The Impact of Price Movements on Real Welfare through the PS-QAIDS Cost of Living Index for Australia and Canada

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Abstract

This paper constructs a cost of living index (CLI) based upon a demographically scaled version of the Quadratic Almost Ideal rank-3 demand system. The construction of a CLI in demographic rank-3 framework allows the index to vary across demographics and expenditure level. The parameters of the CLI are recovered by estimating the demand system based upon a pooled cross section of the Household Expenditure Surveys (HES) and Consumer Price Index (CPI) series from the Australian Bureau of Statistics (ABS) and the Family Expenditure Survey (FES) and CPI series from Statistics Canada (SC). The impact of price changes, in nine broad commodity aggregates, upon real measures of welfare, is examined through the elasticity of CLI with respect to price for Australia and Canada. The variation of the impact is examined across levels of equivalent expenditure and the number of children in the household.

1 Introduction

This paper constructs a cost of living index (CLI) based upon a demographically scaled version of the Quadratic Almost Ideal rank-3 demand system. The construction of a CLI in demographic rank-3 framework allows the index to vary across demographics and expenditure level. The parameters of the CLI are recovered by estimating the demand system based upon a pooled cross section of the Household Expenditure Surveys (HES) and Consumer Price Index (CPI) series from the Australian Bureau of Statistics (ABS) and the Family Expenditure Survey (FES) and CPI series from Statistics Canada (SC). The impact of price changes, in nine broad commodity aggregates, upon real measures of welfare, is examined through the elasticity of CLI with respect to price for Australia and Canada. The variation of the impact is examined across levels of equivalent expenditure and the number of children in the household.

2 Consumer Preference Specification

To construct a CLI in a utility consistent framework requires the specification of a cost function dependent on prices and demographics in addition to base period utility. Estimation of the budget demands derived from the cost function, allow the parameters of the cost function to be recovered and CLI estimated. Section 2.1 contains the specification of the QAIDS cost function used as a basis for estimation of the CLI. To allow for the varying demographic structures of the household used in estimation, an equivalence scale is specified in section 2.2 in a price scaled QAIDS.

2.1 QAIDS Cost Function

The demand system specified in this study for the estimation of equivalence scales and true cost of living indices is QAIDS, a non-linear rank-3 model of Banks, Blundell and Lewbel (1992). QAIDS allows for unique Engel curves that are quadratic in log of household expenditure and thus allow for goods to change from necessities to luxuries across the expenditure distribution. The QAIDS cost function is given in non-demographic form

$$x_R = c_R(u, \mathbf{p}) = e^{\left[a(\mathbf{p}) + \frac{ub(\mathbf{p})}{1 - uc(\mathbf{p})}\right]}$$
(1)

where u is utility, p denotes the vector of prices p_i , and

$$a(\mathbf{p}) = \mathbf{a}_0 + \mathbf{\dot{a}}_i \mathbf{a}_i \log p_i + \frac{1}{2} \mathbf{\dot{a}}_i \mathbf{\dot{g}}_{ij} \log p_i \log p_j$$
 (1a)

$$b(\mathbf{p}) = \prod_{i} p_i^{\mathbf{b}_i} \tag{1b}$$

$$I(p) = \tilde{O}_i p_i^{I_i} , \qquad (1c)$$

with the adding up restrictions $\sum \alpha_i = 1$, $\sum \beta_i = \sum \delta_i = \sum \lambda_i = \sum \gamma_{ij} = 0$, the homogeneity

restriction $\sum_{j} \gamma_{ij} = 0$, and the symmetry restriction $\gamma_{ij} = \gamma_{ji}$ for all i, j. The first term in equation (1), \mathbf{a}_0 is the level of expenditure at the base level prices required for some minimum level of welfare. Since real expenditure is desired to be positive this places an upper bound on $a(\mathbf{p})$. In reference price regime $a(\mathbf{p}) = \mathbf{a}_0$ and if real expenditure is to be positive then $\mathbf{a}_0 \ \mathbf{f} \log (x_{MIN})$. When using x as total expenditure rather than total consumption it is possible for data to report a few levels of expenditure less than even \$1. While such observations are frequently removed in this case they have been included and given a value of \$1, since they are to be included in the nationwide study of inequality. This imposes an upper bound of zero on \mathbf{a}_0 .

2.2 Price Scaled QAIDS and Equivalence Scale Specification

Price scaling (PS), see Ray (1983), the QUAIDS cost function scaled with equivalence scale dependent on prices and household demographics, results in PS-QUAIDS cost or expenditure function is given by

$$x_h = c_h(u, \boldsymbol{p}, \boldsymbol{z}) = c_R(u, \boldsymbol{p}) m_{PS}(\boldsymbol{p}, \boldsymbol{z})$$
 (2)

where $c_R(u, \mathbf{p})$ is given by (6.1.1) and $m_{PS}(\mathbf{p}, \mathbf{z})$ is the equivalence scale. The majority of household equivalence scales are based on household size and composition of its members. This study follows this tradition specifying the price scaling equivalence scale as

$$m_{PS}(\mathbf{p}, \mathbf{z}) = m_{HH}(\mathbf{p}, \mathbf{z}) \tag{3}$$

where $z = \{n_a, n_{k1}, n_{k2}, n_{k3}\},\$

 n_a = the number of adults,

 n_{kl} = the number of infant and young children

 n_{k2} = the number of children

 n_{k3} = the number of older dependents and students.

The specification of the household size and composition equivalence scale $m_{\rm HH}(\boldsymbol{p},z)$ chosen in this study is represented by the product of two terms the first $m_{HHSIZE}(z)$ captures the effect of household size and the second $m_{HHCOMP}(\boldsymbol{p},z)$,

$$m_{\rm HH}(\boldsymbol{p},z) = m_{\rm HHSIZE}(z) m_{\rm HHCOMP}(\boldsymbol{p},z)$$
 (4)

The first term $m_{HHSIZE}(z_h)$ incorporates the varying costs of children and the economies of scale enjoyed by large households. The economies of scale of household size are likely to be significant, see Jorgenson and Slesnick (1987) and Nelson (1998), however are normally specified a priori or ignored. The scale is specified to have a base of a single adult living alone in the base price period such that the scale measures the number of adult 'equivalent persons' living alone, EP. It is defined as

$$m_{HHSIZE} = EP = (n_a + \mathbf{k}_1 n_{k1} + \mathbf{k}_2 n_{k2} + \mathbf{k}_3 n_{k3})^{(1-q)}$$
 (5)

- where k's represent their corresponding resource cost for the three dependent categories, as a proportion of an adult and
 - \mathbf{q} reflects the economies of scale in household size, $\mathbf{q}=0$ indicating no economies of scale.

The second term $m_{HHCOMP}(\boldsymbol{p},z_h)$ captures the effect of household composition price effects via the interaction with prices. Most of the composition effects of the relative cost of adults and different aged dependents has been captured in their size effects in $m_{HHSIZE}(z_h)$ in scaling household expenditure. Thus these sperate effects can't be captured well by $m_{HHCOMP}(\boldsymbol{p},z_h)$. Thus the household composition effects, in light of the specification of $m_{HHSIZE}(z_h)$, are based upon the total number of dependents in the household to capture the effect that they have in shifting a household's budget shares for particular goods in addition to the effects they have of scaling/sharing

total expenditure. Thus the household composition scale is specified,

$$m_{HHCOMP}(\boldsymbol{p}, \boldsymbol{z_h}) = \begin{cases} \mathbf{\hat{c}} \widetilde{\boldsymbol{O}}_{g}^{N_s} p_g^{\mathbf{h}_g} \ddot{\mathbf{\hat{c}}}^{n_k} \\ \mathbf{\hat{c}} g = 1 \end{cases} p_g^{\mathbf{h}_g} \ddot{\mathbf{\hat{c}}}^{n_k}$$

$$(6)$$

where p_g is the price of each good g = 1 to N_g ,

 n_k is the total number of dependents, and

 ${\pmb h}_g$ the parameters to be estimated that have the effect of shifting the budget share demands by ${\pmb h}_g$ for every dependent and ${\dot {\pmb a}}_g {\pmb h}_g = 0$.

In the reference period when all prices are unity then $m_{HHCOMP}(\boldsymbol{p}, \boldsymbol{z_h}) = 1$ and prices do not affect the household equivalence scale. Thus the household equivalence scale is specified in full as:

$$m_{\mathrm{HH}}(\boldsymbol{p}, \boldsymbol{z}_h) = (n_a + \boldsymbol{k}_1 n_{k1} + \boldsymbol{k}_2 n_{k2} + \boldsymbol{k}_3 n_{k3})^{(1-\boldsymbol{q})} \boldsymbol{\xi} \boldsymbol{\tilde{O}}_{\boldsymbol{\xi}} \boldsymbol{\tilde{O}}_{\boldsymbol{g}=1}^{\boldsymbol{N}_s} \boldsymbol{\tilde{o}}_{\boldsymbol{g}}^{\boldsymbol{n}_k}$$
(7)

Which allows the PS-QAIDS budget shares tom be written,

$$s_i = \boldsymbol{h}_i n_k + \boldsymbol{a}_i + \dot{\boldsymbol{a}}_g g_{ig} \log p_g + \boldsymbol{b}_i \log \widetilde{x} + \boldsymbol{I}_i \widetilde{O}_g p_g^{\boldsymbol{I}_g - \boldsymbol{b}_g} \log \widetilde{x}^2$$
(8)

where
$$\log \tilde{x} = \log x - a(p) - \log(n_a + \mathbf{k}_1 n_{k1} + \mathbf{k}_2 n_{k2} + \mathbf{k}_3 n_{k3})^{(1-\mathbf{q})} - n_k \dot{\mathbf{a}} \mathbf{h}_g \log p_g$$
 (9)

3 Cost of Living Index and Prices

Nominal variables need to be divided by a price index for comparisons under different price levels. The CPI series constructed by the Australian Bureau of Statistics (ABS) and Statistics Canada are fixed weighted averages of goods and services where the weights are the budget shares for a working family household, typically one full-time earner, spouse and two children. Using fixed

weights, does not explicitly consider consumer preferences, through the substitution effects of price changes and price-demographic effects. A suitably specified cost of living index allows for such effects.

A cost of living index (CLI), is measured by the ratio of the cost of obtaining a base period level of utility, u_0 at future prices, p_1 , with given household demographics, z, over the cost of the base period level of utility at base level prices, p_0 with the given household composition.

$$CLI = \frac{c(u_0, \boldsymbol{p}_I, \boldsymbol{z})}{c(u_0, \boldsymbol{p}_\theta, \boldsymbol{z})}$$
(10)

The PS-QAIDS model used this study has cost function

$$c_{h}(u, \boldsymbol{p}, \boldsymbol{z}) = c_{R}(u, \boldsymbol{p}) \cdot m_{PS}(\boldsymbol{p}, \boldsymbol{z})$$

$$= \exp \stackrel{e}{\boldsymbol{e}} a(\boldsymbol{p}) + \frac{ub(\boldsymbol{p})}{1 - uc(\boldsymbol{p})} \stackrel{u}{\boldsymbol{u}} \cdot (n_{a} + \boldsymbol{k}_{1}n_{k1} + \boldsymbol{k}_{2}n_{k2} + \boldsymbol{k}_{3}n_{k3})^{(1-\boldsymbol{q})} \stackrel{ae}{\boldsymbol{e}} \stackrel{N_{g}}{\overset{o}{\boldsymbol{e}}} \stackrel{\ddot{\boldsymbol{p}}^{n_{k}}}{\overset{\dot{\boldsymbol{e}}}{\boldsymbol{e}}} \stackrel{\ddot{\boldsymbol{p}}^{n_{k}}}{\overset{\dot{\boldsymbol{e}}}{\boldsymbol{e}}} \stackrel{\ddot{\boldsymbol{e}}^{n_{k}}}{\overset{\dot{\boldsymbol{e}}}{\boldsymbol{e}}}$$

$$(11)$$

Thus the TCLI may be written as

$$CLI = \frac{\exp \stackrel{e}{\hat{\mathbf{e}}} a(\mathbf{p}_{I}) + \frac{u_{0}b(\mathbf{p}_{I})}{1 - u_{0}c(\mathbf{p}_{I})\mathring{\mathbf{u}}} \stackrel{\hat{\mathbf{u}}}{\hat{\mathbf{u}}} \stackrel{\hat{\mathbf{u}}}{\hat{\mathbf{u}}} (n_{a} + \mathbf{k}_{1}n_{k1} + \mathbf{k}_{2}n_{k2} + \mathbf{k}_{3}n_{k3})^{(1-q)} \stackrel{\mathbf{w}}{\hat{\mathbf{v}}} \stackrel{\hat{\mathbf{u}}}{\hat{\mathbf{v}}} \stackrel{\hat{\mathbf{u}}}{\hat{\mathbf{v}}} \stackrel{\hat{\mathbf{u}}}{\hat{\mathbf{v}}} \stackrel{\hat{\mathbf{u}}}{\hat{\mathbf{v}}} \frac{\hat{\mathbf{u}}}{\hat{\mathbf{v}}} \stackrel{\hat{\mathbf{u}}}{\hat{\mathbf{v}}} \frac{\hat{\mathbf{u}}}{\hat{\mathbf{v}}} \stackrel{\hat{\mathbf{u}}}{\hat{\mathbf{v}}} \frac{\hat{\mathbf{u}}}{\hat{\mathbf{v}}} \frac{\hat{\mathbf{u}}}{\hat{\mathbf{v}}} \stackrel{\hat{\mathbf{u}}}{\hat{\mathbf{v}}} \stackrel{\hat{\mathbf{u}}}{\hat{\mathbf{v}}}} \stackrel{\hat{\mathbf{u}}}{\hat{\mathbf{v}}} \stackrel{\hat{\mathbf{u}}}{\hat{\mathbf{v}}}} \stackrel{\hat{\mathbf{u}}}{\hat{\mathbf{v}}} \stackrel{\hat{\mathbf{u}}}{\hat{\mathbf{v}}} \stackrel{\hat{\mathbf{u}}}{\hat{\mathbf{v}}} \stackrel{\hat{\mathbf{u}}}{\hat{\mathbf{v}}} \stackrel{\hat{\mathbf{u}}}{\hat{\mathbf{v}}} \stackrel{\hat{\mathbf{u}}}{\hat{\mathbf{v}}} \stackrel{\hat{\mathbf{u}}}{\hat{\mathbf{v}}} \stackrel{\hat{\mathbf{u}}}{\hat{\mathbf{v}}} \stackrel{\hat{\mathbf{u}}}{\hat{\mathbf{v}}} \stackrel{\hat{\mathbf{u}}}}$$

The base level of utility u_0 can be obtained as a function of prices, demographics and expenditure by using the PS-QAIDS indirect utility function. The base level of utility is given by

$$u_0 = \log \mathbf{y}\left(x_0, \mathbf{p}_{\boldsymbol{\theta}}\right) = \frac{\log x_0^{rs}}{b(\mathbf{p}_{\boldsymbol{\theta}}) + c(\mathbf{p}_{\boldsymbol{\theta}}) \log x_0^{rs}}$$
(13)

where the scaled natural log of expenditure in the base period is given by, $\log x_0^{rs} = \log x_0 - a(p_0) - \log EP - (EP - 1) \sum_k \mathbf{d}_k \log p_{0_k}.$ Substituting in the level of base level of utility into the CLI function gives

$$CLI = \exp \frac{\hat{\mathbf{e}}}{\hat{\mathbf{e}}} a(\mathbf{p}_{I}) - a(\mathbf{p}_{\theta}) + \frac{b(\mathbf{p}_{I}) \log x_{0}^{rs}}{b(\mathbf{p}_{\theta}) + (c(\mathbf{p}_{\theta}) - c(\mathbf{p}_{I})) \log x_{0}^{rs}} - \log x_{0}^{rs} \mathring{\mathbf{u}} \cdot \overset{\bullet}{\mathbf{O}} \overset{\bullet}{\mathbf{e}} \frac{\mathbf{p}_{I}}{p_{0}} \mathring{\mathbf{e}} \frac{\mathring{\mathbf{e}}}{p_{0}} \frac{\mathring{\mathbf{e}}}{\mathring{\mathbf{e}}} g^{n_{k}}$$

$$= \exp \hat{\mathbf{e}} a(\mathbf{p}_{I}) - a(\mathbf{p}_{\theta}) + \frac{(b(\mathbf{p}_{I}) - b(\mathbf{p}_{\theta})) \log x_{0}^{rs} + (c(\mathbf{p}_{I}) - c(\mathbf{p}_{\theta})) (\log x_{0}^{rs})^{2} \mathring{\mathbf{u}}}{b(\mathbf{p}_{\theta}) - (c(\mathbf{p}_{I}) - c(\mathbf{p}_{\theta})) \log x_{0}^{rs}} \mathring{\mathbf{e}} \overset{\bullet}{\mathbf{u}} \cdot \overset{\bullet}{\mathbf{O}} \overset{\bullet}{\mathbf{e}} \frac{\mathbf{p}_{Ig}}{p_{0g}} \mathring{\mathbf{e}} \overset{\bullet}{\mathring{\mathbf{e}}} \overset{\bullet}{\mathbf{e}}$$

$$(14)$$

If in the base period all prices are unity then

$$a(p_0) = \mathbf{a}_0$$
 which is specified as zero in this study

$$b(p_0) = 1$$

$$c(p_0) = 1$$

and the CLI can be more easily written as

$$CLI = \exp \hat{\mathbf{e}}_{\hat{\mathbf{e}}}^{\hat{\mathbf{e}}}(\boldsymbol{p}_{1}) + \frac{(b(\boldsymbol{p}_{1}) - 1)\log x_{0}^{rs} + (c(\boldsymbol{p}_{1}) - 1)(\log x_{0}^{rs})^{2}}{1 - (c(\boldsymbol{p}_{1}) - 1)\log x_{0}^{rs}} \hat{\mathbf{u}} \cdot \tilde{\mathbf{o}}_{g} (\boldsymbol{p}_{1g})^{\mathbf{d}_{g} n_{k}}$$

$$(15)$$

The CLI for the PS-QAIDS model when the base period prices are unity, after some manipulation, can be given by

$$CLI = \exp \frac{\hat{\mathbf{e}}}{\hat{\mathbf{e}}} a(\mathbf{p}_{I}) + \frac{(b(\mathbf{p}_{I}) - 1)\log \tilde{x}_{0} + (c(\mathbf{p}_{I}) - 1)(\log \tilde{x}_{0})^{2} \hat{\mathbf{u}}}{1 - (c(\mathbf{p}_{I}) - 1)\log \tilde{x}_{0}} \hat{\mathbf{u}} \cdot \tilde{\mathbf{O}}_{g} \left(p_{1g}\right)^{\mathbf{d}_{g} n_{k}}$$

$$(16)$$

3.1 Effect of Price Changes on Measures of Welfare

Data on income and more recently expenditure is the most readily available source for an indicator of an individual's or household's level of welfare. To take account of price and demographic variation, the measure of welfare, w, is usually scaled by a price index P and

equivalence scale m to provide a real equivalent measure \widetilde{w} .

$$\widetilde{w} = \frac{w}{P \, m} \tag{17}$$

The effect of price changes on real equivalent welfare can be analysed through price elasticity of welfare with respect to good i, given by

$$e^{\widetilde{W}}_{p_{i}} = \frac{\int \widetilde{w}/\widetilde{w}}{\int p_{i}/p_{i}}$$

$$= \frac{\int w/(Pm)}{\int p_{i}} \frac{p_{i}}{\widetilde{w}}$$

$$= -\frac{\mathcal{E}}{\mathcal{E}} \int p_{i} \frac{1}{P} + \frac{\int m}{\int p_{i}} \frac{1}{m} \frac{\ddot{o}}{\dot{o}} \frac{\widetilde{w}}{1} \frac{p_{i}}{\widetilde{w}}$$

$$= -\frac{\mathcal{E}}{\mathcal{E}} \int p_{i} \frac{p_{i}}{P} + \frac{\int m}{\int p_{i}} \frac{p_{i}}{m} \frac{\ddot{o}}{\dot{o}} \frac{\ddot{o}}{m}$$

$$= -\frac{\mathcal{E}}{\mathcal{E}} \int p_{i} \frac{p_{i}}{P} + \frac{\int m}{\int p_{i}} \frac{p_{i}}{m} \frac{\ddot{o}}{\dot{o}} \frac{\ddot{o}}{m}$$

$$= -\frac{\mathcal{E}}{\mathcal{E}} \int p_{i} \frac{p_{i}}{P} + \frac{\mathcal{E}}{P} \frac{m}{P} \frac{p_{i}}{m} \frac{\ddot{o}}{\dot{o}} \frac{\ddot{o}}{m}$$

$$= -\frac{\mathcal{E}}{\mathcal{E}} \int p_{i} \frac{p_{i}}{P} + \frac{\mathcal{E}}{P} \frac{p_{i}}{P} \frac{\ddot{o}}{m} \frac{\ddot{o}}{\dot{o}} \frac{\ddot{o}}{m}$$

$$= -\frac{\mathcal{E}}{\mathcal{E}} \int p_{i} \frac{p_{i}}{P} + \frac{\mathcal{E}}{P} \frac{p_{i}}{P} \frac{\ddot{o}}{m} \frac{\ddot{o}}{$$

which is the negative of the sum of the price elasticity of the price index P and equivalence scale m. If there are no price demographic effects in the equivalence scale or if they are incorporated into the price index then the elasticity may be written simply as the negative of price elasticity of the price index P.

For a fixed weight price index, such as the CPI, the price elasticity of welfare with respect to good i is equal to the product of the weighting given to i, and the price of good i relative to the price index,

$$e_{p_i}^{\widetilde{W}} = weight_i \frac{p_i}{P}$$
 (19)

With fixed price weights, the elasticity of welfare with respect to good g is constant across households and does not allow for income, substitution or demographic effects

Using the QAIDS CLI and incorporating the price-child effects of specified the equivalence

scale allows the effect of price movements on households to vary across expenditure levels and the number of children. The construction of the price elasticity of the CLI,

$$e_{p_i}^{CLI} = \frac{\P CLI/CLI}{\P p_i/p_i} = \frac{\P \log CLI}{\P p_i} \frac{p_i}{1}$$
 (20)

is aided by specifying the log of the CLI

$$\log CLI = a(\boldsymbol{p}_{1}) + \frac{(b(\boldsymbol{p}_{1}) - 1) \left[\log \widetilde{x}_{0}\right] + (c(\boldsymbol{p}_{1}) - 1) \left[\log \widetilde{x}_{0}\right]^{2}}{1 - (c(\boldsymbol{p}_{1}) - 1) \left[\log \widetilde{x}_{0}\right]} + n_{k} \dot{\boldsymbol{a}} \boldsymbol{d}_{g} \log p_{1g}$$

$$(21)$$

Differentiating the above and multiplying through by p_i gives the elasticity of the CLI with the respect to the price of good i in three parts,

$$e_{p_i}^{CLI} = (I) + (II) + (III)$$
(22)

where

$$(I) = \mathbf{a}_i + \dot{\mathbf{a}} \mathbf{g}_{ij} \log p_j \tag{22a}$$

$$(II) = \frac{\mathbf{b}_i b(P_1) \log \tilde{x}_0}{1 - (c(P_1) - 1) \log \tilde{x}_0} + \frac{\mathbf{I}_i c(P_1) b(P_1) (\log \tilde{x}_0)^2}{[1 - (c(P_1) - 1) \log \tilde{x}_0]^2}$$
(22b)

$$(III) = \mathbf{d}_i n_k \tag{22c}$$

and $\log \tilde{x}_0$, is the real equivalent expenditure in the base period.

The first component of the CLI elasticity (I) can considered the standard income \mathbf{a}_i and substitution $\mathbf{a}_i \mathbf{g}_{ij} \log p_j$ price effects that are invariant to demographics or household expenditure and shall be termed the 'fixed cost' effect. The second effect (II) is the 'utility' effect of price movements that give the impact of prices of households of varying levels of base level expenditure. The third effect (III) is the 'demographic' effect that prices have on households with children.

For infinitely small changes in prices the effects of a change in prices in the base period when all prices are unity the price elasticity of the CLI simplifies to the budget shares for PS-QAIDS in the base period,

$$e_{p_i}^{CLI} = \boldsymbol{a}_i + \left(\boldsymbol{b}_i \log \tilde{x}_0 + \boldsymbol{I}_i \log \tilde{x}_0^2\right) + \boldsymbol{d}_i n_k$$
 (23)

Although this simplification ignores all substitution effects it allows the examination of a change in prices from the base period upon the CLI and thus measures of welfare. The best estimate of base period expenditure for a household that exists outside the base period is provided by

$$\log \tilde{x}_0 = \log x - a(\mathbf{p}) - \log(n_a + \mathbf{k}_1 n_{k1} + \mathbf{k}_2 n_{k2} + \mathbf{k}_3 n_{k3})^{(1-\mathbf{q})} - n_k \dot{\mathbf{a}} \mathbf{h}_g \log p_g$$
(24)

4 The Data and Estimation

4.1 The Data

The data used to estimate the PS-QAIDS for Australia is based on a pooled cross section of the 1975-76, 1984, 1988-89, 1993-94 and 1998-99 Household Expenditure Survey (HES) to provide 32,541 observations on household expenditure and demographic data. This data was combined with broad level price indices by state derived from the ABS's quarterly CPI series. The Canadian data is similarly based upon a pooled cross section of the 1978, 1982, 1986 and 1992 Family Expenditure Survey (FES) from Statistics Canada (SC) and combined with their quarterly CPI series by province.

To aid in the estimation of demand systems goods need to be aggregated into broad expenditure categories. Expenditure has been divided amongst $N_g = 9$ categories in this study, specified in Table 4.1. The Appendix contains tables A.4.1 and A.4.2 which give the expenditure

categories used in terms of the ABS's HES and CPI categories and SC's FES and CPI categories.

Table 4.1 Broad Expenditure Goods

Broad Expenditure Goods	CODE
Food and Non Alcoholic Beverages	FOOD
Accommodation	ACCOM
Electricity and Household Fuel	POWER
Clothing and Footwear	CLOTH
Transport	TRANS
Health and Personal Care	HEALTH
Alcohol and Tobacco	ALCT
Recreation	REC
Miscellaneous and Education	MISC

While the HES and FES are similar in their nature and coverage, they differ in their definition of some variables, including child age categories. Table 4.2 contains the child/dependent age categories used in the specification and estimation of the demographically scaled QAIDS.

Table 4.2 Child/Dependent Categories

Child/Dependent Categories					
		HES	FES (1982, 1986,1992)		
Young Children	n_{kl}	children under 5 years ^a	children under 4 years		
Children	n_{k2}	children 5 to 14 years ^a	children 4 to 15 years		
Dependents (Students)	n_{k3}	dependents 15 to 24 years	persons 16 to 17 years ^a		
Total Children (and dependents)		$n_k = n_{kl+} n_{k2+} n_{k3}$			

Notes: a Used by the 1978 FES

This paper examines variation of the price elasticity of the CLI and hence measures of welfare when deflated by it using 5 levels of base period expenditure based upon the mean and standard deviation of the logarithm of base period expenditure per week, from the 1993-94 HES for

Australia and the 1992 FES for Canada. The five expenditure classes are defined in terms of the mean and standard deviation of the logarithm of the real equivalent expenditure in Table 4.3. Since the distribution of expenditure is skewed and approximately log-normal, the categories may be interpreted as their percentiles from the normal distribution

Table 4.3 Expenditure Classes

Expenditure Class	Definition	Percentile if $\log \tilde{x} \sim N$	Australian Real (1989/90 \$'s) Equivalent Weekly Expenditure 1993/94	Canadian Real (1989/90 \$'s) Equivalent Weekly Expenditure 1992
Very Low	$mean(\log \tilde{x})$ - $2std.dev(\log \tilde{x})$	2.5%	\$103.07	\$125.88
Low	$mean(\log \widetilde{x})$ - $1std.dev(\log \widetilde{x})$	16%	\$179.15	\$203.44
Average	$mean(\log \widetilde{x})$	50%	\$311.38	\$328.81
High	$mean(\log \widetilde{x}) + 1std.dev(\log \widetilde{x})$	84%	\$541.23	\$531.43
Very High	$mean(\log \tilde{x}) + 2std.dev(\log \tilde{x})$	97.5%	\$940.74	\$858.92

5 Results

5.1 Base Price Effects upon the PS-QAIDS CLI and Real Welfare

Table 5.1 and 5.2 show the elasticity of the estimated CLI for Australia and Canada respectively, for 5 expenditure levels. If measures of welfare were to be converted to real measures using the CLI, then the elasticities in Tables 5.1 and 5.2 give the negative of the elasticity of real welfare. For example the elasticity of food for an Australian Household with average real equivalent expenditure, is 0.19, implying that a 1% increase in the price of food will lead to a increase of the CLI by 0.19% and so reduce a real measures of welfare by 0.19%

For both countries there is significant variation in the effect that changes in food prices have upon across real equivalent expenditure. The impact upon households with very low levels of real equivalent expenditure is almost 3 times that for a very high level of real equivalent spending. Even

more dramatic is the variation in the effect of prices on household power for both counties, with the elasticity being only 0.01 for the very high group while approximately 0.05 for the very low group. The effect of rises in the prices of health and personal care products also rises with real expenditure for both countries, but to a much lesser degree than the above

The effect of price rises in accommodation has a large effect across households for all levels of real equivalent expenditure since accommodation consumes a large proportion of the household budget. The effect is greater in Canada since it accommodation spending as a proportion of spending is generally larger. Note that the rank-3 demand system allows for goods to change from necessities to luxuries and back again across levels as expenditure changes, as evident in Australia, with households with a very low or high level of real equivalent expenditure, spending a greater share of their budget on accommodation.

Table 5.1 Australian Price Elasticity of the CLI in the Base Period across Real Equivalent Expenditure

	Australian Real Equivalent Expenditure				
	Very Low	Low	Average	High	Very High
Broad Commodity Group	\$103	\$179	\$311	\$541	\$941
Food and Non Alcoholic Beverages	0.30	0.24	0.19	0.14	0.09
Accommodation	0.27	0.26	0.26	0.26	0.28
Electricity and Household Fuel	0.06	0.04	0.02	0.02	0.01
Clothing and Footwear	0.04	0.05	0.06	0.07	0.07
Transport	0.09	0.13	0.16	0.19	0.21
Health and Personal Care	0.07	0.07	0.07	0.06	0.04
Alcohol and Tobacco	0.07	0.10	0.12	0.14	0.16
Recreation	0.05	0.06	0.06	0.05	0.03
Miscellaneous and Education	0.04	0.05	0.06	0.08	0.11

Note that the very low, low, average, high and very high real equivalent expenditure in base period price, 1988-89 are based upon the 1993-94 HES using the PS-QAIDS a(*p*) price term and equivalence scale.

Table 5.2 Canadian Price Elasticity of the CLI in the Base Period across Real Equivalent Expenditure

			Canadian		
	Real Equivalent Expenditure				e
	Very				Very
	Low	Low	Average	High	High
Broad Commodity Group	\$126	\$203	\$329	\$531	\$859
Food and Non Alcoholic Beverages	0.29	0.24	0.19	0.15	0.11
Accommodation	0.35	0.34	0.32	0.31	0.30
Electricity and Household Fuel	0.10	0.07	0.05	0.03	0.02
Clothing and Footwear	0.04	0.05	0.06	0.07	0.07
Transport	0.06	0.11	0.15	0.19	0.24
Health and Personal Care	0.06	0.06	0.06	0.05	0.05
Alcohol and Tobacco	0.04	0.05	0.07	0.09	0.10
Recreation	0.04	0.05	0.05	0.05	0.05
Miscellaneous and Education	0.02	0.03	0.04	0.05	0.06

Note that the very low, low, average, high and very high real equivalent expenditure in base period price, 1988-89 are based upon the 1992 FES using the PS-QAIDS a(p) price term and equivalence scale.

The effect of price rises upon clothing and footwear, transport, alcohol and tobacco, and miscellaneous and education rises as real equivalent expenditure increases for both Australia and Canada. The elasticities for transport and alcohol and tobacco, vary considerably across spending levels, especially for Canada. The elasticity of the CLI with respect to recreation is relatively constant across real equivalent expenditure for Canada but for Australia, changes in the price of recreation impact most heavily upon households with low and average real equivalent expenditure.

Tables 5.3 and 5.4 show how price changes impact upon households of differing demographics in respect to the number of children for Australia and Canada respectively. The effect of price changes for households with children, on whole is not that much different to a household without children. This is probably due to the broad commodity grouping specified. Further disaggregation of the commodity groups may allow greater child-price effects to be identified in the

demand system estimation. Not surprisingly the most significant impact of children is upon food expenditure in a household. The elasticity of the CLI with respect to food is approximately 0.01 higher per child for Australia and Canada. Also consistent for both Australia and Canada is the decline in the impact of price rises in recreation for households with children

Table 5.3 Demographic Variations in the Australian Price Elasticity of the CLI

	Non Demographic Effect for 'Average' Reference HH	Demographic Effect per child (III)	Price I 'Avera	+(II)+(III) Effect for age' HH vith
Broad Commodity Group	(I)+(II)		1 child	2 children
Food and Non Alcoholic Beverages	0.19	0.009	0.20	0.21
Accommodation	0.26	-0.002	0.25	0.25
Electricity and Household Fuel	0.02	0.000	0.02	0.03
Clothing and Footwear	0.06	0.003	0.07	0.07
Transport	0.16	-0.002	0.15	0.15
Health and Personal Care	0.07	-0.004	0.06	0.06
Alcohol and Tobacco	0.12	-0.004	0.12	0.11
Recreation	0.06	-0.006	0.05	0.05
Miscellaneous and Education	0.06	0.005	0.07	0.07

Table 5.4 Demographic Variations in the Canadian Price Elasticity of the CLI

	Non Demographic Effect for 'Average' Reference HH	Demographic Effect per child (III)	Price I 'Avera	+(II)+(III) Effect for age' HH vith
Broad Commodity Group	(I)+(II)		1 child	2 children
Food and Non Alcoholic Beverages	0.19	0.012	0.20	0.22
Accommodation	0.32	-0.004	0.32	0.31
Electricity and Household Fuel	0.05	0.001	0.05	0.05
Clothing and Footwear	0.06	0.004	0.06	0.07
Transport	0.15	-0.008	0.14	0.13
Health and Personal Care	0.06	0.000	0.06	0.06
Alcohol and Tobacco	0.07	0.000	0.07	0.07
Recreation	0.05	-0.004	0.05	0.04
Miscellaneous and Education	0.04	0.000	0.04	0.04

6 Conclusions

This paper has constructed a CLI based upon a PS-QAIDS, which varies across real expenditure levels and demographics. By differentiating the CLI with respect to price, the price elasticity of the CLI can be obtained and used to assess the implications of price changes on real measures of welfare. Price changes upon households of differing levels of real equivalent expenditure vary significantly for most of the nine broad commodity groups used. Rises in the price of food have a significant impact on real welfare which varies considerably with the impact on very poor households being almost three times than that for very rich households. Changes in the price of accommodation, also has a large effect but does not vary much over a household's level of real equivalent spending. The effect of price rises upon clothing and footwear, transport, alcohol and tobacco, and miscellaneous and education rises as real equivalent expenditure increases for both Australia and Canada. Households with children are more affected by changes in the price of food than average households but less effected by changes in the price of recreation. The childdemographic effects were quantitatively small possibly due to the broad commodity groups used. This study illustrates that there are differing effects of price rises across households, especially across differing levels of real equivalent expenditure. Using a fixed weight index price index, such as the CPI, to adjust for price changes in a household's measure of welfare, is likely to involve significant bias depending on the levels of real equivalent expenditure of the households in question.

Appendix

Table A.4.1 Expenditure Category Specification in terms of the HES and CPI groups

Broad Expenditure Goods	\boldsymbol{G}	HES (ABS) expenditure categories	CPI (ABS) expenditure categories
Food and Non Alcoholic Beverages (FOOD)	1	Food And Non Alcoholic Beverages	Food
Accommodation (ACCOM)	2	Current Housing Costs, Household Furnishings and Household Services & Operation	Housing (less Electricity & Fuel), Household Furnishings, Supplies and Services,
Electricity and Household Fuel (POWER)	3	Domestic Fuel and Power	Electricity & Fuel (Sub-Group)
Clothing and Footwear (CLOTH)	4	Clothing & Footwear	Clothing and Footwear
Transport (TRANS)	5	Transport	Transportation
Health and Personal Care (HEALTH)	6	Medical Care & Health Expenses and Personal Care	Health
Alcohol and Tobacco (ALCT)	7	Alcoholic Beverages and Tobacco Products	Alcohol; Tobacco
Recreation (REC)	8	Recreation	Recreation
Miscellaneous and Education (MISC)	9	Miscellaneous Goods & Services	Education and Miscellaneous

Table A.4.2 Expenditure Category Specification in terms of the FES and CPI groups

Broad Expenditure Goods	G	FES (SC) expenditure categories	CPI (SC) expenditure categories
Food and Non Alcoholic Beverages (FOOD)	1	Food	Food
Accommodation (ACCOM)	2	Shelter; less Electricity and Fuel; Household Operation; Household Furnishings and Equipment	Owned Accommodation; Rented Accommodation; less Water, Fuel and Electricity; Household Operations and Furnishings
Electricity and Household Fuel (POWER)	3	Electricity and Fuel	Water, Fuel and Electricity
Clothing and Footwear (CLOTH)	4	Clothing	Clothing and Footwear
Transport (TRANS)	5	Transportation	Transportation
Health and Personal Care (HEALTH)	6	Health Care and Personal Care	Health Care and Personal Care
Alcohol and Tobacco (ALCT)	7	Tobacco Products & Alcoholic Beverages	Alcoholic Beverages and Tobacco Products
Recreation (REC)	8	Recreation	Recreation
Miscellaneous and Education (MISC)	9	Education; Reading Materials and other Printed Matter; Miscellaneous;	Education and Reading; All-Items

Table A.4.3 Equivalence Scale Estimates

Equivalence	AUS	CAN
Scale Parameters	PS-QAIDS	PS-QAIDS
\mathbf{k}_1	0.2937 (.0331)	0.1675 (.0347)
\mathbf{k}_{2}	0.4481 (.0332)	0.4057 (.0347)
\mathbf{k}_3	0.6074 (.0472)	0.5045 (.0561)
q	0.3700 (.0063)	0.4846 (.0054)

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