(\overset{*}{X}{}^2, \overset{*}{Z}{}^2)} < \ (=, >) \ \frac{V_1^{2\,1}(\overset{*}{X}{}^2, \overset{*}{Z}{}^2)}{V_2^{21}(\overset{*}{X}{}^2, \overset{*}{Z}{}^2)} \tag{48}$$

Note the quality of variety 2 is distorted and skewed if the decisive customer is not representative of the average customer purchasing the variety. In particular, quality is downwardly distorted if the decisive customer’s marginal benefits of quantity and quality do not refect that of the average customer of variety 2. Further, as shown by (48) the quality of variety 2 is skewed if the decisive customer’s MRS differs from the average customer of variety 2’s MRS.

By proposition 1 variety 1 is downwardly (un-,upwardly) skewed if:

$$\frac{NV_2^{11}(\overset{*}{X}{}^1, \overset{*}{Z}{}^1)}{\sum_{\varphi=1}^{n_1} N^{1\varphi}\, V_2^{1\varphi}(\overset{*}{X}{}^1, \overset{*}{Z}{}^1)}\left[\sum_{\kappa=2}^{n_1} N^{1\kappa}V_2^{1\kappa}(\overset{*}{X}{}^1, \overset{*}{Z}{}^1)\left(\frac{V_1^{11}(\overset{*}{X}{}^1, \overset{*}{Z}{}^1)}{V_2^{11}(\overset{*}{X}{}^1, \overset{*}{Z}{}^1)} - \frac{V_1^{1\kappa}(\overset{*}{X}{}^1, \overset{*}{Z}{}^1)}{V_2^{1\kappa}(\overset{*}{X}{}^1, \overset{*}{Z}{}^1)}\right)\right]$$

$$+ N^2\, V_2^{21}(\overset{*}{X}{}^1, \overset{*}{Z}{}^1)\left[\frac{V_1^{1}(\overset{*}{X}{}^1, \overset{*}{Z}{}^1)}{V_2^{1}(\overset{*}{X}{}^1, \overset{*}{Z}{}^1)} - \frac{V_1^{21}(\overset{*}{X}{}^1, \overset{*}{Z}{}^1)}{V_2^{21}(\overset{*}{X}{}^1, \overset{*}{Z}{}^1)}\right] > \ (=, <)\ 0 \tag{49}$$

The distortion in the quality of variety 1 is caused by the firms need to satisfy the self selection constraints and the difference between the decisive customer’s marginal benefits of quantity and quality and that of the average customer of variety 1. The skewness in variety 1 is the result of both the difference between the average MRS of customers purchasing variety 1 and that of decisive type purchasing variety 2, and the weighted difference between the non-decisive customer purchasing variety 1 and the customer on their participation constraint.

Consider the case in which customer type iφ’s preference are iso-elastic in quantity: $V^{i\varphi}(X,Z) = [X\Gamma^{i\varphi}(Z)]^{\omega}$ where $0<\omega<1$ and $\Gamma^{i\varphi\prime}(Z)>0$ and $\Gamma^{i\varphi\prime\prime}(Z)<0$. Such a functional form is arguably a useful representation of preference of customers purchasing packaged goods, such as the jars of coffee discussed in the introduction. If, in addition to these preferences, assume variable cost is iso-elastic in quantity, i.e. cost takes the form $C(X,Z) = F + X^{\chi}\psi(Z)$ where $F\geq 0$ is the fixed cost of a variety, and $\chi>1$, and $\psi'(Z)>0$. Then the contract curve of variety i is:

$$\frac{\sum_{\varphi=1}^{n_i} \varepsilon_\Gamma^{i\varphi}(\hat{Z}).N^{i\varphi}.\Gamma^{i\varphi}(\hat{Z})^\omega}{\sum_{\varphi=1}^{n_i} N^{i\varphi}.\Gamma^{i\varphi}(\hat{Z})^\omega} = \frac{\varepsilon_\psi(\hat{Z})}{\chi} \tag{50}$$

where the LHS is the weighted sum of $\varepsilon_\Gamma^{i\varphi}(Z) \equiv Z\Gamma^{i\varphi\prime}(Z)/\,\Gamma^{i\varphi}(Z)$, the elasticity type iφ's indifference curves. The variety, i, in which the high demand types $N^{i\varphi}\Gamma^{i\varphi}(Z^i)^\omega$ have a relatively low marginal rate of substitution (high $\varepsilon_\Gamma^{i\varphi}$) will tend to have a higher 'unskewed' quality (or contract curve).

The skewness (and hence distortion) of quality of the two varieties could be determined using (48) and (49). However in this instance it is easier to substitute the functional forms and Lagrange multipliers into the first order condition (19). In this case the equilibrium quality of variety 2 is given implicitly by:

$$\chi\varepsilon_\Gamma^{21}(\overset{*}{Z}{}^2) = \varepsilon_\psi(\overset{*}{Z}{}^2) \tag{51}$$

and the equilibrium quality of variety 1 is given implicitly by:

$$\chi\left[\frac{\varepsilon_\Gamma^{11}(\overset{*}{Z}{}^1).N.\Gamma^{11}(\overset{*}{Z}{}^1)^\omega - \varepsilon_\Gamma^{21}(\overset{*}{Z}{}^1).N^2.\Gamma^{21}(\overset{*}{Z}{}^1)^\omega}{N.\Gamma^{11}(\overset{*}{Z}{}^1)^\omega - N^2.\Gamma^{21}(\overset{*}{Z}{}^1)^\omega}\right] = \varepsilon_\psi(\overset{*}{Z}{}^1) \tag{52}$$

Determining the direction of the skewness and distortion of quality of variety i is then a simple matter of comparing the computed values $\overset{*}{Z}{}^i$ and $\hat{Z}^i$. Note that the decisive customers' indifference curve elasticities have a critical role in determining the equilibrium values of quality, and hence on the direction of skewness and distortion of quality. Empirical investigations of the goods that can be described by the above preferences (such as the instant coffee) should find a relationship between the equilibrium level of quality and the elasticity of the decisive customer's indifference curves.

The analysis in this section is limited to considering the skewness of quality when the firm is observed to produce two varieties. The analysis could be readily extended to a larger number of varieties. The extension of the analysis to three or more varieties is not included in this paper for reasons of brevity. However the analysis in this paper indicates how this is achieved.

5. Conclusion

It is common for firms to bundle goods in which a way that customers choose between packages that differ in quantity and quality. Understanding the pricing of such bundles is of fundamental importance. However finding solutions to the general multidimensional screening problem has proved difficult. In contrast to such approaches, this paper provides a characterisation of equilibrium varieties. The concept of quality skewness is used to identify the balance of quantity and quality in each variety of a vertically differentiated good supplied by a monopolist. The difference between the efficient and equilibrium bundle of a particular variety can be decomposed into a skewed and unskewed component. The unskewed component lies along the contract curve – the locus of tangencies between the consumer's indifference curves and the iso cost curve. Quality skewness is defined on the basis of the relationship between equilibrium quality and the contact curve. If the equilibrium quality of a variety is above (below) the contract curve then its quality is upwardly (downwardly) skewed. If a varieties quality is skewed, it is possible to raise social welfare by substituting quality for quantity or visa versa.

Skewing of a variety's quality occurs when there is a difference between its decisive customers' marginal rate of substitution and that of (i) its non-decisive customers (ii) the decisive customers of the varieties that are downwardly bound by it (sometimes also called an 'adjacent varieties'). A useful simplification occurs if all customers have a common marginal rate of substitution. It is noted in section 3 that preferences with this property satisfy a natural generalisation of definition of 'vertical differentiation', which applies to entire bundles rather than simply to the quality of the bundle. These cases are equivalent to those in which the firm has one screening instrument.

The single instrument analyses of Mussa and Rosen (1978) and Maskin and Riley (1984) can be interpreted as special cases of the two instrument analysis in which (i) all customer types have a common MRS and (ii) preferences and cost have particular iso-elastic form. These special cases provide a natural link between the single and multi-instrument analyses. In addition they suggest that all customers have a common MRS for those goods that could be vertically differentiated, but are bundled purely with quantity differentiation (eg rice).

The approach provides very clean results for unbunched equilibrium. This is because the customer type that purchases the variety is decisive, and there are no non-decisive customers to consider in the calculation of skewness. Under uniform ordering low valuation customers have upward skewed quality if the MRS of the low valuation customer is less than the MRS of the upwardly adjacent customer. In the iso-elastic example the contract curve is horizontal, so that the relationship between the efficient and equilibrium quality levels depends only on the direction of skewing. A bundle directed at low valuation customers will have quality that is above the efficient level if the MRS of those consumers is less than that of the upwardly adjacent customers.

Some other cases of unbunched equilibrium are considered. Under diverse ordering there is no quality skewness or distortion. Indeed production is efficient. Under inverse ordering (in which the single crossing/uniform ordering does no hold) a variety's skewness of quality is determined by the relationship between its customers' MRS and that of the customers of the upwardly adjacent variety. The low valuation customer will face downwardly skewed quality if the MRS of the low valuation customer is less than the MRS of the downwardly adjacent customer. In the iso elastic model the low valuation variety have downwardly distorted quality if the if the MRS of its consumers is less than that of the upwardly adjacent (high valuation) customers.

Characterisation of the quality skewness of a bunched variety is not as straightforward that of an unbunched variety. Nevertheless it seems likely that all real world varieties are bunched. This no doubt partly occurs for reasons of screening (Rochet and Choné, 1998). However the production of a particular variety will inevitably involve a fixed cost, and this will also lead to bunching. To provide an application of the analysis, a good produced in two varieties, both of which are bunched, is considered in section 6. In contrast to the cases of unbunched equilibrium, the non-binding variety (labelled variety 2) will be skewed, unless the decisive customer's MRS is representative of all customer types purchasing that variety.

Adopting preferences that are iso-elastic in quantity is a useful and plausible simplification that appears to be capable of capturing significant features of the real world problem. Consider the case in which there are two varieties of instant coffee, each variety with different quality and each sold in differ size jars. If preference and variable cost are iso-elastic in quality then the quality of the varieties could be predicted by equations (51) and (52). In principle these relationships should be observable in empirically. However in practice such an observation would require a richer data source than usually available for empirical investigations. In particular, it would be necessary to identify those customer types that are decisive, and then to measure their MRS (or equivalently elasticity of indifference curve). It would be natural to look to surveys or marketing figures for this type of data.

There is clearly scope for many other applications of the approach adopted in this paper. In these applications, it may sometimes be convenient to adopt utility and variable cost functions that are iso-elastic in quantity. With these restrictions on preferences and cost the contract curve is horizontal. In this case the unskewed and undistorted quality levels are both the same as the efficient quality level (for all levels of equilibrium quantity). This approach thus provides a significant degree of analytic simplification while, in contrast to the ubiquitous unit demand model, also allowing consumers to exhibit substitutability between quantity and quality.

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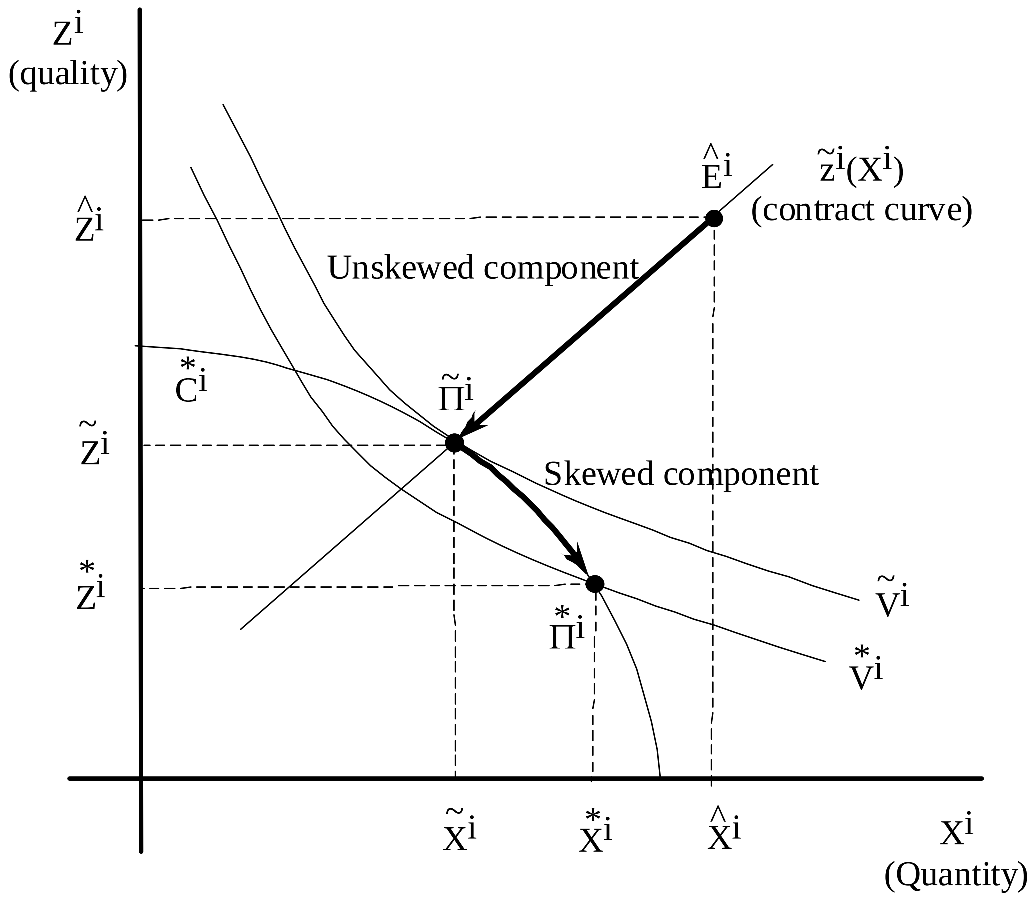

Figure 1: Quality skewness

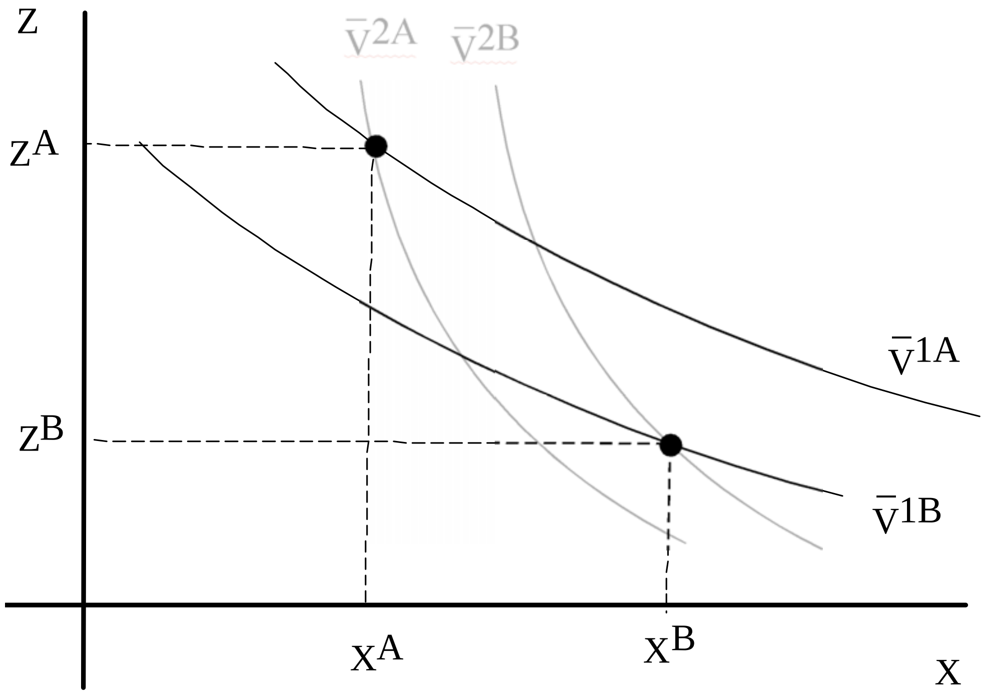

Figure 2: Ranking of bundles

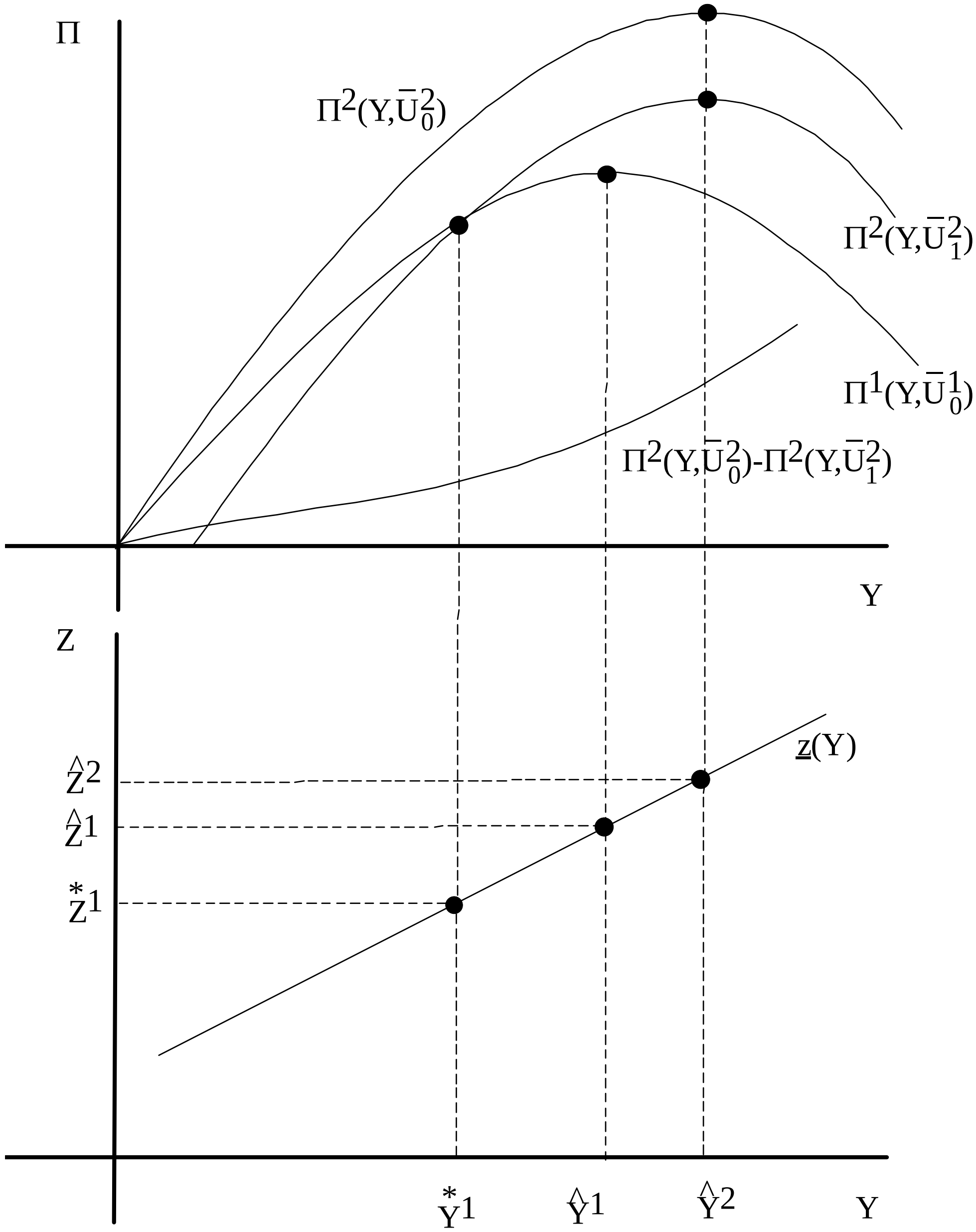

Figure 3: Non-skewed equilibria
with uniform ordering

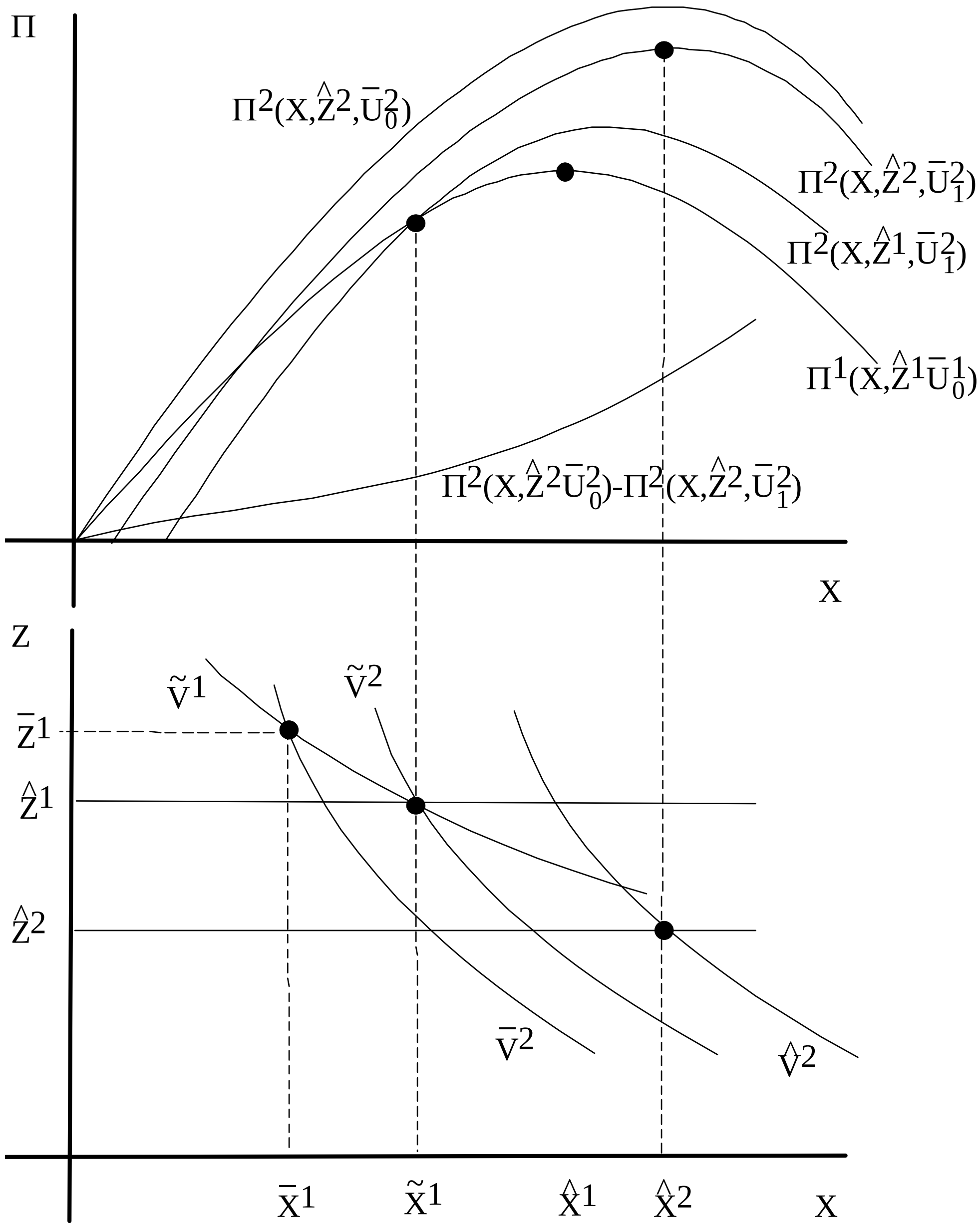

Figure 4: The augmented Maksin-Riley diagram
with the quantity-quality trade-off.

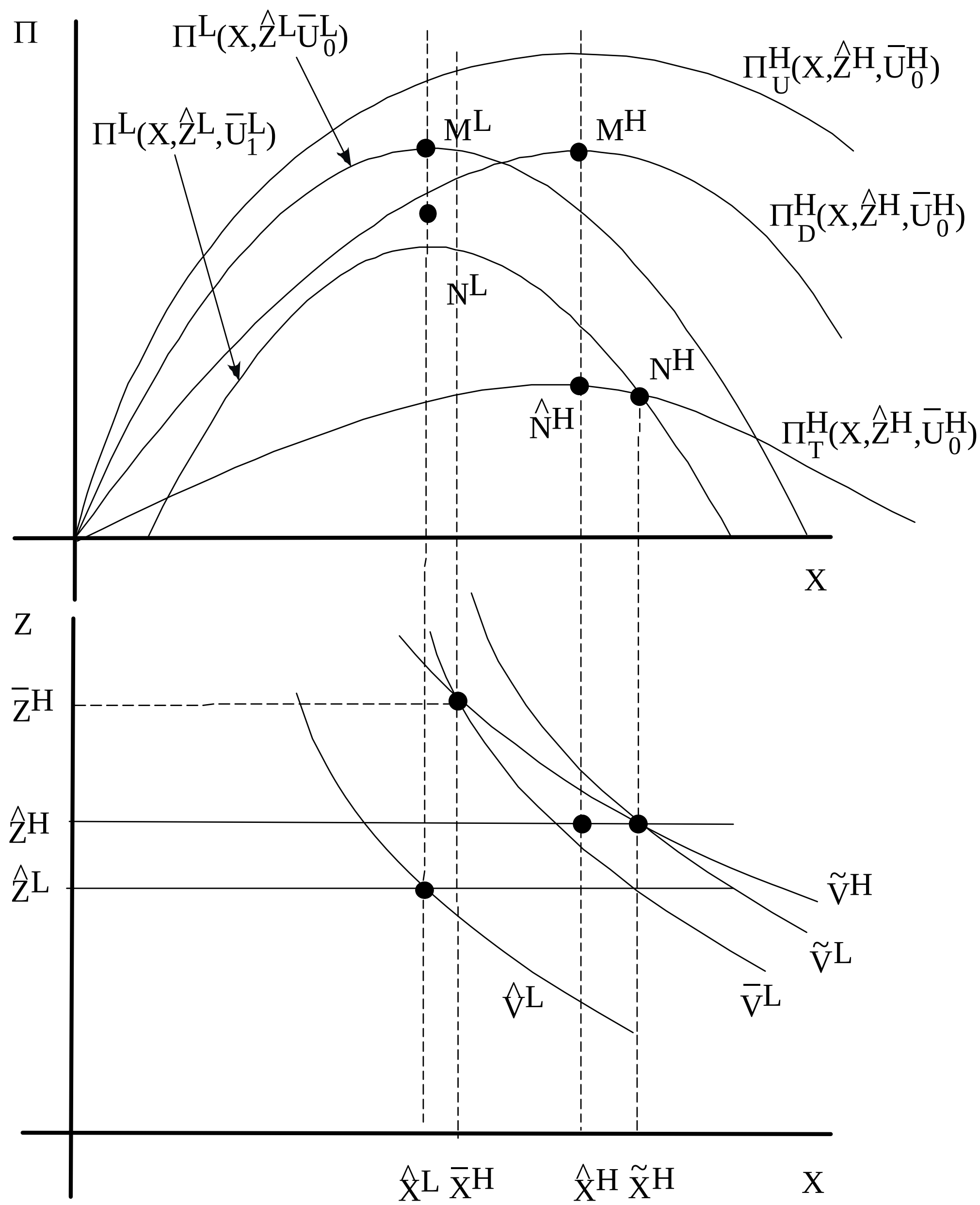

Figure 5: Singly bound equilibrium without uniform ordering

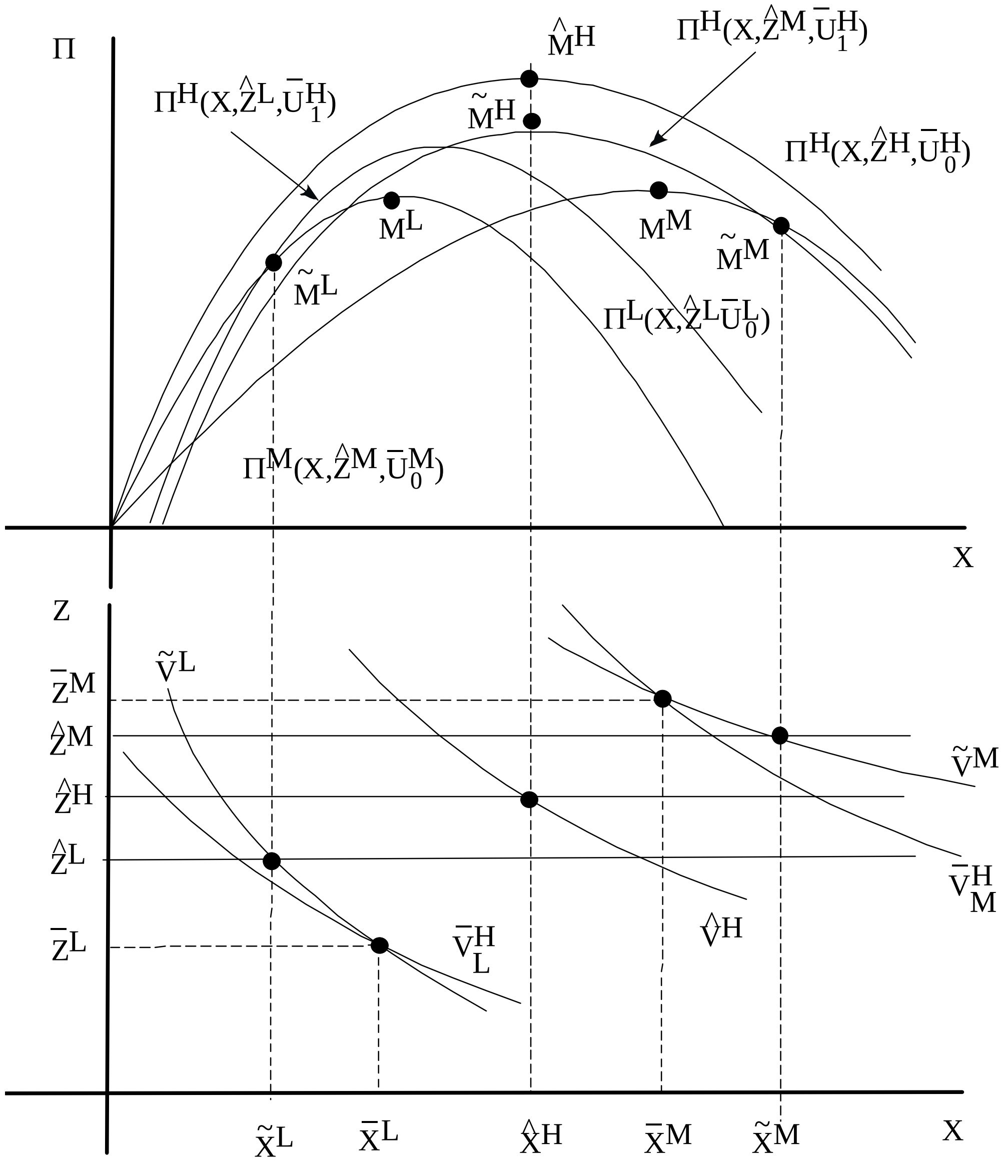

Figure 6: Downwardly bound equilibrium of order 2

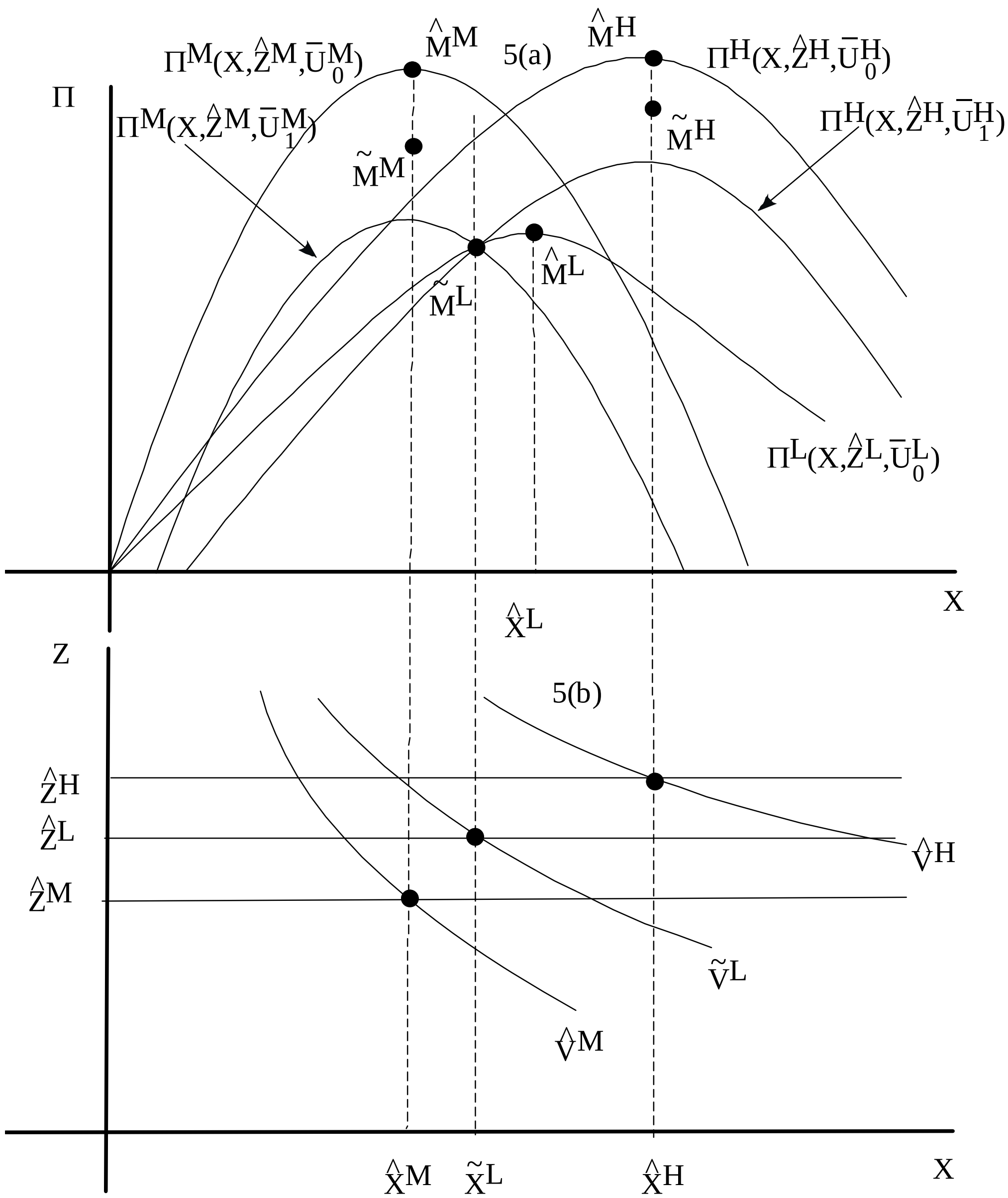

Figure7: Varieties H and M downwardly bound by variety L

Appendix 1: Proof of Corollary 1 (Uniform Ordering and Unbunched Equilibrium)

The proof of corollary 1 requires two lemmas. Lemma A1 shows that only the downward-adjacent incentive compatibility constraints are binding. Lemma A2 shows that only type 1 customer's participation constraint is binding.

<u>Lemma A1</u>: $\lambda^{ij} = 0$ for $j \neq i-1$.

Proof of Lemma A1. The proof proceeds in two steps. (i) Show that $X^i > X^{i-1}$ and $Z^i > Z^{i-1}$; and, (ii) $U^i(X^i, Z^i, T^i) > U^{i-j}(X^{i-j}, Z^{i-j}, T^{i-j})$ for j= 2,...,i-1.

(i) The firm's profit from selling a bundle <X,Z,T> to type i customers is:

$$R^i(X,Z,T) = N^i(X,Z) - U^i(X,Z,T)$$

where $N^i(X,Z) = V^i(X,Z) - C(X,Z)$ is the social surplus from selling to type i customers, which is assumed strictly concave. Let:

$$(\hat{X}^i, \hat{Z}^i) = \underset{X\ Z}{\text{argmax}}\ N^i(X,Z)$$

Along type i's indifference curve, $\bar{U}^i = U^i(X,Z,T)$, profits from type i customers is given by:

$$\bar{R}^i(X,Z) = N^i(X,Z) - \bar{U}^i$$

(Note along the indifference curve T is adjusted to ensure keep U^i constant.) Observe that $\bar{R}^i_1 = N^i_1(X,Z) > 0$ and $\bar{R}^i_2 = N^i_2(X,Z) > 0$ for all $X < \hat{X}^i$ and $Z < \hat{Z}^i$.

Consider the bundle that lie along type i's indifference curve. These can be characterised in the following way:

$$\bar{U}^i = U^i(X,Z,T) = U^i(X+\delta X,Z+\delta Z,T+\delta T)$$

For small variations in X, Z and T the following holds from Taylor's theorem:

$$U^i(X+\delta X,Z+\delta Z,T+\delta T) \approx U^i(X,Z,T) + \delta X V^i_1(X,Z) + \delta Z V^i_2(X,Z) - \delta T$$

Hence along an indifference curve:

$$\delta T = \delta X V^i_1(X,Z) + \delta Z V^i_2(X,Z)$$

Consider the preferences of type j>i customers for bundles along type i's indifference curves:

$$U^j(X+\delta X,Z+\delta Z,T+\delta T) \approx U^j(X,Z,T) + \delta X V^j_1(X,Z) + \delta Z V^j_2(X,Z) - \delta T$$

$$= U^j(X,Z,T) + \delta X V^j_1(X,Z) + \delta Z V^j_2(X,Z) - [\delta X V^i_1(X,Z) + \delta Z V^i_2(X,Z)]$$
$$> U^j(X,Z,T)$$

if $\delta X>0$ and $\delta Z>0$.

Thus if $U^i(X,Z,T) = U^i(X+\delta X,Z+\delta Z,T+\delta T)$ then $U^j(X+\delta X,Z+\delta Z,T+\delta T) > U^j(X,Z,T)$ for $\delta X>0$ and $\delta Z>0$. It may now be shown that a bundle involving $X^i \leq X^{i-1}$ or $Z^i \leq Z^{i-1}$ cannot be an equilibrium bundles. Suppose it was. Then it would satisfy the self-selection constraints for both type i-1 and type i customers, i.e.:

$$U^i(X^i,Z^i,T^i) \geq U^i(X^{i-1},Z^{i-1},T^{i-1}) \text{ and } U^{i-1}(X^i,Z^i,T^i) \leq U^i(X^{i-1},Z^{i-1},T^{i-1})$$

Write $X^i=X^{i-1}+\delta X$, $Z^i=Z^{i-1}+\delta Z$ and $T^i=T^{i-1}+\delta T$. Then these constraints are satisfied if:

$$\delta X[V^i_1(X^{i-1},Z^{i-1}) - V^{i-1}_1(X^{i-1},Z^{i-1})] + \delta Z[V^i_2(X^{i-1},Z^{i-1}) - V^{i-1}_2(X^{i-1},Z^{i-1})] \geq 0 \qquad (1A)$$

This constraint could hold if $\delta X=\delta Z=0$. However δX and δZ could be increased along type i's indifference curve without violating the incentive compatibility constraints. In this case profits will increase. Hence $\delta X=\delta Z=0$ cannot be in the equilibrium bundle.

Similarly, under uniform ordering, (1A) could hold if $\delta X>0$ and $\delta Z<0$. However such a combination cannot be an equilibrium. Suppose that it were. Observe that, for any δX and δZ, it is profit maximizing to ensure that T^i is set so that type i's incentive compatibility constraint is binding, i.e. $U^i(X^i,Z^i,T^i) = U^i(X^{i-1},Z^{i-1},T^{i-1})$. Now if δZ was increased to zero (with δT adjusted to retain indifference), (1A) would continue to hold. However along the indifference curve profits increase. Hence $\delta X>0$ and $\delta Z<0$ cannot be an equilibrium combination. Similarly $\delta X<0$ and $\delta Z>0$ cannot be an equilibrium combination. Thus $X^i > X^{i-1}$ and $Z^i>Z^{i-1}$ in the equilibrium bundle.

(ii) Note that uniform ordering requires that $V^{i+1}(X^{i+1},Z^{i+1}) < V^i(X^i,Z^i)$ if $X^{i+1} < X^i$ and $Z^{i+1} < Z^i$. To show that $U^i(X^i,Z^i,T^i) > U^{i-j}(X^{i-j},Z^{i-j},T^{i-j})$ for j= 2,...,i-1 assume that (X^i,Z^i, T^i), i=1,…,n, represent the equilibrium bundles. In this case the following self-selection constraints hold:

$$V^i(X^i,Z^i) - V^i(X^{i-1},Z^{i-1}) \geq T^i - T^{i-1}$$

and:

$$V^{i-1}(X^{i-1},Z^{i-1}) - V^{i-1}(X^{i-2},Z^{i-2}) \geq T^{i-1} - T^{i-2} \text{ for all } i>2.$$

Then, adding these constraints yields:

$$V^i(X^i,Z^i) - V^i(X^{i-2},Z^{i-2}) + \{V^i(X^{i-2},Z^{i-2}) - V^i(X^{i-1},Z^{i-1}) - [V^{i-1}(X^{i-2},Z^{i-2}) - V^{i-1}(X^{i-1},Z^{i-1})]\}$$
$$\geq T^i - T^{i-2}$$

Note that uniform ordering implies:

$$\{V^i(X^{i-2},Z^{i-2}) - V^i(X^{i-1},Z^{i-1}) - [V^{i-1}(X^{i-2},Z^{i-2}) - V^{i-1}(X^{i-1},Z^{i-1})]\} < 0,$$

Hence:

$$V^{i-1}(X^{i-1},Z^{i-1}) - V^{i-1}(X^{i-2},Z^{i-2}) > T^{i-1} - T^{i-2}$$

for all i>2. Similarly

$$V^i(X^{i+j},Z^{i+j}) - T^{i+j} < V^i(X^i,Z^i) - T^i \text{ for all } j = 1,...n-i. \qquad ||$$

Lemma A2: $\mu^1 > 0$ and $\mu^i = 0$ for $i = 2,...n$.

Proof of lemma A2: Under uniform ordering, any schedule which gives type 1 customers with non-negative utility will provide all other customers with positive utility. Thus only the participation constraint of type 1 customers is binding. ||

Using lemmas A1 and A2 the Lagrangian for the optimization problem (11) is:

$$L = \sum_{i=2}^{n}[N^iT^i - C(N^iX^i,Z^i) + \lambda^i\{V^i(X^i,Z^i) - V^i(X^{i-1},Z^{i-1}) - T^i + T^{i-1}\}]$$

$$+ T^1 - C(N^1X^1,Z^1) + \mu(V^1(X^1,Z^1) - T^1)) \qquad \text{(A1.1)}$$

where $\lambda^i \geq 0$ and $\mu \geq 0$ are the Langrange multipliers. The first order conditions of (A1.1) are:

$$\partial L/\partial T^1 = N^1 + \lambda^2 - \mu = 0$$

$$\partial L/\partial T^i = N^i + \lambda^{i+1} - \lambda^i = 0, \text{ for } i = 2,...,n-1.$$

$$\partial L/\partial T^n = N^n - \lambda^n = 0$$

Hence $\lambda^i = \sum_{j=i}^{n} N^j$, for $i=2,...n$ and $\mu = \sum_{j=1}^{n} N^j$. Note that, as these multipliers are positive, all of their concomitant constraints are binding.

$$\partial L/\partial X^1 = - N^1C_1(N^1X^1,Z^1) + \mu V_1^i(X^1,Z^1) - \lambda^2V_1^2(X^1,Z^1) = 0$$

$$\partial L/\partial X^i = - N^iC_1(N^iX^i,Z^i) + \lambda^iV_1^i(X^i,Z^i) - \lambda^{i+1}V_1^{i+1}(X^i,Z^i) = 0 \text{ for } i=2,...n-1 \qquad \text{(A1.2)}$$

$$\partial L/\partial X^n = - N^nC_1(N^nX^n,Z^n) + \lambda^nV_1^n(X^n,Z^n) = 0 \qquad \text{(A1.3)}$$

$$\partial L/\partial Z^1 = - N^1C_2(N^1X^1,Z^1) + \mu V_2^1(X^i,Z^i) - \lambda^2 V_2^2(X^1,Z^1) = 0$$

$$\partial L/\partial Z^i = - N^iC_2(N^iX^i,Z^i) + \lambda^i V_2^i(X^i,Z^i) - \lambda^{i+1} V_2^{i+1}(X^i,Z^i) = 0 \text{ for } i=2,...n-1 \quad \text{(A1.4)}$$

$$\partial L/\partial Z^n = - N^nC_2(N^nX^n,Z^n) + \lambda^n V_2^n(X^n,Z^n) = 0 \qquad \text{(A1.5)}$$

Dividing (42) by (31) gives:

$$\frac{C_1(N^nX^n,Z^n)}{C_2(N^nX^n,Z^n)} = \frac{V_1^n(X^n,Z^n)}{V_2^n(X^n,Z^n)}$$

Hence variety n exhibits unskewed quality. Dividing (A1.2) by (A1.4) and (A1.3) by (A1.5) gives:

$$\frac{C_1(N^iX^i,Z^i)}{C_2(N^iX^i,Z^i)} = \frac{\sum_{j=i}^{n} N^j V_1^i(X^i,Z^i) - \sum_{j=i+1}^{n} N^j V_1^{i+1}(X^i,Z^i)}{\sum_{j=i}^{n} N^j V_2^i(X^i,Z^i) - \sum_{j=i+1}^{n} N^j V_2^{i+1}(X^i,Z^i)} \qquad i=1,...,n-1$$

Thus the quality of variety i, i=1,..,n-1, is downwardly (un-,upwardly) skewed if:

$$\frac{V_1^i(X^i,Z^i)}{V_2^i(X^i,Z^i)} < (=, >) \frac{\sum_{j=i}^{n} N^j V_1^i(X^i,Z^i) - \sum_{j=i+1}^{n} N^j V_1^{i+1}(X^i,Z^i)}{\sum_{j=i}^{n} N^j V_2^i(X^i,Z^i) - \sum_{j=i+1}^{n} N^j V_2^{i+1}(X^i,Z^i)} \qquad \text{(A1.6)}$$

(37) follows from (A1.6). ||

Appendix 2: Equilibrium Quality and Quantity in the Iso-Elastic Example (Example 1).

Consider equilibrium in which $\phi > \gamma_i$ and $X^i >> \underline{X}$ so that $X^i - \underline{X} \approx X^i$ or $\underline{X}=0$. Substituting the iso-elastic utility and cost functions into (19) yields:

$$\frac{Z^i}{\phi} = \frac{(Z^i-\underline{Z})\left[(\mu^{i1}+\sum_{j\neq i}\lambda^{i1j})A^i(Z^i-\underline{Z})^{\omega\gamma^i} - \sum_{j\neq i}\lambda^{j1i}A^j(Z^i-\underline{Z})^{\omega\gamma^j}\right]}{(\mu^{i1}+\sum_{j\neq i}\lambda^{i1j})\gamma^i A^i(Z^i-\underline{Z})^{\omega\gamma^i} - \sum_{j\neq i}\lambda^{j1i}\gamma^j A^j(Z^i-\underline{Z})^{\omega\gamma^j}}$$

Write:

$$Z^i = \frac{\phi\underline{Z}}{\phi-\gamma^i} - \delta Z^i$$

Then, ignoring second order and higher terms:

$$\delta Z^i = \frac{\sum_{j\neq i}\lambda^{j1i}(\gamma^j-\gamma^i)A^j\left(\frac{\phi\underline{Z}}{\phi-\gamma^i}\right)^{\omega(\gamma^j-\gamma^i)}}{(\mu^{i1}+\sum_{j\neq i}\lambda^{i1j})\gamma^i(\phi-\gamma^i)A^i + \sum_{j\neq i}\lambda^{j1i}\left[\gamma^i(\phi-\gamma^j)+\omega\phi\gamma^j(\gamma^i-\gamma^j)\right]A^j\left(\frac{\phi\underline{Z}}{\phi-\gamma^i}\right)^{\omega(\gamma^j-\gamma^i)}}$$

The quantity of variety i can be determined from (18) as:

$$X^i = \left(\frac{\omega\left[(\mu^{i1}+\sum_{j\neq i}\lambda^{i1j})A^i(Z^i)^{\omega\gamma^i-\phi\chi} - \sum_{j\neq i}\lambda^{j1i}A^j(Z^i)^{\omega\gamma^j-\phi\chi}\right]}{\xi\chi(N^i)^{\chi}}\right)^{\frac{1}{\chi-\omega}}$$

Write:

$$X^i = \underline{X}^i + \delta X^i$$

where $\underline{X}^i$ is the quantity that the firm would produce if $Z^i = \frac{\phi Z}{\phi-\gamma^i}$, so that δX^i represents the adjustment to quantity due to the skewing of quality. In particular

$$\underline{X}^i = \left(\frac{\omega\left[(\mu^{i1}+\sum_{j\neq i}\lambda^{i1j})A^i\left(\frac{\phi Z}{\phi-\gamma^i}\right)^{\omega\gamma^i-\phi\chi} - \sum_{j\neq i}\lambda^{j1i}A^j\left(\frac{\phi Z}{\phi-\gamma^i}\right)^{\omega\gamma^j-\phi\chi} \right]}{\xi\chi(N^i)^{\chi}} \right)^{\frac{1}{\chi-\omega}}$$

and:

$$\delta X^i = \left(\frac{\underline{X}^i}{\chi-\omega}\right)\left(\frac{\phi Z}{\phi-\gamma^i}\right)^{-1} \left(\phi\chi-\omega\gamma^i + \frac{\omega\sum_{j\neq i}\lambda^{j1i}\,(\gamma^j-\gamma^i)A^j\left(\frac{\phi Z}{\phi-\gamma^i}\right)^{\omega\gamma^j-\phi\chi}}{(\mu^{i1}+\sum_{j\neq i}\lambda^{i1j})A^i\left(\frac{\phi Z}{\phi-\gamma^i}\right)^{\omega\gamma^i-\phi\chi} - \sum_{j\neq i}\lambda^{j1i}A^j\left(\frac{\phi Z}{\phi-\gamma^i}\right)^{\omega\gamma^j-\phi\chi}} \right)\delta Z^i$$

Because the equilibrium schedules are assumed to be unbunched and singly binding then $\lambda^{i1j}=0$ for all $j\neq i-1$. The first order condition yield $\lambda^{i\,1\,i-1} = \sum_{j=i}^{n} N^j$, for i=2,...n and $\mu^{11} = \sum_{j=1}^{n} N^j$.

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