Weak Links in the Theory of Weakest-Link Public Goods

Michael Brooks
School of Economics
University of Tasmania
GPO Box 252-85
Hobart 7001
Australia

Email: Michael.Brooks@utas.edu.au
Fax: + 61 3 6226 7587
Tel.: + 61 3 6226 2286

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Michael Brooks[*]
University of Tasmania

Abstract

The orthodoxy on weakest link public goods has a number of weak theoretical links. In most of the examples offered of the weakest-link public goods concept individuals are not restricted to privately consuming the minimal contribution made by some other individual. Once the issue of private provision is broadened to take on the possibility that individuals can privately supply the good independently of the action of others, then the interesting issue of economies of scale from collective provision needs to be taken on board before any pronouncement can be made on whether the market fails.

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[*] School of Economics
University of Tasmania
GPO Box 252-85
Hobart Australia 7001
Michael.Brooks@utas.edu.au
Fax: +61 3 6226 7587
Tel: +61 3 6226 2286

1. INTRODUCTION

One of the more exciting developments in the general theory of public goods over the last two decades has been the development of the weakest-link public goods concept[1]. The development is exciting because it elegantly extends the concept of public goods beyond the summation case technology. Weakest-link public goods are characterised as cases in which $G = \min\{g_i\}$, where G is the total quantity of the public good, and g_i represents individual i's contribution. The prime example of a weakest-link public good was put forward by Hirshleifer (1983) in his seminal paper on the concept. There he imagines the inhabitants of a low-lying island, Anarchia, who are threatened with destruction if the sea level should rise. Each individual can build a dyke as protection from the rising seas but the protection consumed by all equals the height of the lowest dyke. Once the lowest dyke is breached the whole island is flooded. From this colourful example the theory of weakest-link public goods has been developed[2]. A whole raft of additional examples have been made in the literature. Vicary (1990, p. 376) lists a number of them:

> Similar examples would be the protection of a military front, taking a convoy across the ocean going at the speed of the slowest ship, or maintaining an attractive village/landscape (one eyesore spoils the view). Many instances of teamwork involve weak-link elements, for example moving a pile of bricks by hand along a chain or providing a theatrical or orchestral performance (one bad individual effort spoils the whole effect.)

And to complete the list Hirshleifer and Hirshleifer (1998, p.501) raise the following example:

> Suppose the [weakest-link] public good is a chain bridge, each member of the community being responsible for one link of the chain.

In their textbook treatment of public goods, Hirshleifer and Hirshleifer provide perhaps the neatest and certainly simplest presentation of the weakest-link public good.

		Column player	
		Contribute	Do not contribute
Row player	Contribute	1, 1	-1, 0
	Do not contribute	0, -1	0, 0

Consider Table 1 which is replicated from their analysis. Hirshleifer and Hirshleifer (1998, p. 502) capture the unique character of the weakest-link public goods as follows: Here a benefit b = 2 will be received by each only if *both* contribute, at a cost of c = 1 each. And conclude “[t]he *efficient* outcome.. [is] the upper-left cell...” Of the three Nash equilibria, – the pure strategy Nash equilibria (contribute, contribute), (do not contribute, do not contribute) and the mixed-strategy Nash equilibrium [(0.5 contribute, 0.5 do not contribute); (0.5 contribute, 0.5 do not contribute)] - Hirshleifer and Hirshleifer (1998, p. 502) suggest

> [i]t seems “sensible” here for each player to choose contribute. Notice that all the Nash solutions are symmetrical; that is, the players’ choices match one another. If so, it seems plausible that they match at (contribute, contribute) rather than at any of the Pareto-inferior strategy pairs.

A cursory survey of the literature suggests that the concept of the weakest-link public good involves the following basic propositions:

(i) the amount of the good consumed by all is determined by the minimal amount individually provided. (Hirshleifer, 1983).

(ii) All members of the collectivity must contribute, if complete market failure is to be avoided. (Vicary, 1990). And,

(iii) "In contrast with the standard 'summation case', for weakest-link public goods there is at least some likelihood of attaining the efficient solution [under voluntary provision]" as seen in the simple game-theoretic approach. (Hirshleifer and Hirshleifer, 1998, p. 502).

In this paper, I want to expose some weak links in the theory of weakest-link public goods. In doing so, I suggest that the orthodoxy on weakest-link public goods is seriously amiss. It will become evident that the low-lying island case cannot support the weight of the analysis that has been placed on it. And significantly, the analysis of dykes on Anarchia offered here suggests that there are a number of issues which have been overlooked in the orthodoxy.

2. A SIMPLE MODEL OF DYKES

In order to expose the problems in the orthodox analysis of weakest-link public goods, it is necessary to present a specific example. To simplify the discussion of dyke protection on Anarchia suppose the low-lying island is a square which is inhabited by two individuals, A and B. Each individual occupies a rectangular section comprising precisely half the area of the island. Individual A occupies the west coast, B the east, and each has a shore-line equal in length to $x + 2(x/2)$, where x is the length of one side of the island. Consider Figure 1 which shows the individuals' demand for protection, initially represented as D_A and D_B, expressed as willingness to pay for dykes of varying height[3]. Individual A being the more nervous character or perhaps the less able sailor has the larger willingness to pay for protection from the sea.

Up to this point the analysis is standard fare. The cost side requires some elaboration as this is the side of the analysis where some aspects of the weakest-link production technology can be introduced. Initially consider the marginal cost curve, MC. It represents the marginal cost of building dykes of varying heights to the minimal standard along *one side* of the island. For simplicity assume that both individuals share an identical production technology and that neither gains any economies from learning. The marginal

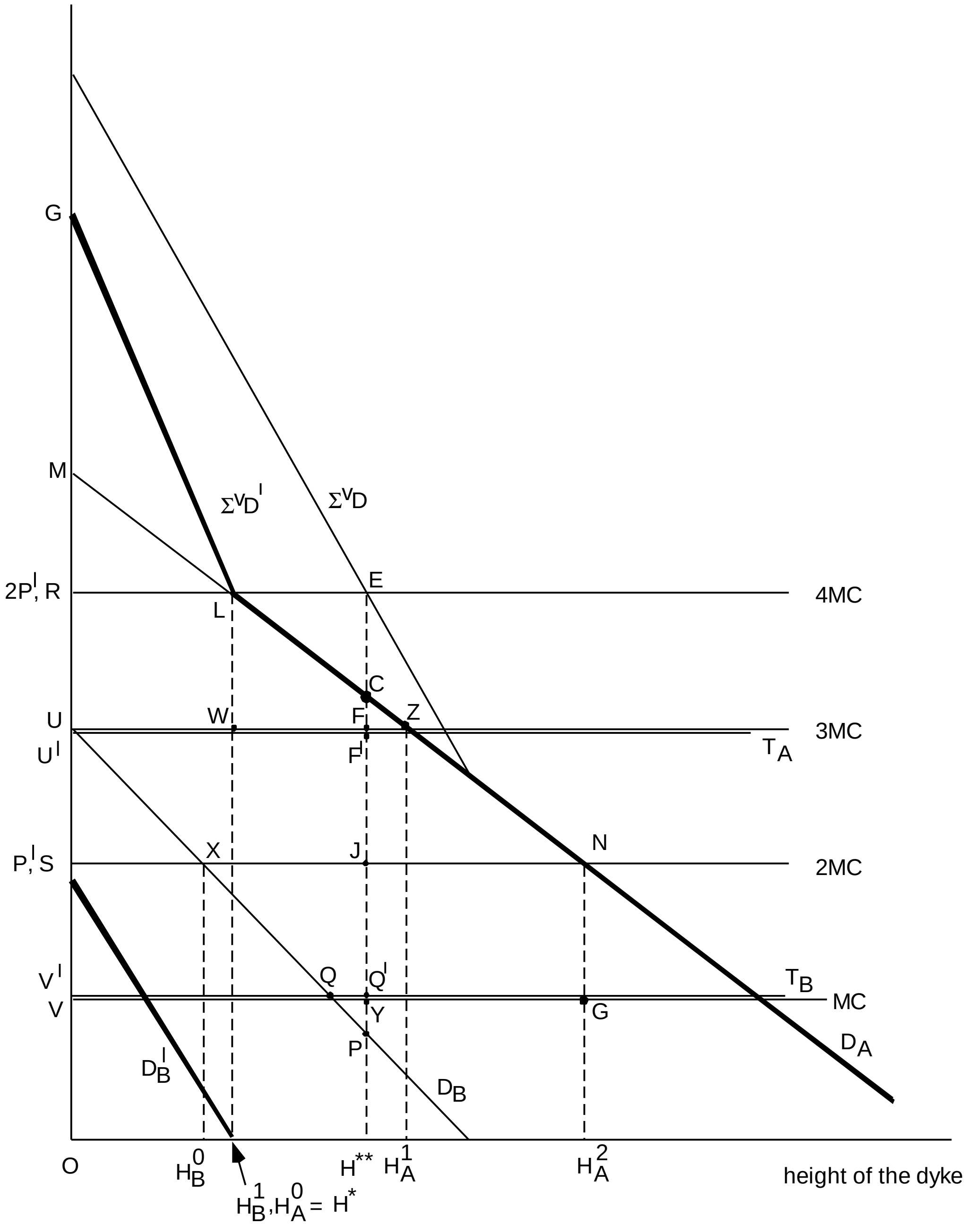

Figure 1

cost of building a dyke of a given height along two sides of the square island is therefore given by 2MC. Equally, the marginal cost of building dykes of varying heights along the entire perimeter of the island is represented by 4MC. In short, the marginal cost of building a dyke of a given height along more than one length of the island is the vertically summed marginal cost curve where the coefficient represents the number of sides of the island that have been fortified from the rising seas.

Following the logic of the standard treatment of the weakest-link public good individual A would like to build a dyke of height H_A^2 along his land bordering the sea. Individual B would like to build a dyke of only height H_B^0 along his sea-frontage. Individual A figures quite reasonably, however, that the additional height $H_A^2 - H_B^0$ offers no further protection: Once the sea reaches a height just above H_B^0, A's land will be flooded even though he has built a higher dyke. If individual A is aware of B's plans, then he will build a dyke of only H_B^0 units in height since the additional building represents a waste of A's resources. The level of private provision of the weakest-link public goods is therefore H_B^0. The amount voluntarily provided equals the minimal contribution towards protection from the sea. Since the good involves equal consumption the Samuelsonian condition for optimality applies and occurs at H*. The level of private provision falls short of the socially optimal dyke.

To take an even more drastic case of market failure, if individual B's demand curve lies everywhere below 2MC, such as that represented by $D_B^{|}$, then individual A would rationally decide to give up the idea of building any dyke along his sea-frontage – a dyke would offer no protection from the rising seas because the land would simply be flooded from B's side. The market fails completely so it seems: no dyke of any description is drastically less than the socially efficient dyke of height H*. In both cases, the market fails to provide the socially efficient dyke.

Although the analysis offers a different geometrical treatment from that found in the literature it does seem to confirm the standard results. The analysis appears to confirm proposition (i) that the amount consumed by all equal the minimal contribution here made by individual B. In regard to proposition (ii) the amount of protection from the rising sea equals the minimal contribution made by individual B. When individual B, for example, decides to completely forgo protection from the sea, individual A has nothing to gain by building any dyke along his land bordering the sea. If he should build a dyke of height H_A^2, then the sea will merely cover his land from B's unprotected coast. Individual A will have wasted the amount of resources represented by area $OSNH_A^2$. Apparently it is the case, that all members of the collectivity must individually provide some units of the good, if complete market failure is to be avoided.

With a slight amendment to the analysis it is possible to show that proposition (iii) holds also: that unlike the standard public goods case there is some possibility that the efficient outcome will be reached under voluntary provision. If B's demand curve is shifted to the right to coincide with A's demand curve, then the socially efficient dyke is one of height H_A^2 whether the dyke is a pure public good or one of the weakest-link variety. Under the orthodox summation case, there is no guarantee at all, that the efficient amount would be reached by voluntary provision. If individual A, for example, provides H_A^2 units of a pure public good, then individual B will have an incentive to free-ride on the amount provided by A. In the case of a pure public good individual B can decide to not contribute safe in the knowledge that he cannot be excluded from the amount provided by A. If each individual attempts to free-ride on the other, then nothing will be provided. The possibility of free-riding on the other's contribution does not arise in the case of weakest-link public goods. The attempt to free-ride would seem to be ruled out by the technology involved in the case of weakest-link public goods. If B attempts to free-ride on A's contribution, then both individuals will end up consuming no protection from the sea. If B builds a dyke too of height H_A^2, then his consumer's surplus is MNS as opposed to no surplus. The possibility that the efficient amount of the weakest-link public good will be voluntarily provided, at least in a world of equals, is not an unreasonable suggestion.

3. THERE IS A HOLE IN THE DYKE: WEAKEST-LINK PUBLIC GOODS LOST

One might be inclined to accept all of the analysis so far as an interesting, but in the end, all too familiar account of weakest-link public goods. The comment is a little too harsh. There is surely some benefit in presenting the familiar properties of weakest-link public goods in terms of the analytical tools that are used so widely in the analysis of pure public goods. In doing so, the differences between pure public goods and weakest-link public goods can be readily seen.

A moment's reflection will indicate, however, that there is more at stake than presenting the standard outcomes of the weakest-link public good using Marshallian partial equilibrium analysis. There is something seriously amiss in the orthodox account of the weakest link public good. The point is obvious once seen, but to this date has been completely overlooked in the literature. There is nothing in the scenario of dykes on Anarchia to stop the individual who seeks more protection from the rising seas to build a dyke around the perimeter of all of his land as opposed to the perimeter of just his land that borders on the sea.

Individual A, for example, is prepared to build at a marginal cost of 3MC a dyke of height H_A^1, along the perimeter of his property. If B should decide to completely forgo dyke production and take to the sea like the central character in *Waterworld,* then A is still offered the protection of a dyke of height H_A^1. Under this alternative setting, individual A's protection from the rising sea is totally unaffected by B's decision on just how much dyke to build. If B's demand curve is represented by D_B, then B will build a dyke of height H_B^0 along his perimeter but A will still continue to consume the protection of dyke H_A^1 along the perimeter of his land. In stark contrast to the orthodoxy on weakest-link public goods, Individuals A and B can consume, fundamentally different amounts of the good. Under the Anarchia example, proposition (i) is evidently at some risk of foundering.

Once it is accepted that the individuals need not consume identical amounts of protection, then it is clear that the Samuelsonian condition for optimality cannot serve as the benchmark for the evaluation of the performance of both market and collective provision. In order to see what precisely is at stake here reconsider Figure 1 and suppose that A's and B's demand curves are represented by D_A and $D_B^{|}$. Under voluntary provision individual A consumes a dyke of height H_A^1 and gains a consumer's surplus of the amount represented by area MZU. Individual B does not build any dyke. Under collective provision both individuals will have to consume a dyke of identical height along the perimeter of their land bordering on the sea. The Samuelsonian condition for optimality occurs at point L and under the requirement of identical provision the socially optimal dyke is one of height H_B^1. A simple comparison of the outcomes under voluntary provision and collective provision reveals that here the market is preferred to the collective outcome. The social surplus under voluntary provision exceeds the social surplus under collective provision; that is MZU is greater than GLR. Manifestly, proposition (ii) which states that *all* individuals must contribute if complete market failure is to be avoided is found wanting. Here under voluntary provision one of the individuals chose not to produce any protection from the rising sea but the market outperforms the collectively provided outcome. The analysis also confirms proposition (iii) but as explained subsequently in this section the reasons why efficiency might be achieved under voluntary provision differs from the orthodox account.

Before examining the status of proposition (iii) it is worth noting that there is no suggestion made here that voluntary provision will always outperform collective provision and that some of basic propositions of the orthodoxy on weakest-link public goods founder. If B's demand curve is shifted to the right to D_B, then under voluntary provision individual B will gain a consumer's surplus of UXS when individual A has already built a dyke of height H^1_A. Individual A will continue to gain a consumer's surplus of amount MZU. Under collective provision, the dyke which satisfies the Samuelsonian condition is one of height H**. It is not difficult to see that collective provision is Pareto preferred to voluntary provision. If individual A is assigned a tax price of T_A (a per unit tax price just under 3MC), then he will be indifferent between voluntary provision and collective provision. Although it is true that A suffers a loss from being afforded less protection under collective provision, it is also true that he pays an assigned tax price less than the price paid under voluntary provision. The lower tax price here is calculated to match the loss of surplus from the reduction in provision. In order to cover the total cost of provision, B's assigned tax-price is T_B (equals $4MC - T_A$) and he gains a consumer's surplus of UQV$^{|}$ minus QPQ$^{|}$. Since the area of UQV minus QPQ$^{|}$ is clearly greater than UXS, it follows that individual B has been made better off under collective provision. Since A is no worse off it follows that collective provision is Pareto preferred to voluntary provision.[4] Under this arrangement it is individual B who captures all the gains from collective provision.

It is worth stressing that it would be wrong to interpret the discussion here of the dykes on Anarchia as nothing more than the analysis of the collective provision of a pure private good. The temptation to believe that the analysis centres on pure private goods is almost irresistible. After all, under voluntary provision individuals A and B are free to adjust the individual quantity of dyke consumed which is a characteristic unique to private goods. But there the similarity ends, on at least two accounts.

First, once individual A builds the dyke along the perimeter of his property bordering on the sea, individual B is free to consume the portion of A's dyke running across the centre of the island. The external benefit is reflected in the observation that individual B can build a dyke of height H_B^0 along his land bordering the sea at a marginal cost of 2MC when individual A has built either a dyke of a height at least equal to H_B^0 along the entire border of his land or a dyke along the sea bordering land of height at least H_B^0. If individual A does not build his dyke, then individual B faces a higher marginal cost of 3MC when he chooses to afford himself any protection. Individual A's private provision therefore provides an external benefit to individual B.

Second, the dykes retain the central characteristic of the weakest-link public good, at least under the collective provision regime. In order to see the point without the need for an additional diagram, a slight re-interpretation of Figure 1 is required. Suppose the price of the private good produced under constant marginal cost in the market is $P^{|}$ (equals 2MC). The marginal cost of supplying an equal amount of the private good under collective provision is $2P^{|}$ (equals 4MC). When a pure private good is collectively supplied at a level of H** and each individual faces a tax-price of $P^{|}$, individual A has an incentive to privately supplement his consumption with $H_A^2 - H**$ units at a marginal cost of $P^{|}$. Individual A can improve his consumer's surplus by area CJN. In the case of the dykes there is no such possibility here to supplement the collectively provided amount with private provision. If the collectivity has built a dyke of height H** along the perimeter of the island bordering the sea, then A's effort to extend the heights of the dyke along his sea-frontage at a marginal cost of 2MC will offer no additional protection. If the sea level should rise above H**, then A’s land would be flooded by the water flowing over the lower segment of the dyke on B’s land. Evidently under collective provision the dykes on Anarchia retain the central characteristic of a weakest-link public good, at least over a range of outcomes, that is protection is limited to the minimal albeit *collectively* provided contribution.[5]

There are a number of important lessons to be drawn form the analysis of the optimal provision of dykes. First, the source of the gains from the collective provision of the dykes on Anarchia differs substantially from the standard interpretation offered in the literature on weakest-link public goods. In the orthodox literature the gains from collective provision arise from the solution of a coordination game at the Pareto efficient outcome. This is most evident in Hirshleifer and Hirshleifer’s interpretation of the weakest-link public goods game offered in Table 1. Although there is some possibility that the individuals themselves will solve the game at the pure strategy Nash equilibrium (contribute, contribute) there can be no guarantee that each individual will independently of the other decide to play the contribute strategy. Collective provision resolves the coordination dilemma in favour of the Pareto-preferred equilibrium. The analysis conducted here in terms of partial equilibrium tools suggests that the source of the gains from collective provision do not rest alone on solving some coordination game. There is an additional gain from the economies of scale from collective production. When the individuals each privately provide their own protection from the rising seas, and A first builds a dyke along the perimeter of his land of height $\mathrm{H}_{\mathrm{A}}^{1}$, he faces a marginal cost of 3MC. Since individual B does not have to build a dyke along the perimeter of his entire property free-riding on A’s provision, he builds a dyke of height $\mathrm{H}_{\mathrm{B}}^{0}$ facing a cost at the margin of 2MC. The total marginal cost of private provision of protection from the sea on Anarchia is therefore 5MC.

Under collective provision the individuals face a total marginal cost of 4MC. If the individuals can reach agreement on their dyke building efforts there is little need for individual A to build the segment of the dyke along the internal perimeter of his land.[6] There are a number of tax-arrangements which provide each individual with a share of the economies of scale in production resulting from collective provision. Although the individuals prefer different amounts of provision under the collective arrangement, here the gain from the economies of scale is sufficient to offset the inefficiency from not being able to use Lindahl prices[7]. The possibility that there are economies of scale in production to be had from collective provision is a feature of the weakest-link public goods concept that simply does not figure in the orthodox account, at least as captured in the coordination game. Evidently, the weakest-link public goods concept has a number of subtleties, some of which have been overlooked in standard accounts.

The second lesson to be drawn from the analysis is that the case of the dykes on Anarchia is clearly a poor example of the weakest-link public good commonly interpreted. If analysts want to restrict the concept to those cases in which the amount consumed by all equals the minimal contribution made under voluntary provision, then the example of dykes on Anarchia is far from satisfactory.

4. WEAKEST-LINK PUBLIC GOODS REGAINED?

It might be argued that even if it is accepted that the case of the dykes on the low-lying island turns out to be a poor choice for the prime example of the concept, the weakest-link concept as it is commonly interpreted is still a useful concept. There are, after all, a number of examples of the weakest-link public goods concept which have been raised in the literature. Cannot the analyst exploit some other example to demonstrate the central propositions of the weakest-link public good as it is commonly interpreted?

Take the case of convoys across the Atlantic Ocean during the Second World War. The theory would have it that the convoy was restricted to the speed of the slowest ship. A straight forward interpretation of the theory of weakest-link public good suggests that the protection to all was equal to the slowest ship. If one ship reduced its speed due to, say, engine problems, the weakest-link public good concept suggests that protection could only be provided if all other ships in the convoy similarly reduced their speed. A cursory examination of the historical record suggests that there is a certain ring of truth to this example: convoys were organised to match the speed of the ships requiring protection. But there is nonetheless a good deal of slippage between the empirical record and the orthodox theory. The historical record indicates that in the case of engine problems the convoy would continue its crossing leaving the “struggler” in its wake. Equally, it was not necessary for all ships to enter into some collective arrangement to be offered some protection from the U-boats. Just as individual A could privately provide himself with protection from the rising seas by building a dyke around the perimeter of all of his land, ships such as the Queen Mary crossed most of the Atlantic run without any accompanying ships, afforded protection from the U-boats by its sheer speed.

Alternatively take the case of the protection of a military front. In line with the theory of weakest-link public goods the example presumably means that a front is only as strong as the weakest point of defence. The Maginot line would seem to fit the bill here: the line of forts erected by the French as a barrier to the Germans afforded the French little or no protection from the German Army as it was able to by-pass the Maginot line by invading France via Belgium. Even so the case of the protection of a military front is not, however a general example of a weakest-link public good. In the First World War, for example, segments of a front could be over-run by the enemy but this did not necessarily lead to the collapse of the entire front. Manifestly, the suggestion that a military front is an example of a weakest-link public good on the argument that the front is only as strong as its weakest unit is not a very robust example.

Consider the example of a theatrical or orchestral performance where it is claimed that "...one bad individual effort spoils the whole effect" (Vicary 1990, p. 376). It is certainly possible that a poor performance can seriously mar the overall effect of a group. But this clearly need not be the case. One frequently hears comments that a group's guitarist did not have his chops together but the rhythm section was really tight.[8] Evidently, it is possible to distinguish between the various instrumentalists in a group and give credit where it is due: it is not necessarily the case that one bad individual effort need bring all aspects of the performance down to the lowest level. Much the same comment can be made to Vicary's example of "...maintaining an attractive/landscape, one eyesore spoils the view." (1990, p. 376) as an example of a good containing weakest-link elements. One hears the comment, shame about the one spot, but the rest of the view was simply magnificent. Finally, consider Hirshleifer and Hirshleifer's case of the chain bridge as a example of a weakest-link public good. This appears to be a good example. All of us are acquainted with the aphorism that a chain is only as strong as its weakest link. The trouble with the example is that bridges were not built from chain. Early bridges used vines or ropes that were twisted together to form a strong support. If one vine or rope failed, then the remaining vines or ropes offered sufficient strength to support the load. Modern bridges use steel cables. Chain bridges do not provide a good example of a weakest link public good.

Manifestly, there is a substantial gap between the empirical record set by the examples offered in the extant literature on weakest-link public goods and the orthodox theoretical presentation of the weakest-link public good concept. It is simply not the case that when one individual performs poorly in the provision of the putatitve weakest-link public good that all other individuals will also end up consuming little or no units of the good. The empirical record suggests that in some cases an individual acting alone can privately provide the good at hand. And of the other examples offered in the extant literature on weakest-link public goods they do not appear to be adequately described by the Leontief-like technology first aired in Hirshleifer's (1983) paper on weakest-link public goods. A poor effort by a single person need not mar the consumption of all other individuals.

5. CONCLUSIONS

Undoubtedly it is possible to carry out an analysis of a good in which each and every individual must consume an identical amount of the good as determined by the minimal contribution under both private and collective provision. The problems raised in this paper would be thereby ruled out of court. This endeavour might be of some analytical interest to the pure theoretician but it comes at the cost of creating extremely lean pickings for the public economist who is interested in applying the concept to resolving practical issues. And of greater note, under the more realistic setting explored here the strong conclusions of the orthodoxy on just what the weakest-link public goods concept entails no longer hold. In the case of most of the examples offered of the weakest-link public goods concept individuals can privately consume different amounts of the good: individuals are not restricted to privately consuming the minimal contribution made by some other individual. And once the issue of private provision is broadened to take on the possibility that individuals can privately supply the good independently of the action of others, then the interesting issue of economies of scale from collective provision needs to be taken on board before any pronouncement can be made on whether the market fails in the provision of pseudo weakest-link public goods.

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ENDNOTES

[1] See the papers by Hirshleifer (1983, 1985) for the seminal contributions to the literature of public goods.

[2] See Vicary (1990) and Conybeare, Murdoch and Sandler (1994) for further analysis of the concept.

[3] The analysis of weakest-link public goods in terms of a simple partial equilibrium framework is, to the best of my knowledge, a novel aspect of this note. Hirshleifer's initial exposition (1983, 1985) was conducted, for example, in terms of truncated lexicographic indifference curves and kinked reaction paths, certainly a novel treatment even for public economics which has a history of using geometrical treatments which at times border on the arcane. Hirshleifer's most recent treatment with David Hirshleifer (1998) is carried out in terms of the normal form found in game theory. The proposed in the paper here has two advantages over existing treatments. It allows a straight forward comparison to be made to the more familiar summation case. And, in particular, it allows the production side to be directly featured in the analysis.

[4] There is, of course, no requirement that all of the gain from collective provision should go to just individual B. Other tax-sharing arrangements are possible in which both individuals gain from collective provision. Individual A need only be assigned a slightly lower tax-price than T_A for A to gain from collective provision. Individual B's gain will be, of course, somewhat less as he now pays a higher tax-price. The limit of this adjustment occurs where B's loss from paying the higher tax price just matches the gain from being able to consume at a lower tax-price than 2MC and a higher dyke than what he could provide himself.

[5] Even this conclusion may require some qualification. Individual A *could* extend the height of the collectively-provided dyke along his segment of dyke bordering on the sea to H_A^2 by spending the amount represented by $H^{**}JN\,H_A^2$, and building at the same time an interior wall of matching height at a total cost of $0VG\,H_A^2$. On the assumption that an individual can carry out additional building on the collectively provided dyke, one has to admit that the amount consumed of a weakest-link public good under collective provision is not necessarily restricted to the amount provided on the basis of the summation of all preferences. A moment's inspection of Figure 1 should indicate that individual A *will not* extend his protection here since the additional benefits are outweighed by the additional cost, i.e., $H^{*}CN\,H_A^2$ is less than $H^{**}JN\,H_A^2$ plus $0VG\,H_A^2$.

[6] Even within the confines of the simple example put forward here the economies of scale from collective provision can exceed 5MC - 4MC equals 1MC. If, for example, each individual occupies an equal triangular area, then the economies of scale from collective provision is $\left[2\mathrm{MC} + \sqrt{2\mathrm{MC}} + 2\mathrm{MC}\right] - 4\mathrm{MC}$ equals $\sqrt{2\mathrm{MC}}$. Many scenarios are possible depending on the assumptions made about the shape of the island and the distribution of the land between the occupants. The economies of scale from collective provision can also exceed 1MC, at least over a range of output, under different assumption about the sequence in which the dyke is built. In the main part of the paper I have set aside all strategic problems by assuming that individual A with the higher demand for protection first builds a dyke around the whole perimeter of his land under voluntary provision. Individual B can then exploit A's provision in that he needs to only build a dyke around the perimeter of his land bordering on the sea. But other scenarios are possible. Suppose, for example, that in the absence of any knowledge about the other person's preferences, individual A waits for B to build his dyke first. If B goes ahead with building, then he will build a dyke around the perimeter of his land at a marginal cost of 3MC. If individual A has a preference for a higher dyke than B, then he will have to build a higher dyke around the perimeter of his land at a marginal cost of 3MC. The gain from being able to achieve coordination under collective provision is 6MC - 4MC equals 2MC.

[7] If individual A is asked to pay a Lindahl tax for the collectively provided dyke of height H**, then he will prefer the market arrangement. Under the collective arrangement, not only would A face a higher per unit price than he bears under the market, he also consumes a smaller quantity of protection. Under these circumstances individual B will not be able to convince individual A to give up the freedom of independent action.

[8] In less colloquial terms, the guitarist played poorly but the percussionist and drummer kept the beat.