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Hugh Sibly

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Hugh Sibly
School of Economics and Finance
University of Tasmania
Private Bag 85
Hobart Tas 7001

Email: HSIBLY@POSTOFFICE.UTAS.EDU.AU
Ph: (03) 62 262825
Fax: (03) 62 267587
(IDD code +61+3+)

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ABSTRACT

This paper considers a monopolist that conducts vertical product differentiation. Previous analyses that assume customers have unit demand or firms conduct non-linear pricing. In contrast to these studies customers purchase multiple units at a linear price. Customers differ in their income and preferences, particularly their willingness to substitute between quantity and quality. The model distinguishes those aspects of customer demand that are sources of vertical differentiation (income and preferences) from those aspects that cause quality distortion. It is demonstrated that under uniform ordering vertical differentiation only causes quality distortion when consumer demand is such that there is a material difference in the mark-up of different varieties. Under non-uniform ordering a variety of patterns of quality distortion are possible.

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* I would like to thank Ann Marsden for useful comments on an earlier draft. All errors remain my responsibility.

Vertical Product Differentiation with Linear Pricing

Vertical product differentiation is profitable for firms when its customers agree on desirability of a products characteristics, or quality, but differ in their valuation of these characteristics. If customer groups (or types) are readily identified the monopolist can utilise third degree price discrimination. However if customer types cannot be readily identified, monopolists can use product differentiation as a 'self selection' mechanism to identify customers' groups. The received wisdom on the impact of vertical (quality) differentiation follows the pioneering analysis of Mussa and Rosen (1978) (MR). MR consider customer that have a unit demand for a good. Increases in quality linearly increase the willingness to pay for that unit. Customer differ in the extent to which increases in quality increases the willingness to pay. In that analysis vertical differentiation leads to inefficiently low quality in all but the highest quality variety. The spectrum of quality levels is predicted to widen as a result.

The unit demand model introduced by MR allows for a straightforward analysis of vertically differentiated markets. Thus it has become imbedded in the subsequent literature.[1] The unit demand model might be defended as a useful approximation to more general cases. However it is not immediately obvious whether the unit demand model is approximating linear or non-linear pricing. Maskin and Riley (1984) show that there is a direct correspondence between non-linear pricing of quantity and vertical differentiation in the unit demand model. In addition, the requirement in the unit demand model that consumer utility is cardinal suggests that MR's analysis is closer to an analysis of non-linear pricing than linear pricing. Subsequent literature modelling vertical differentiation focuses on non-linear pricing (Armstrong, 1996, Armstrong and Rochet, 1999, Rochet and Choné, 1998, Rochet and Stole, 2002, Sibley and Srinagesh, 1997, Stole, 1995).

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[1] For example the recent textbooks by Carlton and Perloff (2005), Church and Ware (2000) and Pepall, Richards and Norman (2005) each describe the MR analysis as the outcome of vertical product differentiation.

In practice, firms use linear pricing is often used to sell vertically differentiated goods. For example restaurants often vertically differentiate their meals and drinks, though few provide a discount for multiple purchases. A great many grocery items also are priced in this way. For instance bakers often sell multiple varieties (of differing quality levels) of bread loaf. Supermarkets are selling vertically differentiated goods under their own (private) labels (Dobson and Chakraborty, 2008). Coffee shops sell cups of 'fair trade' and 'standard' coffee. Indeed many of the items discussed in the classic literature of goods quality – light globes and razor blades – are also sold using liner pricing. (Even when these goods are bundled, the extent to which non-linear pricing used is quite limited. For example a customer might be forced to choose between 1 or 6 light globes.) Recalling Dupuit's classic example (see Ekelund, 1970), railways set a linear price for first and economy class tickets. Similarly airlines set linear prices for first, business and economy class tickets.

Do low quality varieties have sub-optimal quality under linear pricing (as might be thought by extrapolating the results of MR) or, rather, are they fit for purpose? To answer this question, this paper introduces a model consumers purchase multiple units at a linear price. The two commonly cited reasons why customers differ in their demand for quality are explicitly modelled: differences in preferences and income. Customers are divided into 'types' that depends on their preference for quality and income. Specifically customers within a type have a common substitutability of quantity for quality and a common hourly wage rate. (Customer types with a high substitutability of quantity for quality are those whose demand is most influenced by quality changes.) Each customer has an elasticity of demand (elasticity of quantity with respect to price) which need not be related their substitutability of quantity for quality. There need not be a common elasticity of demand within a type; the model allows for the possibility that demand elasticities may vary across customer within a type. The customer type's elasticity of demand is the weighted sum of the individual customers' elasticities of demand within the type.

Under uniform ordering high income types are also the types with a higher preference for quality. Vertical differentiation results in the high (low) quality varieties being purchased by customer types with relatively high (low) substitutability of quantity for quality and/or high (low) hourly wage. When the firm can costlessly identify customer types (third degree price discrimination) then, under the assumptions of the model, it provides the efficient quality (though not quantity) for each variety. When, however, the firm cannot identify a given customer's type, it may distort the quality of a variety to satisfy self-selection. The model is used to find general conditions under which vertical differentiation causes quality distortion under linear pricing and, just as importantly, when it doesn't cause quality distortion. It is shown that under uniform ordering this mechanism only causes quality distortion if there is sufficient difference in the mark-up on different varieties. (The precise definition of differences in the mark-up of varieties is given in this paper.) In particular a variety has distorted quality if its customers have a relatively low mark-up. This only occurs if that variety's customers have relatively high elasticity of demand or relatively low wage. If the mark-up on all varieties does not differ greatly, then no variety exhibits quality distortion.

Non-uniform ordering occurs when high-income types are the types with a relatively low preference for quality. It does not appear to be possible to make many general claims about quality distortion under non-uniform ordering. However it is demonstrated that a variety with a high equilibrium quality level may be one with a low efficient quality level under non-uniform ordering. In addition, when there are two customer types both may purchase a single variety. This type of 'bunching' only occurs under uniform ordering when (i) there are three or more customer types and (ii) one of the 'middle' demand types are readily unprofitable (usually because there are a low number of such customer types). Neither of these requirements is necessary for bunching to occur under non-uniform ordering.

Section 1 of the paper models consumers' substitutability of quantity for quality and elasticity of demand. Section 2 analyses a firm that can conduct third degree price discrimination. This analysis is used as a benchmark for the analysis of a firm that undertakes vertical differentiation. This latter analysis is undertaken in section 3. The cases in which there are two and three types of customers are studied in detail. Section 4 concludes the paper.

1. Customers

1.1 The Substitutability of Quantity for Quality

The product under consideration can be vertically differentiated along a continuum y. The measure y is called quality of a unit of the good. Different varieties or models of the product can be produced which differ purely in their quality levels. The quality of each variety is assumed to be common knowledge and is exogenous from the point of view of each consumer.

Assume consumers are divided into 2 types, and let type i have m^i individuals. Customers within a type have a common substitutability of quantity (units) for quality. To capture differences in the substitutability of quantity for quality across customer types, let customer j, who is a member of type i (customer ij), measure their satisfaction of consuming variety k, x^{ijk}, using the relationship $x^{ijk}=X^{ijk}g^i(y)$, where: (i) X^{ijk} is the number of units of variety k consumed and (ii) $g^i(y)$ measures the satisfaction per unit of type i. The elasticity of this customer's "iso-satisfaction curve", $dx^{ijk}=0$, is:

$$\frac{y}{X^{ijk}}\frac{dX^{ijk}}{dy} = -\varepsilon^i_{gy}(y) \tag{1}$$

where $\varepsilon^i_{gy} \equiv y g^{i\prime}(y)/g^i(y)$, i=1,2,..,n, be the elasticity of i's satisfaction per unit with respect to quality. By (1) $\varepsilon^i_{gy}(y)$ also measures the substitutability of quantity and quality. Note this substitutability depends only on customer type. Assume that type 1 consumers are more sensitive to quality than type 2 consumers, i.e. $\varepsilon^1_{gy}(y)>\varepsilon^2_{gy}(y)$ for all y. Assuming $g^{i\prime\prime}(y)<0$ ensures that customer's indifference curves are convex.

1.2 Consumer Demand and its Elasticity

In this section consumer utility is specified, and used to identify the monopolist's demand from each customer type. First it is shown that customer ij purchases variety i. Then, by determining customer ij's demand for variety i, the total demand for each variety is specified.

Suppose that there are 2 varieties. This ensures that there are sufficient varieties to target one variety at each customer type. Let variety k have quality of y^k. Customers take the satisfaction provided by each variety as perfect substitutes for one another. The utility function of customer ij is $U^{ij}(g^i(y^1)X^{ij1}+g^i(y^2)X^{ij2},L^{ij})$, where L^{ij} is the 'other activity', which is will be called 'leisure', undertaken by the customer. The model allows for preference differences within a customer type in order to allow customers to differ in their elasticity of demand. Consumption of the each unit takes (non-leisure) time of a hours. Each consumer has T hours available. Consumer ij's budget constraint is thus:

$$\sum_{k=1}^{2} P^k X^{ijk} = w^i(T-L^{ij}-a\sum_{k=1}^{2} X^{ijk}) + I^i \qquad (2)$$

where P^k is the price per unit of variety k, w^i is the hourly wage rate and I^i is the non-labour income of type i customers. Under 'uniform ordering' $w^1>w^2$ and under 'non-uniform' ordering $w^2>w^1$.

The customer ij's maximisation function is:

$$\text{Max } U^{ij}(\sum_{k=1}^{2} g^i(y^k)X^{ijk},L^{ij}) \text{ subject to: } \sum_{k=1}^{2}\left(\frac{P^k+aw^i}{g^i(y^k)}\right)g^i(y^k)X^{ijk} + w^iL^{ij} = w^iT + I^i \qquad (3)$$

The price of satisfaction of variety k to customer ij is defined as $(P^k+aw^i)/g^i(y^k)$. As customers treat the satisfaction provided by each variety as perfect substitutes, the customer chooses the variety with the lowest price of satisfaction. Assume that (as will be justified below) it is variety i is that type i customers find has the lowest unit price of satisfaction. Thus type i will purchase variety i exclusively. Then the consumer ij's optimisation problem becomes:

$$\text{Max } U^{ij}(x^{ij},L^{ij}) \text{ subject to } p^ix^{ij}/w^i + L^{ij} = T+I^i/w^i \qquad (4)$$

where $x^{ij} \equiv g^i(y^i)X^{iji}$ is the satisfaction provided to the customer by variety i, and $p^i \equiv (P^i + aw^i)/g^i(y^i)$ is the price of satisfaction of variety i.

It is natural to consider the consumer problem in the formulation (4) rather than (3). In the formulation (4), satisfaction is the "good" which the consumer must forgo to consume leisure. All type i customers face the common prices and income p^i, w^i, and I^i. Thus standard consumer theory can be used to determine the demand for satisfaction and demand for units from type i customers as:

$$x^i(p^i/w^i, I^i/w^i) = \sum_{j=1}^{m^i} x^{ij}\,(p^i/w^i, I^i/w^i)$$

$$\Leftrightarrow X^i(P^i, g^i(y^i), w^i, I^i/w^i) = \sum_{j=1}^{m^i} \frac{1}{g^i(y^i)}\, x^{ij}\left(\frac{P^i + aw^i}{w^i g^i(y^i)}, I^i/w^i\right) \qquad (5)$$

where reference to T is suppressed as it is common to all customer types. Type i's elasticity of demand is given by:

$$\varepsilon_x^i(p^i/w^i, I^i/w^i) \equiv -\,(p^i/x^i)(\partial x^i\, \partial p^i) = -\,(p^i/(w^i x^i)).x_1^i(p^i/w^i, I^i/w^i) = ((P^i + aw^i)/P^i)E_X^i \quad (6)$$

where $E_X^i \equiv -(P^i/X^i)(\partial X^i/\partial P^i)$. Each consumer, j, of type i has an elasticity of demand that is defined analogously to (6), and these elasticities may differ across consumers of type i. The elasticity, ε_x^i, is the weighted sum of the individual type i consumers' elasticities of demand. The elasticity of demand is usually measured empirically by E_X^i, however it is ε_x^i that accurately represents the preferences of consumers. Thus it is ε_x^i that is used in the analysis below to represent the elasticity of demand of each consumer type.

2. Third Degree Price Discrimination

This section considers a firm that can costlessly identify customer type. By offering members of a particular customer type only one variety the firm can undertake third degree price discrimination. This analysis proves necessary to develop and interpret the model presented in the next section, in which customers self select the variety they purchase. The firm may choose to produce different quality levels to sell to each type of customer. A common technology, summarised by the cost function $C(X^i,y^i)$, is used to produce each variety. The firm's cost of producing a variety increases with the number of units produced and the quality. More specifically, the cost function is assumed to take the often-adopted 'constant returns to scale' form:

$$C(X^i,y^i) = X^i\psi(y^i) \tag{7}$$

where $\psi'(y) > 0$ and $\psi''(y) > 0$. Note that, for simplicity it is assumed there are no fixed cost, and no economies of scope across varieties. Further this enables the analysis in the next section to focus on the role of customer self selection on the distortion of quality.

A distinct price per unit, P^i, is charged to members of customer type i. The profit of the firm, π, is:

$$\pi = \sum_{i=1}^{2}[P^iX^i - X^i\psi(y^i)] = \sum_{i=1}^{2}[p^ix^i - f^i(x^i,y^i)] \tag{8}$$

where, using (5), $f^i(x^i,y^i) \equiv aw^ix^i/g^i(y) + x^i\psi(y)/g^i(y)$ and where x^i is defined by (5). It is useful to interpret $f^i(x^i,y^i)$ as the total cost (i.e. cost to customer and producer) of providing satisfaction of x^i to type i customers as a function of quality. The minimum cost of providing x^i to type i customers, $y^i_e(x^i)$, is determined by:

$$y^i_e(x^i) = \underset{y^i}{\operatorname{argmin}}\ f^i(x^i,y^i) \tag{9}$$

The following proposition follows directly from the first order conditions of (8).

Proposition 1: Under third degree price discrimination the profit maximising quality minimises the cost of providing the profit maximising satisfaction to each type. Specifically quality is given by (9). The level of satisfaction of each customer type is given by:

$$p_t^i = \frac{(aw^i+\psi(y_e^i))/g^i(y_e^i)}{1-(1/\varepsilon_x^i)} \Leftrightarrow P_t^i = \frac{aw^i}{\varepsilon_x^i-1} + \frac{\psi(y_e^i)}{1-(1/\varepsilon_x^i)} \tag{10}$$

where p_t^i is the monopolist's price of satisfaction, and P_t^i is the monopolist's price per unit, under third degree price discrimination.

To identify the relationship between cost minimising quality and customer type, consider the first order condition of (9):

$$f_2^i(x,y) = -X^i(g^{i\prime}(y)/g^i(y)^2)(aw^i + \psi(y)) + X^i\psi'(y^i)/g^i(y) = 0 \tag{11}$$

or equivalently:

$$\varepsilon_{\psi y}(y_e^i)[\psi(y_e^i)/(aw^i + \psi(y_e^i))] = \varepsilon_{gy}^i(y_e^i) \tag{12}$$

where $\varepsilon_{\psi y}(y) \equiv y\psi'(y)/\psi(y)$. Note that (12) is a function of y_e^i alone, and thus cost minimising quality is independent of the number of units produced. This is a requirement of Swan invariance. Equation (12) shows that cost minimising quality of a customer type is determined by their substitution of quantity for quality and their hourly wage rate. The following result follows from (12) and the adoption of the uniform ordering condition:

Lemma 1: $y_e^1 > y_e^2$ under uniform ordering.

The proof of this result, and subsequent results, is given in a mathematical appendix. Under third degree price discrimination, the firm therefore sets higher quality for customer types with higher sensitivity to quality changes, or those with higher hourly wages.

Note the critical role played by a, the unit consumption time. A relatively high value of the unit consumption time means that the price per unit, P^i, is a relatively small part of the price of satisfaction. Thus the firm perceives that if faces a relatively inelastic demand curve.[2] Thus a high value of the unit consumption time is associated with a high price, as indicated by (10). Similarly, a higher unit consumption time means that consumers benefit more from quality and, as indicated by (12), will thus demand higher quality. The unit consumption time acts as a disincentive to the substitution of quantity and quality. A high opportunity cost of time (wage) tends to be translated into a higher demand for quality rather than quantity. Again this is indicated by (12).

Note from equation (12) that if a=0 hourly wage differences do yield differences in the cost minimising quality. Thus wage differences do not yield vertical differentiation. Similarly, from (12), the cost minimising quality of a customer type is independent of their non-labour income. Thus differences in non-labour income across types do not play a critical role in vertical product differentiation. Thus, for brevity, it is assumed below that $I^i=0$ for all i.

An increase in the hourly wage of type i customers has a different effect on the price per unit, P^i_t , than an increase in the substitutability of quantity for quality of type i customers. By (12) an increase in the substitutability of quantity for quality (i.e. $\varepsilon^i_{gy}(y^i_e)$) increases quality, y^i_e. Thus marginal cost, $\psi(y^i_e)$, is increased and by (10) so is P^i_t. An increase in w^i also increases y^i_e and thus marginal cost. However the increase in wage has an additional effect. The increase in w^i means that the price per unit is a relatively smaller part of the price of satisfaction. Thus the firm perceives a reduction in the elasticity of its demand curve. It thus increases price. This effect is represented by the first term on the RHS of (10).

In the next section it is necessary to understand the differing affects of an increase in the hourly wage and the substitutability of quantity for quality on p^i_t. To assess these impacts first observe that from (10):

$$(p^i_t/w^i)(1-(1/\varepsilon^i_x(p^i_t/w^i)) \ = (aw^i+\psi(y^i_e))/(w^i g^i(y^i_e)) \qquad (13)$$

[2] The firm will perceive the elasticity of the demand curve it faces as $[P^i/(P^i+aw^i)]\varepsilon^i_x$. The greater is the term aw^i the more inelastic is the demand curve.

Consider an increase in the substitutability of quantity for quality, $\varepsilon^i_{gy}(y^i_e)$, that leaves $g^i(y^i_e)$ unaffected. By (12) the quality, y^i_e, is increased. However, as y^i_e is the cost minimising quality (i.e. satisfies (9)), the change in the RHS of (13) is negligible. Hence there is no change in p^i_t/w^i, and thus p^i_t and $x^i(p^i_t/w^i)$. However by noting that $X^i=x^i(p^i_t/w^i)/g^i(y^i_e)$, the level of quantity falls. Intuitively, an increase in the substitutability of quantity for quality does not influence the total level of satisfaction, but rather causes substitution of quality for quantity.

From (13) an increase in w^i lowers p^i_t/w^i. Thus an increase in the wage increases the demand for satisfaction. However the quality also increases. Thus the effect of a wage increase on the equilibrium quantity is ambiguous. A wage increase raises (lowers) the equilibrium number of units when the increased demand for satisfaction (does not) outweighs the substitution of quality for quantity.

From (10) an increase in w^i has two effects on p^i_t. The first is that it increases type i customers' demand and thus p^i_t. The second effect is that it reduces the relative price p^i_t/w^i. When the elasticity of demand is increasing in p^i_t/w^i both these effects act to increase p^i_t . However when the elasticity of demand is decreasing in p^i_t/w^i both these effects act in opposite directions. The following lemma summarises the net effect of an increase in w^i on p^i_t :

<u>Lemma 2</u>: Let $\eta^i_x(p_t/w)\equiv\left(\frac{p_t}{w}\right)\left(\frac{\varepsilon^{i\prime}_x(p_t/w)}{\varepsilon^i_x(p_t/w)}\right)$. Then the elasticity of the third degree discriminating price is given by:

$$\left(\frac{p^i_t}{w^i}\right)\left(\frac{dp^i_t}{dw^i}\right)=\frac{\eta^i_x(p^i_t/w^i)+\dfrac{aw^i\varepsilon^i_x(p^i_t/w^i)}{p^i_t g^i(y^i_e)}}{\eta^i_x(p^i_t/w^i)+\dfrac{[aw^i+\psi(y^i_e)]\varepsilon^i_x(p^i_t/w^i)}{p^i_t g^i(y^i_e)}} \tag{14}$$

The second order conditions require that the denominator of the RHS is positive, so that an increase in w^i increases p^i_t provided:

$$\eta^i_x(p^i_t/w^i) > -\frac{aw^i\varepsilon^i_x(p^i_t/w^i)}{p^i_t g^i(y^i_e)} \tag{15}$$

The term $\eta^i_x(p^i_t/w^i)$ represents the rate at which the elasticity of demand is increasing with p^i_t/w^i. The expression $-\eta^i_x(p^i_t/w^i)$may thus be viewed as representing the extent of concavity of the demand curve. Lemma 2 indicates that an increase in w^i increases p^i_t unless the demand curve is sufficiently concave. In particular if $\overline{\eta}(p^i_t/w^i)\geq 0$ then an increase in w^i increases p^i_t. From (14) that the dp^i_t/dw^i is associated with a higher value of $\eta^i_x(p^i_t/w^i)$.

The efficient level of satisfaction and quality are those that maximise the total surplus. They correspond with the satisfaction level and quality of the third degree price discriminating firm that faces perfectly elastic demand from all customer types. Hence:

Proposition 2: The social welfare maximising firm chooses the efficient quality, given by (12), and the efficient satisfaction level is given by:

$$p^i_e = (aw^i+\psi(y^i_e))/g^i(y^i_e) \Leftrightarrow P^i_e = \psi(y^i_e) \tag{16}$$

where p^i_e is the efficient price of satisfaction and P^i_e is the efficient price per unit. Equation (16) states that the efficient firm provides satisfaction and (equivalently) units so that price equals marginal cost. Propositions 1 and 2 collectively imply that Swan Invariance holds under third degree price discrimination. That is, the third degree price discriminating firm produces the efficient quality for each variety.

3. Self Selection of Varieties

3.1 Profit Maximisation with Self Selection

Suppose the firm cannot identify customer types. Type i customers elect to purchase variety i if it has a lower price of satisfaction than any other variety, specifically if:

$$(P^i+aw^i)/g^i(y^i) \leq (P^j+aw^i)/g^i(y^j) \Leftrightarrow p^i \leq p^j\theta^{ji}(y^j) + a(w^i-w^j)/g^i(y^j) \qquad (17)$$

for $j\neq i$, where $\theta^{ji}(y) \equiv g^j(y)/g^i(y)$. Note $\theta^{ji\prime}(y) < (>) 0$ when $j>(<)i$. Note that $p^j\theta^{ji}(y^j)$ is the price of satisfaction of variety j from type i customers' perspective. The term $a(w^i-w^j)/g^i(y^j)$ measures the time cost to type i customers of purchasing a unit of satisfaction from variety j. It is useful to note the following result:

<u>Lemma 3</u>: The efficient quantity (equivalently satisfaction) and quality satisfies the self-selection constraints.

Note that this result holds under both uniform and non-uniform ordering.

When customers can self select the firm's optimisation problem is:

$$\max_{p^i\, y^i} \pi \text{ s.t. } p^i \leq p^j\theta^{ji}(y^j)+ a(w^i-w^j)/g^i(y^j) \text{ for } i=1,2 \text{ and } j\neq i. \qquad (18)$$

The Lagrangian for this optimisation problem is:

$$L = \sum_{i=1}^{2} [p^ix^i - f^i(x^i,y^i)+\sum_{j\neq i}\lambda_{ij}(p^j\theta^{ji}(y^j)+ a(w^i-w^j)/g^i(y^j) -p^i)] \qquad (19)$$

where λ_{ij} are the Lagrange multipliers. A self-selection constraint is binding if $\lambda_{ij}>0$ and is not binding if $\lambda_{ij}=0$. The first order conditions of (19) yield:

$$p^1(1-(1/\varepsilon_x^1)) = (aw^1+\psi(y^1))/g^1(y^1) + w^1[\lambda_{12}-\lambda_{21}\theta^{12}(y^1)]/x^{1\prime}(p^1/w^1) \qquad (22)$$

$$p^2(1-(1/\varepsilon_x^2)) = (aw^2+\psi(y^2))/g^2(y^2) + w^2[\lambda_{21} - \lambda_{12}\theta^{21}(y^2))]/x^{2\prime}(p^2/w^2) \tag{23}$$

$$f_2^1(x^1,y^1) = \lambda_{21}[\theta^{12\prime}(y^1)p^1 + a(w^1-w^2)g^{2\prime}(y^1)/g^2(y^1)^2] \tag{24}$$

$$f_2^2(x^2,y^2) = \lambda_{12}[\theta^{21\prime}(y^2)p^2 - a(w^1-w^2)g^{1\prime}(y^2)/g^1(y^2)^2] \tag{25}$$

and the self-selection constraints are:

$$p^1 \leq s(p^2,y^2) \tag{26}$$

and

$$p^1 \geq s(p^2,y^1) \tag{27}$$

where $s(p^2,y) \equiv \theta^{21}(y)p^2 + a(w^1-w^2)/g^1(y)$. Note that under uniform ordering $s_2(p^2,y)<0$.

3.2 Uniform Ordering

This sub-section considers the firm's optimal pricing when customers exhibit uniform ordering and can self select varieties. The following proposition broadly describes the impact of self-selection of varieties on price and quality.

Proposition 3: Under uniform ordering:

(i) both varieties' quality cannot be simultaneously distorted,

(ii) variety 1 can exhibit either undistorted or upwardly distorted quality

(iii) if variety 1's quality is distorted, then its equilibrium price of satisfaction, p_d^1, is greater than under third degree price discrimination, i.e. $p_d^1 > p_t^1(y_e^1)$. Further the price of satisfaction of variety 2, p_d^2, is less than under third degree price discrimination, i.e. $p_d^2 < p_t^2(y_e^2)$,

(iv) variety 2 can exhibit either undistorted or downwardly distorted quality.

(v) if variety 2 has distorted quality, then $p_d^1 < p_t^1(y_e^1)$ and $p_d^2 > p_t^2(y_e^2)$.

According to proposition 3, it is possible that neither of the self-selection constraints is binding or, alternatively, that only one of the self-selection constraints is binding.

Consider the case when neither self-selection constraint is binding. This requires $\lambda_{12}=\lambda_{21}=0$, and thus (22)-(25) are equivalent to (10) and (12). That is, the price and quality of each variety is identical to that chosen under third degree price discrimination. In particular the firm sets the cost minimising qualities y_e^1 and y_e^2. Figure 1 can be used to illustrate this possibility. Observe that type 1 customers correctly self select if the point representing the profit maximising prices of satisfaction lies below the line $p^1 = s(p^2,y_e^2)$. Type 2 customer correctly self select if the point representing the profit maximising prices of satisfaction lies above the line $p^1 = s(p^2,y_e^1)$. An example of this possibility is price of satisfaction combination represented by the point A in figure 1.

Quality distortion occurs when third degree price discrimination is not consistent with the self-selection constraints. Suppose, for example, the third degree price discriminating prices of satisfaction is as represented by point C in figure 1. In this case the third degree discriminating price of satisfaction of variety 2 is high relative to that of variety 1. If the quality of variety 1 were undistorted (i.e. $y^1=y_e^1$), the self-selection constraint (27) would not hold: type 2 customers would prefer to purchase variety 1. One way to satisfy the self-selection constraint would be to sufficiently raise the quality of variety 1, while simultaneously adjusting the prices of a unit of varieties 1 and 2 to maintain their prices of satisfaction. Graphically this would be represented by a shift downward of the line $p^1=s(p^2,y^1)$ in figure 1, until it cut the point C.[3] This action raises the price of satisfaction of variety 1 to type 2 customers, because type 2 customers are less sensitive to quality increases than type 1 customers. A sufficient increase in the quality of variety 1 would deter type 2 customers from purchasing variety 1. Alternatively the self-selection constraint could be satisfied if the price of satisfaction (or equivalently the price of a unit) of variety 1 was raised, or the price of satisfaction of variety 2 was lowered. The profit maximising trade off between these three actions occurs at point D in figure 1, where the quality of variety 1 is y_d^1 (and the self selection constraint is thus represented by the line $p^1= s(p^2,y_d^1)$), the price of satisfaction of variety 1 is $p_t^1(y_d^1)$ and the price of variety 2 is p_d^2.

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[3] Note that $s_2(p^2,y^i)<0$, so an increase in the quality shifts the curve $p^1=s(p^2,y^i)$ downward.

Figure 1 can be used to show why proposition 3(ii) holds: specifically why, when the self-selection constraint (27) is binding ($\lambda_{21}>0$, $\lambda_{12}=0$), the price of satisfaction of variety 1 must be less than under third degree price discrimination. Given quality levels $y^1_d > y^1_e$ and y^2_e the respective profit maximising prices of satisfaction are $p^1_t(y^1_d)$ and $p^2_t(y^2_e)$. These prices are indicated by the point E in figure 1. Note that these prices do not satisfy the self-selection constraint (27). The prices of satisfaction that maximise profit and satisfy the self-selection constraints are those for which the isoprofit curve, $\overline{\pi}(y^1_d,y^2_e)$, is tangent to the self-selection constraint. Note that the isoprofit curves are vertical along the line $p^1=p^1_t(y^1_d)$ and horizontal along the line $p^2=p^2_t(y^2_e)$. Thus $p^1_d> p^1_t(y^1_e)$ and $p^2_d <p^2_t(y^2_e)$.

The class of third degree discriminating prices of satisfaction represented by point C occurs when the third degree discriminating price of variety 2 is relatively high compared to variety 1. In equilibrium the self-selection constraints are “downwardly” binding and variety 1 is upwardly distorted. Similarly, the self-selection constraints also do not hold for the class of third degree discriminating prices of satisfaction represented by point B in figure 1. In this example the third degree discriminating price of satisfaction of variety 1 is relatively high compared to variety 2. The self-selection constraints are “upwardly” binding, and variety 2 exhibits downwardly distorted quality.

Before moving on to analyse the determinants of quality distortion, it is worthwhile considering the relationship between distortion under linear pricing (as represented by proposition 3) and that predicted by the unit demand model. MR finds that, in the unit demand model, customer self-selection is to broaden the spectrum of quality levels produced when there is a uniform ordering of the absolute and marginal willingness to pay for quality (single crossing property). Proposition 3(i) indicates that under uniform ordering the spectrum of quality levels is not narrowed. MR also finds that the low quality variety is distorted downward. Proposition 3(iv) indicates that the low quality variety (variety 2) does not have upwardly distorted quality. Proposition 3(v) indicates that type 1 customers’ gain (relative to third degree price discrimination) in those instances when variety 2 is downwardly distorted. This is consistent with the findings of MR. However type 2 customers are strictly worse off, which contrasts with MR. (In MR’s analysis type 2 customers are always on their participation constraint.)

Furthermore in MR's analysis the price per unit of variety 2 is lower than third degree price discrimination. This need not be the case under linear pricing.

In MR's analysis the high quality variety does not exhibit quality distortion. MR's analysis (in the case of two customer types) is extended by Donnenfeld and White (1988), Donnenfeld and White (1990), Srinagesh and Bradburd (1989) and Srinagesh Bradburd and Koo (1992), who modify the utility function adopted by MR. While they retain the assumption of unit demand from all customers, they make the willingness to pay of customers nonlinearly dependent on quality. By adopting particular functional forms for the willingness to pay, these models can generate more general patterns of quality distortion than found by MR. In particular Donnenfeld and White (1988) and Srinagesh and Bradburd (1989) shows that, in the unit demand model, quality may be upwardly distorted in *some* instances when the absolute and marginal willingness to pay for quality are inversely ordered. (In this event the single crossing property does not hold.) Proposition 3(ii) also shows that the high quality variety can be upwardly distorted. Figure 1 suggests this occurs when the third degree discriminating price of variety 1 is sufficiently low.

The following proposition identifies the relative mark-up as the determinant of quality distortion.

Proposition 4: If variety j has (un) distorted quality then (i) under third degree price discriminating prices and quality $\mu^i(\varepsilon_x^j, w^i, w^j, y_e^i, y_e^j) > (\leq) \chi^i(\varepsilon_x^i, \varepsilon_x^j, w^i, w^j, y_e^i, y_e^j)$ for $i \neq j$, and (ii) in equilibrium $\mu^i(\varepsilon_x^j, w^i, w^j, y_d^i, y_d^j) > (\leq) \chi^i(\varepsilon_x^i, \varepsilon_x^j, w^i, w^j, y_d^i, y_d^j)$ for $i \neq j$, , where:

$$\mu^i(\varepsilon_x^j, w^i, w^j, y^j) \equiv \frac{a(w^i - w^j)\varepsilon_x^j}{g^i(y^j)(\varepsilon_x^j - 1)} - \frac{a(w^i - w^j)}{g^i(y^j)} = \frac{a(w^i - w^j)}{g^i(y^j)(\varepsilon_x^j - 1)} \tag{28}$$

and:

$$\chi^i(\varepsilon_x^i, \varepsilon_x^j, w^i, w^j, y^i, y^j) \equiv \left[\frac{(aw^i + \psi(y^j))\varepsilon_x^j}{g^i(y^j)(\varepsilon_x^j - 1)} - \frac{(aw^i + \psi(y^i))\varepsilon_x^i}{g^i(y^i)(\varepsilon_x^i - 1)}\right] \tag{29}$$

Broadly speaking, proposition 4 states that variety j exhibits quality distortion if the mark-up on variety i is relatively high compared to variety j. To explain this, it is necessary to provide interpretations of the terms μ^i and χ^i. The expression χ^i consists of the difference in two terms, each of which represents a price of satisfaction evaluated from the perspective of type i customers. The second term on the RHS of (29) simply represents the price of satisfaction of variety i. The first term represents the price of satisfaction of variety j (from type i's perspective) if type j customers had the same hourly wage as type i customers. This term thus excludes the mark-up that is due to differences in wage levels (as shown by the RHS of (10)). Note that χ^i is always positive when type i and j customers share a common elasticity of demand. This is because the quality of variety i is chosen to minimise (average) cost. Further, χ^i is negative if the elasticity of demand of type i customers is sufficiently greater than that of type j customers.

The expression μ^i captures two effects. The first is the increase in the mark-up on the price of satisfaction of variety i relative to variety j (from the perspective of type i customers). The second effect is time cost to type i customers from purchasing variety j (which is the second term on the RHS of (17)). Note that the first effect dominates in determining the sigh of μ^i. Proposition 4 may be interpreted as saying that variety 1 is distorted if its mark-up is relatively large, due either to wage differences (high χ^1) or if type 1 customers have relatively inelastic demand (low μ^1).

Proposition 4 thus implies that difference in the elasticity of demand between customer types is one source of quality distortion. The following result formalises this.

Corollary 1: Neither variety exhibits distorted quality if the demand of both customer types is sufficiently elastic. If variety j has distorted quality then (i) under third degree price discriminating prices and quality $\varepsilon_x^j > E^i(\varepsilon_x^i, y_e^i, y_e^j)$ for $i \neq j$, and (ii) in equilibrium $\varepsilon_x^j > E^i(\varepsilon_x^i, y_d^i, y_d^j)$ for $i \neq j$, , where:

$$E^i(\varepsilon_x^i, y^i, y^j) = \begin{cases} \dfrac{\eta^{ij}}{\eta^{ij}-1} \text{ if } \eta^{ij}<1 \\ \infty \text{ otherwise} \end{cases} \tag{30}$$

and where $\eta^{ij} \equiv \dfrac{g^i(y^j)[aw^i+\psi(y^i)]\varepsilon_x^i + g^i(y^i)a(w^j-w^i)(\varepsilon_x^i-1)}{g^i(y^i)[aw^j + \psi(y^j)](\varepsilon_x^i-1)}$

If the demand of both customer types is sufficiently elastic, then μ^i is approximately zero. Then, as χ^i is positive, proposition 4 indicates that neither variety exhibits quality distortion. In this event the prices of each variety do not differ greatly from marginal cost. Thus marginal cost pricing satisfies self-selection.

The elasticities of demand in Corollary 1(ii) are evaluated for equilibrium prices of satisfaction and quality. This provides a test for the presence of quality distortion, i.e. a variety has distorted quality if it is observed to have a demand elasticity greater than E^i. If $E^i = 1$ then, as in equilibrium $\varepsilon_x^j > E^i$, variety j necessarily has distorted quality. However if $E^i = \infty$ then variety j cannot exhibit quality distortion.

Corollary 1 shows that the self-selection constraints are not an obstacle to achieving efficiency when customers have a common wage. Provided government policy ensures the firm produces the efficient number of units and quality for each variety, customers will appropriately self select the variety corresponding to their type. Furthermore, under Cournot competition an increase in the elasticity of a firm's residual demand curve occurs with an increase in competition. Thus, to the extent that an increase in the elasticity of demand can be related to increased competition, corollary 1 suggests that quality distortion will not occur in sufficiently competitive markets. Under this interpretation, the allocation under perfect competition (with the elasticity of demand for both types being perfectly elastic) would correspond with the efficient one identified in proposition 2.

To understand the implications of proposition 4 it is first useful to consider the case in which customers differ in their substitutability of quantity for quality but have a common wage. Firstly:

Corollary 2: Suppose $w^1=w^2$. Variety j does not exhibit distorted quality if $\varepsilon_x^i(p_t^i/w^i) \geq \varepsilon_x^j(p_t^j/w^j)$ for $i \neq j$.

If customer types have a common elasticity of demand then neither variety exhibits quality distortion. In this case there is a common mark-up of varieties 1 and 2 because customers have a common wage and elasticity of demand. In this case the third degree discriminating prices of satisfaction of varieties 1 and 2 have the same relative values as the efficient prices. They therefore satisfy the self-selection constraints.

To consider the implications of the differing wages of customer types on quality distortion it is useful to adopt the following the following assumptions on functional form: $x^i(p/w) = \Phi^i(p/w)^{\phi^i}$, $g^i(y) = Gy^{\theta^i}$ and $\psi(y) = \Psi y^{\xi}$, where Φ^i, ϕ^i, G, θ^i Ψ and ξ are positive parameters. Note that $\varepsilon_x^i = \phi^i$, $\varepsilon_{\psi y}= \xi$ and $\varepsilon_{gy}^i= \theta^i$. By assumption $\theta^1 \geq \theta^2$. Under this 'iso-elastic model':

> Corollary 3: Consider the iso-elastic model with $w^1/w^2>1$ and $\theta^1=\theta^2$. (i) For sufficiently large w^1/w^2 neither variety exhibits quality distortion. (ii) Variety 2 has downwardly distorted quality when $\phi^1 \leq \phi^2$ provided w^1/w^2 is sufficiently small. (iii) Variety 1 has upwardly distorted quality if ϕ^1 sufficiently greater than ϕ^2 and w^1/w^2 is sufficiently small. (iv) Variety j does not exhibit quality distortion as $\phi^i \rightarrow \infty$.

An increase in the wage of type 1 consumers relative to type 2 consumers has two effects that are relevant to interpreting corollary 3. First the increase in the wage of type 1 consumers raises the mark-up on variety 1, thus raises μ^1. However it also raises the quality of variety 1, thus causes the price of satisfaction of variety 1 to fall, which raises χ^1. In the isoelastic model the latter effect is greater for large w^1/w^2, while the former effect dominates for w^1/w^2 close to one. Hence there is no quality distortion for large w^1/w^2, while variety 2 is downwardly distorted for w^1/w^2 close to one.

When w^1/w^2 is sufficiently small, μ^i is approximately zero in the isoelastic model. Further more if ϕ^1 is sufficiently larger than ϕ^2 then χ^2 is negative. In this case variety 1 exhibits quality distortion. As the elasticity of demand of type i customers increase, the mark-up of variety i under third degree price discrimination is reduced. Thus χ^i is increased. In the isoelastic model this effect is sufficient to ensure that variety j is undistorted.

Proposition 4 indicates that the relative values of the elasticity of demand is an important determinant of quality distortion. The customer types' elasticities of demands can differ in equilibrium because their demand curves (and thus elasticities) differ or because the elasticities of demand vary along their demand curve. To assess the potential of the latter effect to cause quality distortion it is natural to consider the case in which all customer types have a common demand curve, denoted $\overline{x}(p^i/w^i)$. In this case, difference across types is solely due to either (i) differences in the substitutability of quantity and quality across types or (ii) differences in the hourly wage. In the first of these cases:

Corollary 4: Suppose customer types have a common wage, w, and common demand function. Then if the elasticity of demand is

(i) constant neither variety exhibits distorted quality

(ii) increasing in p_t^i/w then the quality of variety 1 is undistorted and the quality of variety 2 is either undistorted or downwardly distorted

(iii) decreasing in p_t^i/w then the quality of variety 2 is undistorted and the quality of variety 1 is either undistorted or upwardly distorted

When customer types have a common constant elasticity of demand the firm sets a common mark-up over marginal (average) cost for each variety. As noted in corollary 2, with a common mark-up type i customers prefer variety i.

Observe that the third degree price discriminating price of satisfaction of type 1 customers is lower than that of type 2 customers. Thus if elasticity is increasing (decreasing) in relative price, then type 1 (2) customer's elasticity of demand is lower than that of type 2 (1) customers. There is therefore the potential for the quality of variety 2 (1) to be downwardly distorted.

Now consider the case in which customer types differ only in their wages:

Corollary 5: Suppose customer types have a common substitutability of quantity for quality and common demand function. Then:

(i) if the elasticity of demand is non-decreasing in p_t^i/w^i then the quality of variety 2 is downwardly distorted for w^1/w^2 sufficiently close to 1.

(ii) if the elasticity of demand is decreasing sufficiently in p_t^i/w^i so that $dp_t^i/dw^i < 0$, and w^1/w^2 is in the neighbourhood of 1, then the quality of variety 1 is upwardly distorted.

(iii) if demand is perfectly elastic both varieties have undistorted quality.

Corollary 5 indicates that, for small differences in w^1 and w^2, variety 2 exhibits downward quality distortion for a wide class of demand functions. For instance, a linear demand function or any convex demand function would ensure that variety 2 exhibited quality distortion. However variety 1 only exhibits quality distortion when customers possess a highly concave demand function, one that is unlikely to describe many, if any, real markets.

If the elasticity of demand of type 2 customers is relatively elastic, the prices of satisfaction under third degree will be given by a point such as B in figure 1. The quality of the low quality variety, variety 2, will be downwardly distorted. The equilibrium prices of satisfaction would be given by a point such as H in figure 1. In response to an inefficiently low quality, a government might be tempted to impose a minimum quality requirement on producers. If a minimum quality standard of y_e^2 is imposed, producers would face the self-selection constraint $p^1= s(p^2,y_e^2)$. In this event the firm would choose the profit maximising prices of satisfaction along this curve. These prices are given by the point F, where the iso profit curve $\overline{\pi}(y_e^1,y_e^2)$ is tangent to the self-selection constraint. It is possible that the price of satisfaction facing type 2 customers is lower at point H than point F. If this is the case, the imposition of minimum standard regulations would have lowered the welfare of the consumers of the low quality variety.

As noted in corollary 3 if the elasticity of demand is constant there is no quality distortion if w^1/w^2 is sufficiently great. However corollary 5 does not guarantee that quality is not distorted if w^1/w^2 is sufficiently great when the elasticity of demand increasing in p_t^i/w^i. The following corollary indicates why increasing the difference between w^1 and w^2 may not eliminate quality distortion if the elasticity of demand increasing in p_t^i/w^i.

<u>Corollary 6</u>: Consider linear demand, that is $x(p^i/w^i)=A-B(p^i/w^i)$ where A,B>0. Assume also that $y_e^1\rightarrow\infty$ as $w^1\rightarrow\infty$. Then, for sufficiently large w^1/w^2, neither variety (variety 2) is (downwardly) distorted if $a/g(y_e^2)>$ ($<$) A/2B.

An increase in the w^1 (holding w^2 constant) increases p_t^1. The elasticity of demand increases with A/B. Hence the lower is A/B the higher p_t^1 rises for a unit increase in w^1. The time cost to type 1 customers from purchasing variety 2 increases by $a/g(y_e^2)$ for a unit increase in w^1. When $a/g(y_e^2)>A/2B$ then time cost dominates the increase in p_t^1 if w^1 is sufficiently great, and the third degree discriminating price satisfies the self selection constraints. Intuitively, if y_e^2 is too high it is not possible for the increase in w^1 to raise y_e^1 to sufficiently differentiate the varieties so that self-selection holds at the third degree discriminating prices.

The above results suggest that a greater the difference in customer types the greater is the potential for non-distorted quality. When there are both differences in the taste and income of customers, these differences tend to reinforce each other. For instance:

> Corollary 7: Suppose, under uniform ordering, customer types differ in both their substitutability of quantity for quality and wage. Suppose also that customer types have a common constant elasticity of demand. Then neither variety exhibits quality distortion if the elasticity of demand is sufficiently great.

In this case the presence of the heterogeneity in taste differentiates the varieties sufficiently to ensure that the third degree discrimination prices satisfy the self-selection constraints.

An increased difference in either the substitutability of quantity for quality or wage between customer types increases the vertical differentiation of the varieties. The above analysis indicates that, given sufficiently small differences in the elasticity of demand across customer types, this increased differentiation reduces the difference in mark-up across varieties and thus reduces the tendency toward quality distortion. In principle proposition 4 (or subsequent corollaries) could predict whether there is sufficient differentiation to avoid the use of quality distortion to satisfy self-selection. However in practice that approach is almost certainly impractical, given the requirements for data to estimate the elasticity of demand, the substitutability of quantity and quality, and wages. A more straightforward test of whether a variety's quality is distorted under uniform ordering would be to determine whether the self-selection constraint (26) or (27) is binding. Specifically, under uniform ordering, variety 1 would exhibit downwardly distorted quality if type 1 customers are virtually indifferent between purchasing variety 1 and variety 2 and type 2 customers strictly prefer to purchase variety 2.

3.3 Non-uniform Ordering

Under non-uniform ordering the efficient quality level of variety 1 can either be greater or less than that of variety 2: the difference in substitutability of quality for quantity causes y_e^1 to be greater than y_e^2 whereas the difference in wages causes y_e^2 to be greater than y_e^1. The in the quality of the varieties depends on which affect dominates.

The analysis of the case in which $y_e^1 > y_e^2$ is qualitatively similar to the analysis of the previous section. Thus this section focuses on the case in which the wage differences dominates the taste difference and thus it is assumed $y_e^2 > y_e^1$.

<u>Proposition 5</u>: (i) If only variety 1 has distorted quality then $p_d^2 < p_t^2$. If additionally variety 1 exhibits upwardly distorted quality then $p_d^1 > p_t^1$. (ii) If only variety 2 has distorted quality then $p_d^1 > p_t^1$. If additionally variety 2 exhibits upwardly distorted quality then $p_d^2 < p_t^2$.

Proposition 5(i) is illustrated in figure 2. In figure 2 the self-selection constraints with undistorted quality are drawn. Under non-uniform ordering these self-selection constraints cross. As variety 1 has the lower quality the line representing the boundary of the set (27) is steeper than the line representing the boundary of the set (26). Suppose the third degree discriminating prices are represented by the point F. At this point the self-selection constraint (27) does not hold. (Type 2 consumers would purchase variety 1.) When the quality of variety 1 is raised the boundary of (27) rotates clockwise (toward the boundary of (26)). First suppose the quality level is raised to y_d^1. As is the case with uniform ordering, the self-selection constraints can also be satisfied by raising the price of satisfaction of variety 1 and lowering that of variety 2. The profit maximising trade off between these three actions occurs at point D in figure 2.

Note that, because a change in quality rotates the boundary of (27), it may be optimal for the firm to lower the quality of variety 1. In this case the point E in figure 2 would lie below that of point F. The price of equilibrium satisfaction of variety 1 may therefore be lower than the third degree price discriminating price of satisfaction. However equilibrium price of satisfaction of variety 2 remains lower than the third degree price of satisfaction.

There do not appear to be many general conclusions that can be drawn under uniform ordering. It is useful to instead consider the behaviours that can be observed under non-uniform ordering that do not occur under uniform ordering. The iso-elastic model is used to demonstrate these behaviours occur. In the calculations reported below it is assumed that $\Phi^i = G = \Psi = 1$ and $\xi = 1.2$.

As both $w^2 > w^1$ and $y_e^2 > y_e^1$ it would appear that points such as A and F in figure 2 are often representative of the third degree discriminating prices of satisfaction when customer types have a common elasticity of demand. In these cases the self-selection constraint (27) does not hold when $y^1=y^1_e$. In this case the firm distorts quality. However it is possible that either $y^1_d<y^2_d$ or $y^1_d>y^2_d$. In the latter case there is a reversal of the equilibrium ordering of quality from the efficient ordering. The calculations in table 1 and table 2 show that both possibilities can occur. In table 1, $y^2_d>y^1_d$ for $w^2>1.30$, so the equilibrium quality ordering of varieties is that of the efficient ordering. This is the case depicted in figure 2. In table 2 $y^2_d<y^1_d$ for the parameter values for which $y^2_e>y^1_e$, thus a reversal of quality ordering has occurred for these parameter values. This case could be depicted in figure 2 by assuming the quality of variety 1 makes a greater adjustment (and price a lesser adjustment) and the point D would be closer to the point F. In this event the self-selection constraint $p^1=s(p^2,y^1_d)$ would lie below the constraint $p^1=s(p^2,y^2_e)$.

Bunching occurs when both customer types purchase one variety. Under uniform ordering bunching cannot occur when there are fewer than three customer types. With three or more customers a customer type does not warrant a variety of their own if they do not generate sufficient demand. (In the case of the unit demand model this occurs when a particular type has too few members. See Mussa and Rosen. 1978.) However, as is indicated by table 1, bunching can occur with non-uniform ordering with just two customer types and when neither type has particularly low demand. In particular, with the parameter values assumed in table 1, bunching occurs for w^2 = 1.25, and 1.3. In fact bunching occurs over the range $1.21\leq w^2\leq 1.30$. Over this range of w^2 variety 1 is upwardly distorted and variety 2 is downwardly distorted. Intuitively the firm wants to distort the quality of variety 1 upward, and the quality of variety 2 downward in order to satisfy self-selection. The firm does not reverse the ordering of the quality of its varieties in such a case, as this would represent unnecessary distortion of the quality of one of the varieties. Thus it bunches them and produces just one variety.

The calculations in table 2 and table 1 are conducted for a common elasticity of demand. However the analysis of the previous sub-section suggests that differences in elasticity of demand can be important in determining whether quality is distorted. Table 3 reports solutions to the isoleastic model for selected values of ϕ^2 and assuming that $\phi^1=3$, $\theta^1=0.8$, $\theta^2=0.75$, $w^1=1$ and $w^2=1.3$. Figure 3 graphs the efficient and equilibrium quality levels of the varieties in the iso-elastic model as ϕ^2 varies. Bunching occurs in the range

$2.0 \leq \phi^2 \leq 3.1$ for the reasons discussed above. Bunching also occurs for $\phi^2 \geq 14.2$. In this instance bunching occurs because the profitability of type 2 customers is very low, and it is not in the interests of the firm to provide them with a distinct variety. This is the reason bunching occurs in the uniform ordering case.

Over the range $4.1 \leq \phi^2 \leq 5.7$ the two varieties exhibit the efficient quality level. In this case price assumes the burden of satisfying the self-selection constraints. This corresponds to cases in which the prices of satisfaction are analogous to point C in figure 2. Between $3.2 \leq \phi^2 \leq 3.5$ both varieties are downwardly distorted, while for $3.6 \leq \phi^2 \leq 4$ variety 2 is downwardly distorted and variety 1 is undistorted. Variety 2 is downwardly distorted because the third degree discriminating prices of satisfaction can be represented by points such as B in figure 2, and the self-selection constraint can be satisfied by lowering the quality of variety 2.

4. Conclusion

This paper analyses the potential quality distortion that accompanies vertical differentiation. A model is developed in which (in contrast to the existing literature) customers purchase multiple units of the good at a linear price. Vertical differentiation occurs when there is more than one customer type, i.e. groups of customers whose hourly wage rate and/or substitutability of quantity and quality is common within the group but differs across groups. These differences cause customer types to demand different combinations of quality and quantity. However these two sources of vertical differentiation are expressed differently in consumer demand. A higher substitutability of quantity and quality tends to cause customers to demand quality at the expense of quantity. A high wage increases the opportunity cost of time and, because each unit requires time to consumer, increase the demand for quality. (The consequent substitution of quality for quantity is ameliorated by the increased demand for satisfaction accompanying a higher wage.) The higher wage also has the effect of reducing the elasticity of demand perceived by the firm, thus is accompanied by an incentive for the firm to raise price.

The firm responds to the differing demands for quality by customer types by producing a number of varieties, each of which is designed to appeal to a particular customer type. This vertical differentiation alone, however, is not sufficient evidence that there is quality distortion. For instance, if demand is sufficiently elastic no variety exhibits quality distortion. In this case type i customers perceive the price of satisfaction of variety i as the lowest price of satisfaction and hence purchases that variety. The higher quality of variety 1 offsets its higher price per unit for type 1 customers, but not for type 2 customers. An implication of this result is that the self-selection constraints are satisfied in the efficient allocation. Quality distortion, therefore, only occurs when the exercise of market power by the monopolist alters the relative balance of each variety's prices of satisfaction (from that given by efficient levels) in such a way that the self-selection constraints no longer hold.

Under uniform ordering the relative mark-up of varieties is a determinant of quality distortion. Differences in the mark-up of varieties are due to the differences in the elasticity of demand across customer types or differences in the wage across customer types. Much of the detail in the analysis developed in this paper is required to account for the way in which the elasticity of demand may differ across customer types: Differences in the elasticity of demand between customer types could occur either because the demand function differs across types or the elasticity of demand varies along customers' demand curve. However to summarise the findings of the analysis it is useful to consider the case in which two customers types have a common demand curve. Further to rule out variations in the mark-up because of variations in the elasticity of demand along the demand curve it is useful to consider the case in which customers types have a common constant elasticity of demand. If customer types differ only in the substitutability of quantity for quality then no variety exhibits quality distortion as there is a common mark-up on each variety.

Suppose, instead, that customer types have different wages and a common substitutability of quantity and quality. The relatively higher wage of type 1 has two effects. As noted above, it causes type 1 customers to have a higher demand for quality relative to quantity. It also provides an incentive for the firm to set a relatively higher mark-up to type 1 customers than type 2 customers (by increasing the elasticity of demand of the demand curve the firm faces). Consequently wage differences may cause an upwardly violation of the self-selection constraints and thus downward quality distortion of variety 2. When customer types have a common constant elasticity of demand this effect dominates over the separation of types due to quality differences for small differences in the wages of customer types. However when the wage difference is sufficiently large this effect is relatively small and neither variety exhibits quality distortion.

In summary, a striking conclusion from the above analysis is that markets in which consumer are characterised by uniform ordering do not exhibit quality distortion when the firm (using linear prices) sets a common mark-up across varieties. Thus, in contrast to the conclusions of MR, quality distortion need not be – and is unlikely to be – ubiquitous. Indeed a straightforward empirical test for whether any variety exhibits quality distortion is to determine whether the self-selection constraints (26) and (27) are binding. If neither is binding, none of the varieties has distorted quality.

In such empirical investigations, is also necessary to identify whether uniform ordering holds. If it does, the above straightforward conclusions regarding quality distortion can be drawn. If it does not, a wide range of quality distortion patterns are possible. For instance table 3 indicates that it is possible that (in contrast to the uniform ordering case) both self-selection constraints can simultaneously hold. In this event it is possible that either one or both varieties have downward distorted quality. Indeed it is possible that one self-selection constraint holds and the quality of both varieties is undistorted. Furthermore, bunching may occur, in which case there is a sub-optimal number of varieties. In this case both customer types purchase a variety that has, for them, an inefficient quality level. Thus, under non-uniform ordering, it is not straightforward to draw conclusions regarding the distortion of the quality of varieties.

The literature on quality choice is closely related to that on durability choice. In his survey, Waldman (2003) quotes the results of MR as a source of distortion of durability away from its efficient level (specifically low durability goods have inefficiently low durability). However the analysis in this paper indicates that the presence of product varieties with different durability levels need not imply inefficient durability levels. Consider the classic example of light globes. All consumers presumably have a common substitutability of quantity for durability (as all consumers are interested in is the total quantity of light). However high wage consumers would prefer high durability light globes. From the analysis of this paper it can be concluded that the durability of a variety of light globe is only inefficient if there is a material difference in mark-ups between the various varieties with different durability levels. As noted above, this need not be the case.

The analysis of this paper is concerned only with the question of whether vertical product differentiation causes quality distortion. However there are other sources of quality distortion that could be present in real world markets that are not considered by the analysis in this paper. In this paper product quality is assumed common knowledge. Of course, it is well known that asymmetric information about quality can lead to inefficiently low quality (Akerlof, 1970). Second, if the cost function does not exhibit constant returns to scale, Swan invariance does not hold. In this case a monopolist's actions in restricting output of each variety to drive up price will cause the cost minimising quality level to change. Similarly if there are economies of scope in the production of varieties, a distortion in the quality level of one variety may distort the quality level of another variety (Kim and Kim, 1996). Finally, there may be a fixed cost to producing varieties. In this case there may be insufficient customers of a given type to produce a separate variety for them. Customers of different types (characteristics) might purchase a single variety. This type of bunching leads to quality distortion when the elasticity of demand of customer types differ (Sibly, 2007).

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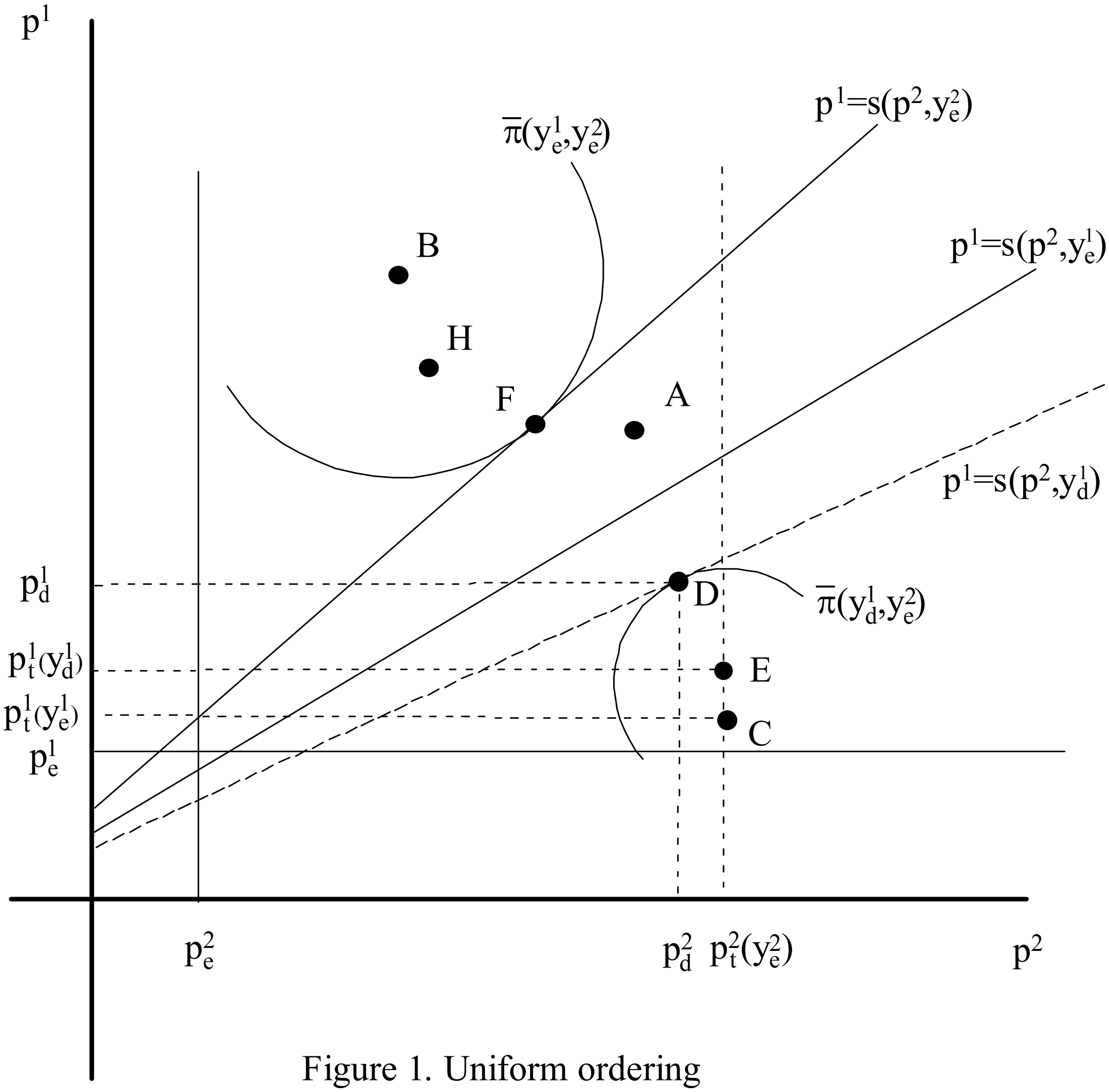


Figure 1. Uniform ordering

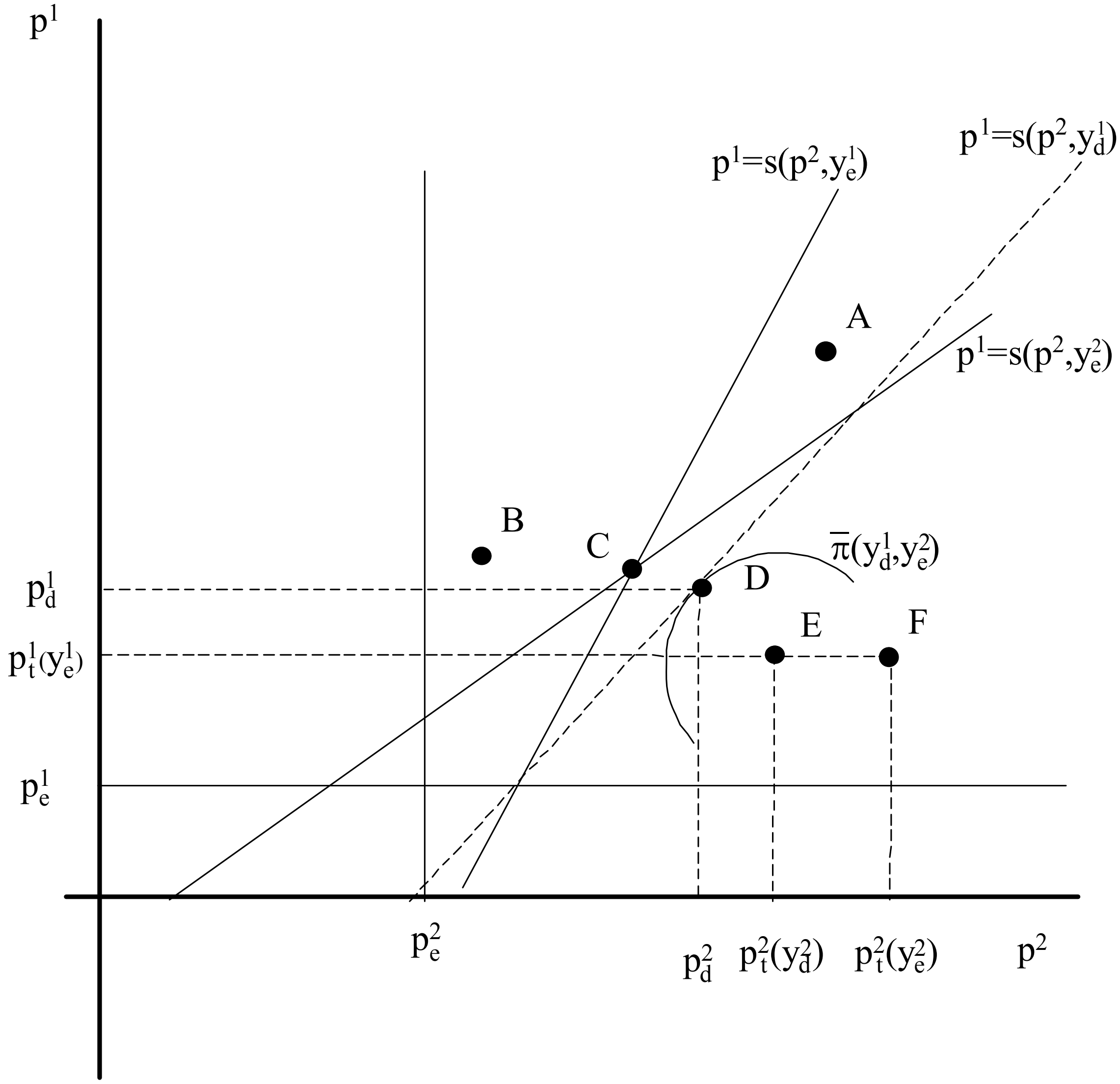
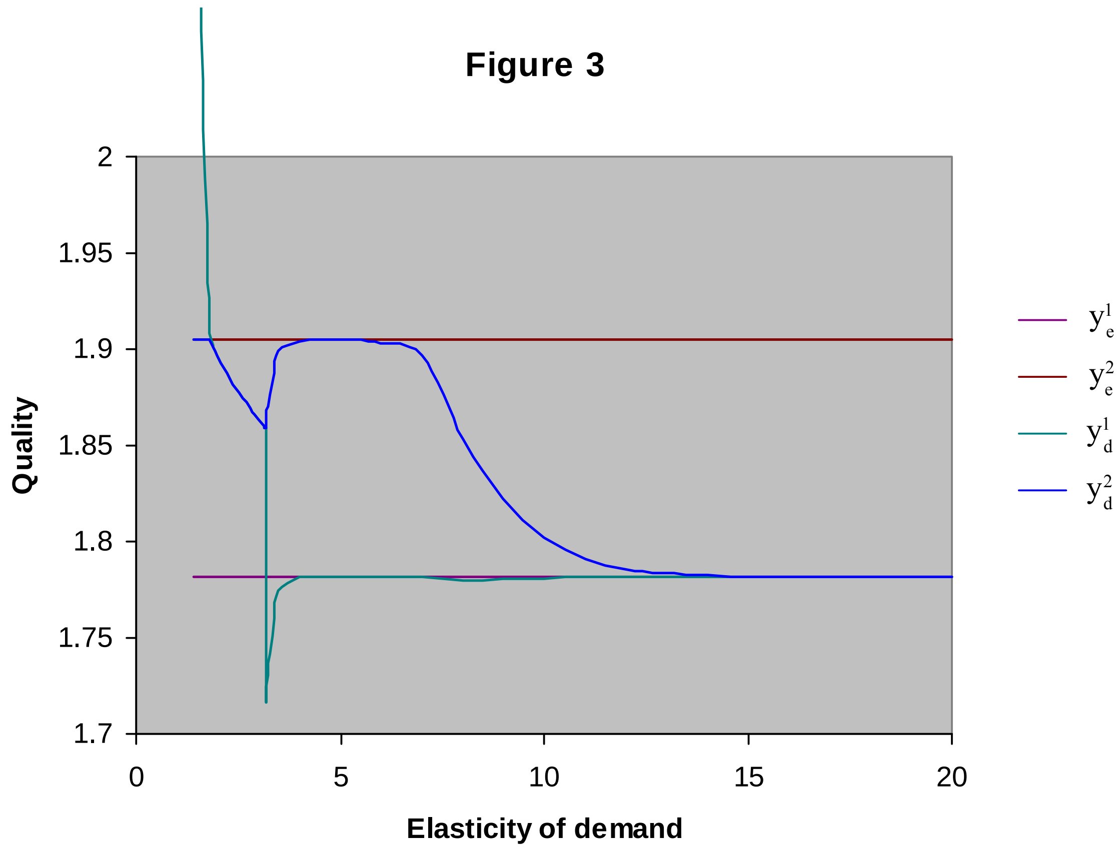


Figure 2.Non-uniform ordering with $y^2_e > y^1_e$

Figure 3



2
1.95
1.9
1.85
1.8
1.75
1.7
Quality
0
5
10
15
20
Elasticity of demand
y_e^1
y_e^2
y_d^1
y_d^2

Table 1: Isoelastic model with ϕ^1=ϕ^1=3, θ^1=0.8, θ^2=0.75, w^1=1							
w^2	1.1	1.2	1.25	1.3	1.31	1.35	1.4
p_t^1	2.835	2.835	2.835	2.835	2.835	2.835	2.835
p_t^2	3.012	3.112	3.16	3.207	3.216	3.253	3.298
p_d^1	2.85	2.871	2.881	2.892	2.887	2.901	2.911
p_d^2	2.999	3.085	3.129	3.172	3.173	3.213	3.256
p_c^1	2.871	2.898	2.912	2.926	2.929	2.941	2.956
p_c^2	2.983	3.048	3.08	3.112	3.119	3.145	3.177
y_e^1	1.782	1.782	1.782	1.782	1.782	1.782	1.782
y_e^2	1.657	1.782	1.843	1.905	1.917	1.966	2.026
y_d^1	1.785	1.785	1.822	1.863	1.739	1.779	1.775
y_d^2	1.657	1.782	1.822	1.863	1.895	1.966	2.026
λ_{12}	0	0	0.019	0.14	0.122	0	0
λ_{21}	0.0005	0.001	0.02	0.137	0.12	0.002	0.002
P^1	3.531	3.565	3.657	3.758	3.494	3.6	3.607
P^2	3.28	3.558	3.657	3.758	3.813	3.984	4.129

Table 2: Isoelastic model with ϕ^1=ϕ^1=3, θ^1=0.8, θ^2=0.4, w^1=1				
w^2	3.5	4	4.5	5
p_t^1	2.835	2.835	2.835	2.835
p_t^2	6.535	7.143	7.727	8.289
p_d^1	3.265	3.388	3.506	3.62
p_d^2	6.442	7.048	7.633	8.199
p_c^1	3.701	3.78	3.858	3.936
p_c^2	5.556	5.953	6.35	6.746
y_e^1	1.782	1.782	1.782	1.782
y_e^2	1.594	1.782	1.966	2.146
y_d^1	2.498	2.681	2.851	3.008
y_d^2	1.594	1.782	1.966	2.146
λ_{12}	0	0	0	0
λ_{21}	0.0046	0.0049	0.005	0.005
P^1	5.79	6.457	7.107	7.736
P^2	4.263	4.88	5.502	6.127

Table 3: Isoelastic model with ϕ^1=3, θ^1=0.8, θ^2=0.75, w^1=1, w^2=1.3												
ϕ^2	1.5	2	2.5	3	3.5	4	5	6	9	12	15	20
p_t^1	2.835	2.835	2.835	2.835	2.835	2.835	2.835	2.835	2.835	2.835	2.835	2.835
p_t^2	6.415	4.276	3.564	3.207	2.993	2.851	2.673	2.566	2.405	2.333	2.291	2.251
p_d^1	5.402	3.648	3.127	2.892	2.768	2.705	2.672	2.703	2.811	2.832	2.835	2.835
p_d^2	5.791	3.952	3.414	3.172	3.044	2.979	2.944	2.976	3.088	3.11	3.112	3.112
p_c^1	6.032	3.962	3.271	2.926	2.719	2.581	2.409	2.305	2.15	2.079	2.039	2
p_c^2	3.112	3.112	3.112	3.112	3.112	3.112	3.112	3.112	3.112	3.112	3.112	3.112
y_e^1	1.782	1.782	1.782	1.782	1.782	1.782	1.782	1.782	1.782	1.782	1.782	1.782
y_e^2	1.905	1.905	1.905	1.905	1.905	1.905	1.905	1.905	1.905	1.905	1.905	1.905
y_d^1	2.29	1.897	1.878	1.863	1.775	1.782	1.782	1.782	1.781	1.782	1.782	1.782
y_d^2	1.905	1.897	1.878	1.863	1.899	1.904	1.905	1.903	1.822	1.786	1.782	1.782
λ_{12}	0	0.007	0.044	0.14	0.031	0.005	0.006	0.005	0.01	0.01	0.0096	0.0096
λ_{21}	0.006	0.016	0.049	0.137	0.028	0	0	0	0.0095	0.0098	0.0094	0.0093
P^1	9.481	5.087	4.177	3.758	3.381	3.295	3.241	3.291	3.46	3.496	3.5	3.5
P^2	8.089	5.087	4.177	3.758	3.623	3.529	3.475	3.523	3.541	3.504	3.5	3.5

Mathematical Appendix : Proofs

Proof of lemma 1: From (11):

$$f_2^i(x,y) = [x/yg^i(y)][\varepsilon_{\psi y}(y)\psi(y) - (aw^i + \psi(y))\varepsilon_{gy}^i(y)]$$

$$< [g^\iota(y)/g^i(y)][x/yg^\iota(y)][\varepsilon_{\psi y}(y)\psi(y) - (aw^\iota + \psi(y))\varepsilon_{gy}^\iota(y) + a(w^\iota - w^i)\varepsilon_{gy}^\iota(y)]$$

$$= [g^\iota(y)/g^i(y)]f_2^\iota(x,y) + a(w^\iota - w^i)x\varepsilon_{gy}^\iota(y)/yg^i(y)$$

Hence $f_2^\iota(x,y_e^1) > 0$ when $f_2^i(x,y_e^\iota) = 0$. Given $f_{22}^i > 0$, this implies $y_e^i > y_e^\iota$. ||

Proof of Proposition 2: Integrating consumer i's demand curve, (5), yields their consumer surplus, $v^{ij}(p^i)$, where $v^{ij}(p^i) = \int_{P^i}^{\infty} X^{ij}(r,y)dr = \int_{p^i}^{\infty} x^{ij}(r)dr$. Consumer i's benefit can be expressed as:

$$B^{ij}(P^i) = \int_{P^i}^{\infty} X^{ij}(r,y)dr + P^i X^{ij}$$

$$= b^{ij}(p^i) - aw^i X^{ij}$$

where $b^{ij}(p^i) = v^{ij}(p^i) + p^i.x^{ij}(p^i)$. Let $b^i(p^i) = \sum_{j=1}^{m^i} b^{ij}(p^i)$. Note that $b^{i\prime}(p^i) = p^i x^{i\prime}(p^i)$. Write:

$$f^i(x^i(p^i),y^i) = \sum_{j=1}^{m^i}[aw^i X^{ij} + X^{ij}\psi(y^i)] = x^i(p^i)[aw^i + \psi(y^i)]/g^i(y^i)$$

The surplus may thus be expressed as:

$$S = \sum_{i=i}^{n}\sum_{i=i}^{m^i}\left(B^{ij}(P^i)-X^{ij}\psi(y^i)\right) = \sum_{i=i}^{n}\left(b^i(p^i)-f^i(x^i(p^i),y^i)\right)$$

Now, the first order condition for price of satisfaction to maximise the surplus is:

$$\partial S/\partial p^i = b^{i\prime}(p^i)-f_1^i(x^i(p^i),y^i) = p^i x^{i\prime}(p^i) - x^{i\prime}(p^i)[aw^i + \psi(y^i)]/g^i(y^i) = 0$$

This yields (16). The first order condition for quality to maximise the surplus is:

$$\partial S/\partial y^i = -f_2^i(x^i(p^i),y^i) = 0$$

which yields (12). ||

<u>Proof of proposition 3:</u> (i) Note that (24) shows that there is no distortion of quality for variety 1, i.e. $y^1=y_e^1$, if $\lambda_{21}=0$. In this event the self-selection constraint (26) is not binding. Similarly there is no distortion in the quality of variety 2 if $\lambda_{12}=0$. It is not possible for both varieties to have distorted quality. For this to be the case both self-selection constraints must be binding. In this event, it is necessary that $y^1=y^2$. However, if the constraint (27) is binding, $\lambda_{21}>0$, and hence (24) implies that $y^1>y_e^1$. Similarly (25) implies that $y^2<y_e^2$. As $y_e^2<y_e^1$, this implies that $y^1>y^2$. Hence both self-selection constraints cannot both hold simultaneously.

(ii) Note $\theta^{12\prime}(y^1)p^1+ a(w^1-w^2)g^{2\prime}(y^1)/g^2(y^1)^2 >0$. Hence by (24) $f_2^1(x^1,y^1)>0$ if $\lambda_{21}>0$, hence $y^1>y_e^1$.

(iii) By (24), variety 1's quality is (upwardly) distorted when $\lambda_{21}>0$. Let the equilibrium value of y^1 be $y_d^1 > y_e^1$. In this case the constraint (27) is binding and the constraint (26) is not. Thus $\lambda_{12}=0$. The first order condition (25) becomes:

$$f_2^2(x^2,y^2) = 0$$

In equilibrium there is no distortion of the quality of variety 2, i.e. $y^2 = y_e^2$. From (23) when $\lambda_{21}>0$ and $\lambda_{12}=0$ the equilibrium price, p_d^2, is given by:

$$[p_d^2(1-(1/\varepsilon_x^2)) - (aw^2+\psi(y_e^2))/g^2(y_e^2)]x^{2\prime}(p_d^2) = \lambda_{21} > 0 \qquad \text{(A1)}$$

Note that (A1) is equivalent to $\pi_1^2(p_d^2,y_e^2)>0$. On the assumption that the profit function is concave, this implies that $p_d^2 < p_t^2(y_e^2)\equiv p_t^2$. Further, by (22) the equilibrium price of variety 1 ,p_d^1, is given by:

$$[p_d^1(1-(1/\varepsilon_x^1)) - (aw^1+\psi(y_d^1))/g^1(y_d^1)]x^{1\prime}(p_d^1) = - \lambda_{21}\theta^{12}(y_d^1) \qquad \text{(A2)}$$

Equation (A2) is equivalent to $\pi_1^1(p^1,y_d^1)<0$, which implies that $p_d^1 > p_t^1(y_d^1)$ Marginal cost is greater when quality is y_d^1 rather than y_e^1, hence $p_t^1(y_d^1)> p_t^1(y_e^1)$, and thus $p_d^1 > p_t^1(y_e^1)$.

(iv) Note $\theta^{21\prime}(y^2)p^2 - a(w^1-w^2)g^{1\prime}(y^2)/g^1(y^2)^2 <0$, hence $f_2^2(x^2,y^2)<0$ if $\lambda_{12}>0$. Hence the quality of variety 2 is downwardly distorted.

(v) This proof proceeds as does part (ii). ||

Proof of Proposition 4: (i) From (17) there is upward (no) distortion of variety j's quality if:

$$(P_t^i+aw^i)/g^i(y_e^i) > (\leq)(P_t^j+aw^i)/g^i(y_e^j)$$

Equivalently:

$$\frac{(aw^i+\psi(y_e^i))\varepsilon_x^i}{g^i(y_e^i)(\varepsilon_x^i-1)} > (\leq) \frac{(aw^j+\psi(y_e^j))\varepsilon_x^j}{g^i(y_e^j)(\varepsilon_x^j-1)} + \frac{a(w^i-w^j)}{g^i(y_e^j)}$$

or:

$$\frac{(aw^i+\psi(y_e^i))\varepsilon_x^i}{g^i(y_e^i)(\varepsilon_x^i-1)} - \frac{(aw^i+\psi(y_e^j))\varepsilon_x^j}{g^i(y_e^j)(\varepsilon_x^j-1)} > (\leq) \frac{a(w^j-w^i)\varepsilon_x^j}{g^i(y_e^j)(\varepsilon_x^j-1)} + \frac{a(w^i-w^j)}{g^i(y_e^j)}$$

or:

$$\mu^i(\varepsilon_x^i,\varepsilon_x^j,w^i,w^j,y_e^i, y_e^j) > (\leq)\ \chi^i(w^i,w^j,y_e^i,y_e^j)$$

(ii) The proof then proceeds as part (i). ||

<u>Proof of Corollary 1:</u> (i) From (17) there is upward distortion of variety j's quality if

$$p_t^i > p_t^{ij}\theta^{ji}(y_e^j) + a(w^i-w^j)/g^i(y_e^j)$$

Substituting (10) into this inequality yields:

$$D^i\frac{\varepsilon_x^i}{\varepsilon_x^i-1} > D^j\frac{\varepsilon_x^j}{\varepsilon_x^j-1} + B^{ij}$$

where:

$$D^i \equiv \frac{(aw^i+\psi(y_e^i))}{g^i(y_e^i)}$$

$$D^j \equiv \frac{(aw^j+\psi(y_e^j))}{g^i(y_e^j)}$$

and

$$B^{ij} \equiv \frac{a(w^i-w^j)}{g^i(y_e^j)}$$

Rearranging yields:

$$[D^i\varepsilon_x^i - B^{ij}(\varepsilon_x^i-1) - D^j(\varepsilon_x^i-1)]\ \varepsilon_x^j > D^i\ \varepsilon_x^i - B^{ij}(\varepsilon_x^i-1)$$

or:

$$\varepsilon_x^j > \frac{\eta^{ij}}{\eta^{ij}-1}$$

where $\eta^{ij} \equiv \dfrac{D^i\varepsilon_x^i - B^{ij}(\varepsilon_x^i\text{-}1)}{D^j(\varepsilon_x^i\text{-}1)} = \dfrac{g^i(y^j)[aw^i+\psi(y^i)]\varepsilon_x^i + g^i(y^i)a(w^j\text{-}w^i)(\varepsilon_x^i\text{-}1)}{g^i(y^i)[aw^j + \psi(y^j)](\varepsilon_x^i\text{-}1)}$

provided:

$$D^i\varepsilon_x^i - B^{ij}(\varepsilon_x^i\text{-}1) - D^j(\varepsilon_x^i\text{-}1) > 0$$

or:

$$D^i\varepsilon_x^i - B^{ij}(\varepsilon_x^i\text{-}1) > D^j(\varepsilon_x^i\text{-}1)$$

or:

$$\eta^{ij} > 1.$$

(ii) If variety j is distorted in equilibrium then (20) yields:

$$p^i(1\text{-}(1/\varepsilon_x^i)) = (aw^i+\psi(y^i))/g^i(y^i) + \lambda_{ij}/x^{i\prime}(p^i)$$

and

$$p^j(1\text{-}(1/\varepsilon_x^j)) = (aw^j+\psi(y^j))/g^j(y^j) - \lambda_{ij}\theta^{ji}(y^j)\,/x^{j\prime}(p^j)$$

Hence:

$$p^i(1\text{-}(1/\varepsilon_x^i)) < (aw^i+\psi(y^i))/g^i(y^i)$$

and

$$p^j(1\text{-}(1/\varepsilon_x^j)) > (aw^j+\psi(y^j))/g^j(y^j)$$

Further in equilibrium the self-selection constraint holds:

$$p^i = p^j\theta^{ji}(y^j) + a(w^i\text{-}w^j)/g^i(y^j)$$

Hence:

$$\frac{aw^i+\psi(y^i)}{g^i(y^i)(1\text{-}(1/\varepsilon_x^i))} > \frac{(aw^j+\psi(y^j))}{g^i(y^j)(1\text{-}(1/\varepsilon_x^j))} + \frac{a(w^i\text{-}w^j)}{g^i(y^j)}$$

The proof then proceeds as part (i). ||

Proof of corollary 2: The third degree discriminating price and quality satisfy the self-selection constraint (17) if:

$$\frac{aw+\psi(y_e^i)}{g^i(y_e^i)(1-(1/\varepsilon_x^i))} < \frac{aw+\psi(y_e^j)}{g^i(y_e^j)(1-(1/\varepsilon_x^j))} \tag{A3}$$

In this case λ_{ij}=0, and there is no distortion in the quality of variety j. Note that:

$$\frac{aw+\psi(y_e^i)}{g^i(y_e^i)} < \frac{aw+\psi(y_e^j)}{g^i(y_e^j)}$$

because y_e^i is the cost minimising quality for variety i. Hence (A3) holds when $\varepsilon_x^i(p_t^i/w^i) \geq \varepsilon_x^j(p_t^j/w^j)$ for $i \neq j$. ||

Proof of corollary 3:

(i)Assume $\theta^i=\theta^j=\theta$. Then:

$$\eta^{ij} = \left(\frac{\phi^i}{\phi^i-1}\right)\left(\frac{w^i}{w^j}\right)^{\frac{\xi-\theta}{\xi}} + \left(\frac{\xi-\theta}{\xi}\right)\left(1-\left(\frac{w^i}{w^j}\right)\right)$$

It is readily confirmed from (30) that $E^1 = \infty$ for all $(w^1/w^2)>\omega^1$, where $\omega^1>0$ satisfies:

$$\left(\frac{\phi^1}{\phi^1-1}\right)(\omega^1)^{\frac{\xi-\theta}{\xi}} + \left(\frac{\xi-\theta}{\xi}\right)\left(1-(\omega^1)\right) = 1$$

Similarly $E^2 = \infty$ for all $(w^1/w^2)>\omega^2$, where $\omega^2>0$ satisfies:

$$\left(\frac{\phi^2}{\phi^2-1}\right)\left(\frac{1}{\omega^2}\right)^{\frac{\xi-\theta}{\xi}} + \left(\frac{\xi-\theta}{\xi}\right)\left(1-\left(\frac{1}{\omega^2}\right)\right) = 1$$

(ii) Write $w^1/w^2 = 1 + \delta_w$. For sufficiently small δ_w:

$$\eta^{12} \approx \frac{\xi\phi^1+(\xi-\theta)\delta_w}{\xi(\phi^1-1)}$$

and hence:

$$E^1 \approx \phi^1 - \frac{(\xi-\theta)(\phi^1-1)\delta_w}{\xi + (\xi-\theta)\delta_w} < \phi^1.$$

Thus if $\phi^1 \leq \phi^2$ then $E^1 < \phi^2$ and thus variety 2 exhibits quality distortion.

(iii) Write $w^2/w^1 = 1 - \delta_w$. For sufficiently small δ_w:

$$\eta^{21} \approx \frac{\xi\phi^2-(\xi-\theta)\delta_w}{\xi(\phi^2-1)}$$

and hence:

$$E^2 \approx \phi^2 + \frac{(\xi-\theta)(\phi^2-1)\delta_w}{\xi + (\xi-\theta)\delta_w}.$$

Variety 1 has distorted quality if $E^2 < \phi^1$ or if $\phi^1-\phi^2 > \frac{(\xi-\theta)(\phi^2-1)\delta_w}{\xi + (\xi-\theta)\delta_w}$.

(iv) If $\phi^i \to \infty$ then $\omega^i \to 1$. Hence variety i does not exhibit distorted quality for any $w^1/w^2 > 1$. ||

Proof of corollary 4: Let:

$$\Gamma^{ji} = p^i - p^j\theta^{ji}(y_e^j) = \frac{(aw+\psi(y_e^i))}{g^i(y_e^i)(1-(1/\varepsilon_x^i))} - \frac{(aw+\psi(y_e^j))}{g^i(y_e^j)(1-(1/\varepsilon_x^j))} \text{ for } i\neq j$$

where $\varepsilon_x^i = \overline{\varepsilon}(p_t^i/w)$. From the self selection constraints, (17), variety j exhibits distorted quality if $\Gamma^{ji}>0$.

(i) If $\overline{\varepsilon}(p_t^i/w)$ is constant then

$$\Gamma^{ji} = (1-(1/\varepsilon))^{-1}\left(\frac{(aw+\psi(y_e^i))}{g^i(y_e^i)} - \frac{(aw+\psi(y_e^j))}{g^i(y_e^j)}\right) < 0$$

as y_e^i is the cost minimising quality for type i customers.

(ii) Note that $p_t^1 < p_t^2$. If $\overline{\varepsilon}(p_t^i/w)$ is increasing then $\varepsilon_x^1 < \varepsilon_x^2$. In this case $\Gamma^{12}<0$ and variety 1 has undistorted quality. However if ε_x^1 is sufficiently close to 1 (relative to ε_x^2) then $\Gamma^{21}>0$ and variety 1 exhibits downwardly distorted quality.

(iii) If $\overline{\varepsilon}(p_t^i/w)$ is decreasing then $\varepsilon_x^1 > \varepsilon_x^2$. In this case $\Gamma^{21}<0$ and variety 2 has undistorted quality. However if ε_x^1 is sufficiently close to 1 (relative to ε_x^2) then $\Gamma^{21}>0$ and variety 1 exhibits upwardly distorted quality.

Proof of corollary 5:

(i) Let $\Gamma^{ji} = p_t^i - p_t^j - a(w^i-w^j)/g(y_e^j)$. The quality of variety j is distorted if $\Gamma^{ji}>0$. Note that if $w^1=w^2$ then $\Gamma^{ji}=0$. Now:

$$\partial\Gamma^{21}/\partial w^1 = dp_t^1/dw^1 - a/g(y_e^2)$$

When this expression is evaluated for $w^1=w^2$ then:

$$\partial\Gamma^{21}/\partial w^1 = \frac{a}{g(y_e)(1-(1/\overline{\varepsilon}))} + \frac{(aw+\psi(y_e))}{g(y_e)(1-(1/\overline{\varepsilon}))^2}\frac{\overline{\varepsilon}'}{\overline{\varepsilon}^2} - \frac{a}{g(y_e)}$$

where $\overline{\varepsilon}$ is the elasticity of demand evaluated at the common price of satisfaction and y_e is the common quality. If $\overline{\varepsilon}'>0$ this expression is positive.

(ii) As above:

$$\partial\Gamma^{12}/\partial w^2 = dp_t^2/dw^2 - a/g(y_e^1)$$

If $dp_t^2/dw^2 < 0$ in the neighbourhood of $w^1=w^2$, then $\Gamma^{12}>0$ for w^1/w^2 sufficiently close to 1. In this case variety 1 exhibits upwardly distorted quality.

(iii) If the demand of both customer types is perfectly elastic:

$\Gamma^{ji} = p_t^i - p_t^j - a(w^i-w^j)/g(y_e^j).$

$$= \frac{aw^i+\psi(y_e^i)}{g(y_e^i)} - \frac{aw^i+\psi(y_e^j)}{g(y_e^j)}.$$

In this case $\Gamma^{ji}<0$ because y_e^i is the cost minimising quality for type i customers. ||

<u>Proof of Corollary 6</u>: If demand is given by $x(p^i/w^i)=A-B(p^i/w^i)$ where $A,B>0$

Then

$$p_t^i = Aw^i/(2B) + (aw^i+\psi(y_e^i))/(2g^i(y_e^i))$$

Hence:

$\Gamma^{21} = p_t^1 - p_t^2 - a(w^1-w^2)/g(y_e^2).$

$= Aw^1/(2B) + (aw^1+\psi(y_e^1))/(2g^1(y_e^1)) - Aw^2/(2B) + (aw^2+\psi(y_e^2))/(2g^2(y_e^2)) - a(w^1-w^2)/g(y_e^2)$

$= (w^1-w^2)[A/(2B)-a/g(y_e^2)] + (aw^1+\psi(y_e^1))/(2g^1(y_e^1)) - (aw^1+\psi(y_e^2))/(2g^2(y_e^2))$

The quality of variety 2 is distorted if $\Gamma^{21}>0$. Note that if $w^1=w^2$ then $\Gamma^{21}=0$. Now:

$$\partial\Gamma^{21}/\partial w^1 = (1/2)[A/B + a/g^1(y_e^1)] - a/g(y_e^2)$$

Further:

$$\frac{\partial(\Gamma^{21})^2}{\partial^2 w^1} = \frac{-ag^{1\prime}(y_e^1)}{2g^1(y_e^1)^2}\frac{\partial y_e^1}{\partial w^1} < 0.$$

Assuming that $y_e^1 \to \infty$ as $w^1 \to \infty$ then, for sufficiently large w^1

$$\partial\Gamma^{21}/\partial w^1 \approx (1/2)[A/B] - a/g(y_e^2)$$

If $(1/2)[A/B] - a/g(y_e^2) > 0$ then $\partial\Gamma^{21}/\partial w^1 > 0$ for all $w^1 \geq w^2$. Hence the quality of variety 2 is always downwardly distorted. If $(1/2)[A/B] - a/g(y_e^2) < 0$, then $\partial\Gamma^{21}/\partial w^1 = 0$ for some value of w^1, say w^{1*}. Then $\partial\Gamma^{21}/\partial w^1 < 0$ for all $w^1 > w^{1*}$. However as $\partial(\Gamma^{21})^2/\partial^2 w^1 < 0$ there must be some w^1, say $w^{1**} > w^{1*}$, for which $\Gamma^{21} = 0$. Thus $\Gamma^{21} < 0$ for all $w^1 > w^{1**}$. In this case neither variety exhibits quality distortion for all $w^1 > w^{1**}$.

<u>Proof of Corollary 7</u>:

With both a heterogeneity in taste and wages:

$$\Gamma^{ji} = p_t^i - \theta^{ji}(y_e^j)p_t^j - a(w^i - w^j)/g^i(y_e^j).$$

With constant elasticity of demand:

$$\partial\Gamma^{21}/\partial w^1 = \frac{a}{g^1(y_e^1)(1-(1/\phi))} - \frac{a}{g^1(y_e^2)}$$

Further:

$$\frac{\partial(\Gamma^{21})^2}{\partial^2 w^1} = \frac{-ag^{1\prime}(y_e^1)}{(1-(1/\phi))g^1(y_e^1)^2}\frac{\partial y_e^1}{\partial w^1} < 0$$

Let $y^1_e(w^2)$ be the cost minimising quality when w^1=w^2. Observe that if:

$$\phi > \frac{1}{1-[g^1(y^2_e)/g^1(y^1_e(w^2))]}$$

then $\partial\Gamma^{21}/\partial w^1 < 0$ when w^1=w^2. As $\partial(\Gamma^{21})^2/\partial^2 w^1$<0 this implies that $\partial\Gamma^{21}/\partial w^1 < 0$ for all w^1>w^2. Hence Γ^{21}<0 for all $w^1 \geq w^2$ and variety 2 does not exhibit distorted quality. ||

<u>Proof of Proposition 5</u>:

(i) By assumption only variety 1 is distorted. By (24), variety 1's quality is distorted when λ_{21}>0 and by (25) variety 2's quality is not distorted when λ_{12}=0. Thus y^2_e is the equilibrium quality of variety 2. Let y^1_d be the equilibrium quality of variety 1. From (23) when λ_{21}>0 and λ_{12}=0:

$$[p^2_d(1-(1/\varepsilon^2_x)) - (aw^2+\psi(y^2_e))/g^2(y^2_e)]x^{2\prime}(p^2_d) = \lambda_{21} > 0$$

Note that this equation is equivalent to $\pi^2_1(p^2_d,y^2_e)$>0. On the assumption that the profit function is concave, this implies that the profit maximising price, p^2_d, satisfies $p^2_d < p^2_t$. Further, (22) becomes:

$$[p^1_d(1-(1/\varepsilon^1_x)) - (aw^1+\psi(y^1_d))/g^1(y^1_d)]x^{1\prime}(p^1_d) = - \lambda_{21}\theta^{12}(y^1_d)$$

This equation is equivalent to $\pi^1_1(p^1,y^1_d)$<0, which implies that $p^1_d > p^1_t(y^1_d)$. If y^1_d>y^1_e then $p^1_d > p^1_t(y^1_d) > p^1_t$.

(ii) This proof proceeds as does part (i)

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