The Simple Macroeconomics of a Monopolised Labour Market

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Abstract

This paper examines the problem of unemployment in the context of a monopolised labour market. It is shown that a rigid real wage serves the maximisation of the expected utility of the labour force. It is shown that a labour monopoly that chooses *both* the wage rate *and* work practises may choose to both overman and impose a wage floor. Both socially wasteful unemployment and wasteful excess manning emerge as features of uncompetitive labour markets.

Socially costly unemployment constitutes a central intellectual difficulty for macroeconomics. It does so on account of the apparent incongruence of such unemployment with the central model of economics, competitive markets. It may be said that a large amount of macroeconomics has been concerned to dissolve this appearance of incongruence of competitive markets with socially costly unemployment. This concern embraces, for example, both Keynesian macroeconomics and New Classical macroeconomics. However, different they may be their assumptions ('ad hoc' propensities versus optimisation), however different they may be in their explanations (effective demand versus monetary surprises) and however different may be their policy conclusions (policy activism versus policy rules), they both seek to reconcile the existence of socially costly unemployment and competitive labour markets.

An alternative response to this problem of the incongruence of socially costly unemployment with competitive markets is to stop trying to solve it. An alternative response is to search for a comprehension of unemployment in terms of uncompetitive labour markets. The theory of monopoly suggests this may be a promising undertaking, as this theory famously maintains that a monopolised market is consistent with surplus and wasted resources. This paper, therefore, undertakes to re-examine the problem of unemployment in the context of a monopolised labour market. It undertakes to scrutinise the equilibrium and comparative statics of a suite of simple one period macro models, in which output is produced by the standard two factor neoclassical aggregate production function, and in which the entire labour force is monopolised.

The analyses of this paper implies that unemployment and wage rigidity may characterise the monopoly union economy. The monopoly union that chooses the wage rate may put a floor on the wage rate, such that a sufficient decline in labour demand will generate unemployment, rather than wage reductions.

However, the analysis also reveals that a monopoly union that exploits its monopoly power by choosing the degree of overmanning, rather than the wage rate, will secure an outcome that is, by its own criterion, equal to or superior to the outcome of the wage-floor model, and at no cost to unemployment. Consequently, the phenomenon of overmanning ("work practises") jeopardises any labour monopoly explanation of unemployment. The issue of overmanning cannot be discounted as just an add-on in any analysis of a monopoly labour market. It must be assimilated.

In the last section we show that a labour monopoly that chooses *both* the wage rate *and* work practises may choose to both overman and impose a wage floor. Both socially wasteful unemployment and wasteful excess manning emerge as features of uncompetitive labour markets.

1. The Literature

Before 1980 the literature on the macroeconomic significance of monopolisation of labour markets was slight.

Prior to the Keynesian Revolution wage rigidity was a pat explanation of unemployment, and unions would figure casually as a significant cause of that rigidity. But little was done to pursue this hypothesised connection of unions and wage rigidity. A telling illustration is that Pigou's *Theory of Unemployment* of 1933, which might be expected to say *something* about the issue of unions and unemployment, seems to say exactly nothing.[1]

From 1936 the thesis that labour monopoly is a source of unemployment was not just neglected, but almost proscribed. Any notion that wages were the source of unemployment was squarely rejected by Keynes at the opening of the *General Theory*. Consequently, any hypothesis that the lack of competition in labour markets could be the source of unemployment could not be entertained. In that book's

[1] In the second decade of the 20th c two book with very similar titles (*Unemployment and Trade Unions*, *Unemployment and American Trade Unions*) appeared on opposite sides of the Atlantic, each written by a young author, who it appears, never published in economics (or anything) again. (Cyril Jackson (1910) *Unemployment and Trade Unions,* London: Longmans, Green; and David Paul Smelser (1919) *Unemployment and American trade unions,* Baltimore : Johns Hopkins Press).

marvellously detailed 2,300 entry index there are precisely 3 references to trade unions.

The Monetarist counter-revolution, and its New Classical extension, did nothing to reverse this lack of interest in monopolisation of labour markets. One tenet that Keynes and Chicago held in common was the macroeconomic insignificance of imperfect competition in labour (or product) markets. During the period of Monetarist versus Keynesian controversy, interest in labour monopoly and unemployment came from those outside the 'normal science' of macroeconomics, such as F.A. Hayek (1980).

It was with the exhaustion of the Keynesian/Monetarist dispute that interest in unions and an unemployment has acquired life. Since about 1980 the macroeconomic significance monopolised labour markets has been the subject of substantial research (see Sanfey (1995) for a survey of some literature, and Creedy and MacDonald (1991)). Topics that have received particular attention include,

- The notion that unions serve 'insiders' to the cost of outsiders'
- Specification of the monopoly union's 'utility function'
- The outcome of bargaining in a bilateral monopoly, with capital on one side and labour on the other.)

I would suggest that there is something of the mood of 'new Keynesian' economics, (ie "new neo-Keynesian") about this surge of interest in unions and labour monopoly. It is "new neo-Keynesian" in that it takes stickiness seriously (that is the neo-Keynesian part), and it takes stickiness seriously by providing optimising microfoundations for it (that is the 'new' part). That 'new neo-Keynesian' mood is underlined by other work done at this time (eg macroeconomic significance of imperfect competition in product markets.). This surge of interest in unions and employment is one part of what Solow described in 1985 as a "true renaissance" in the search for "adequate" labour microfoundations for "mainstream macroeconomics", "with alternative models appearing monthly" (Solow 1985, 411).

As a macroeconomist my only grumble about this literature is that there has be too much microfoundations and not enough macroeconomics. The flavour of many such papers is labour economics seasoned with industry economics. And this is ‘no accident’: the spirit of many (not all) of these models is to not to add to macroeconomics: but to rationalise what is already there; to rationalise wage rigidity, and thereby rationalise what Solow called “accepted” macroeconomic theory. My interest, by contrast, is in seeing how labour monopoly might contribute to macroeconomic theory. The research strategy of this paper, therefore, is to insert a labour monopoly boldly into a simple macroeconomic model, and see what happens.

2. The Assumptions

The paper uses a very simple model.

- There is one period. Intertemporal issues are ignored.

- Output is produced by capital and labour under constant returns to scale.

$$\frac{Y}{K} = q(\frac{L}{K}) \qquad q' > 0 \quad q'' < 0 \qquad (1)$$

Y = output, L = labour, K = capital

- The product market is perfectly competitive.
- All decision makers are perfectly informed.
- The labour force is composed of Σ identical members.
- Every member of the labour force has an L shaped labour supply function. The kink occurs at 1 unit of labour supply.
- Leisure is valued at an exogenous amount, b (< w = the real wage).
- Each member of the labour force is either employed for one unit, or unemployed. Any unemployment is randomly allocated across members of the labour force.
- The labour market is completely ‘monopolised’. The terms of the sale of all labour is the prerogative of one unitary agency, the labour monopolist. This agency is

mostly easily thought of as a union that has been endowed by the legislature with these prerogatives. But the agency might also be thought of the legislature itself.

- All members of the labour force share equally in the control of the labour monopolist ("union"). This union is a universalist union. There is no insider/outsider issue, because there are no outsiders .

Finally, we assume the labour monopolist can either set a minimum wage rate, or it can set maximum amount of work per worker.

3. The wage fixing monopoly union

We begin with a labour monopoly that fixes the wage rate.

The expected utility of every member of the monopoly union is,

$$EU = pU(w) + [1-p]U(b) \tag{2}$$

where

b = value of liesure (both non-pecuniary and non pecuniary)
p = the probability of employment.
w = real wage

Therefore w is chosen to maximise (2).

We explore the implications of maximisation in four cases.

3.1 Risk neutrality and valueless leisure

The simplest case assumes b = 0, and risk neutrality. The maximand then becomes,

$$\max_{w} : EU = \frac{L}{\Sigma} w \tag{3}$$

L = employment

Σ = labour supply

First and Second Order Conditions

Maximisation yields as a First Order Condition,

$$\frac{\partial EU}{\partial w} = L[\frac{w \partial L}{L \partial w} + 1] = 0 \tag{4}$$

or,

$$\frac{\partial EU}{\partial w} = L[1 - e] = 0 \tag{5}$$

e = the positived elasticity of the demand for labour to the wage rate

Consequently,

e = 1 First Order Condition (6)

The Second Order Condition requires,

$$\frac{\partial e}{\partial w} > 0 \qquad \text{Second Order Condition} \qquad (7)$$

Figure 1 plots a relation between w and such that the two maximisation conditions are satisfied. In Figure 1 the competitive (full employment) wage is w_c , and the monopoly wage is w_m. But neither the First Order Condition, nor the Second Order Condition, need be satisfied. **Figures 2** and **3** show cases where neither the First Order, nor Second Order, Condition is satisfied: there is no stationary point. **Figure 4** shows a case where the First Order Condition (e=1) is satisfied, but the Second Order Condition (∂e/∂w > 0)is not. If e= 1 then the wage bill is *minimimised.*

It proves convenient to express the First and Second Order Conditions in terms of the elasticity of substitution, and the profit share. It is an easy matter to show,

$$\frac{s}{p} = e \qquad (8)$$

σ = elasticity of technical substitution

and

$$if \quad s > 1 \quad then \quad \frac{\partial p}{\partial w} > 0 \qquad (9)$$

$$if \quad s < 1 \quad then \quad \frac{\partial p}{\partial w} < 0$$

Thus the Conditions may be restated,

$$\frac{s}{p} = 1 \qquad \text{First Order Condition} \qquad (10)$$

$$\sigma < 1 \qquad \text{Second Order Condition}$$

Figure 5 plots W (=wL) and e against w when $\sigma < 1$. **Figure 6** plots W and e against w when $\sigma = 1$. **Figure 7** plots W and e against w when $\sigma > 1$.

Clearly there is no monopoly equilibrium if $s \geq 1$. In these cases wage rises will only reduce the wage-bill, and the expected wage. In these cases the monopolist will maximise the wage bill by obtaining full employment, through setting the wage equal to the competitive level.

Further, it is also clear that there may be no monopoly equilibrium even if $\sigma < 1$. Even if $\sigma < 1$, there will be no monopoly equilibrium if $\sigma/\pi > 1$ at full employment. In that situation wage rises above the full employment level will only reduce the wage-bill. If $\sigma/\pi > 1$ the monopolist will maximise the wage bill by obtaining full employment, and setting the wage equal to the competitive level.

Thus a monopolised labour market may yield results identical to the competitive market, ie full employment. This is no embarrassment to the model. Obviously, episodes of full employment have occurred, and some have been lengthy. It would be more embarrassing if the model predicted the necessity of unemployment.

Henceforth we will concern ourselves with monopoly equilibria distinct from competitive equilibria.

Table 1 shows the magnitude of monopoly gains in the wage rates associated with various rates of optimising unemployment, given an elasticity of substitution of 0.5.

Table 1: A Calibration of the Impacts of Monopolisation

The deviation from competitive outcomes corresponding to ‘optimal’ magnitudes of unemployment

σ=0.5

Unemployment	5 percent	10 percent	20 percent
Wage	5.3	11.4	26.6
Wage bill	0.1	0.6	2.4
Expected utility	0.1	0.6	2.4

The gains of Table 1 are small. If we allowed risk aversion the gains would be still smaller, as the risk of unemployment would be costly in utility terms. On the other hand, if we allowed for leisure the gains would be larger, as the loss in wage income would be partly compensated for by the increase in leisure.

But although the gains in Table 1 are small, they are, nevertheless, gains. The expected utility of each worker has risen, despite the existence of unemployment. Thus the monopoly model casts quite a different light on the “problem” of unemployment. For the workforce this problem is not a problem: it is their strategy for maximising of their expected utility. For ‘society as a whole’ the unemployment is a problem, as national product is reduced. But for workers the abolition of unemployment by means of a wage reduction would reduce their expected utility. It is true that a monopoly wage increases only the expected utility: and that those workers who end up missing out on work have a lower utility than under full employment. But these workers are comparable to persons who have made a gamble at favourable odds, but have (despite the favourable odds) lost. Such persons will repeat the gamble if the opportunity is re-presented. And the workers who have lost (ie become unemployed) are content to repeat the gamble (ie have the wage set at the monopoly level) next period.

Comparative statics

The comparative-statics turn on the First Order Condition, that determines the labour: capital ratio.

$$s = p\left(\frac{L}{K}\right) \qquad (11)$$

With the labour:capital ratio determined by (11) the wage rate is determined by the marginal productivity of labour. And with the labour:capital ratio determined by (11) L is determined, by the exogenous magnitude of the capital stock. We can further infer,

> Employment is unit elastic to the capital stock.
>
> The wage has a zero elasticity to the capital stock
>
> Employment and the wage have a zero elasticity to Σ.

These comparative-statics flow from the equilibrium labour/capital ratio being unchanged by either K or Σ.

These comparative-statics also suggest how labour monopoly exhibits some manifestations of real wage rigidity. The rate of unemployment is no way figures in the determination of the wage: the wage rate is completely insensitive to its magnitude. An increase in Σ will not depress the wage. A decline in the capital stock will not depress the wage. The demand for labour curve will be shifted leftwards by a decline in capital, but the only labour market consequence is a rise in unemployment. Conversely, an increase in capital, will shift the labour demand curve rightwards but the wage shall not be increased, and unemployment shall fall. Once the demand for labour has shifted sufficiently rightwards that full employment has been reached, the wage shall rise. Thus for a given technology, a monopoly union puts a floor on the wage, and an L shaped relation between the wage rate and employment emerges (**Figure 8).**

Monopoly, we may say in summary, provides an explanation of real wage rigidity: a rigid real wage serves the maximisation of the wages bill.

The wage rigidity of labour monopoly can be further understood by extending the analysis to include technical change. If $\dot{l}$ = rate of labour augmentation and $\dot{k}$ = rate of capital augmentation, the First Order Condition becomes,

$$s = p\left(\frac{[1+\dot{l}\,]L}{[1+\dot{k}\,]K}\right) \qquad (12)$$

We may infer;

$\frac{\partial w}{w\partial \dot{l}}$ = 1. The wage has a unit elasticity to the labour augmenting technical progress

$\frac{\partial L}{L\partial \dot{l}}$ = -1. Employment has a negative unit elasticity to labour augmenting technical progress.

The wage is increased by the rate of labour augmenting technical progress; and employment is reduced by that rate. Effectively, the wage floor is being increased by labour augmenting technical progress[2], but as labour demand schedule is shifted neither right nor left [3], employment falls. (**Figure 9**).[4]

To sum up; in a monopolised labour market the wage rate is shaped is not factor supplies (K, Σ), but labour augmenting technical progress alone. The monopoly wage rate will grow according the rate of labour augmenting technical progress. Any upward trend in λ will entail an upward trend in the wage floor, and consequent upward pressure on unemployment, that can only be countered by an upward trend in K .

Some further insight into the character of wage rigidity in the monopoly model is supplied by the consideration of taxation.

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[2] No other parameter affects the wage floor. Increases in K and $\dot{k}$ will shift the demand curve but not increase the floor.

[3] If e =1 then changes in $\dot{l}$ have no impact on the labour demand curve. The reduction in the marginal product of work is just matched by the increase in the amount of work done by a unit of labour, leaving the marginal product of labour unchanged. The demand curve shifts neither left or right.

[4] We may further infer from the First Order Condition,

$\frac{\partial L}{L\partial \dot{k}}$ = 1; Employment is unit elastic to the capital augmenting technical progress.

$\frac{\partial w}{w\partial \dot{k}}$ = 0. The wage has a zero elasticity to the capital augmenting technical progress.

These results turn on the effective labour:capital ratio being tied down by the First Order Condition, and unchanged by any technical progress. The preservation of this ratio implies actual employment to grow by the amount of capital augmenting progress ( ); and the wage to be unaffected by capital augmenting technical progress.

In the competitive model, with a zero elasticity of labour supply, both income taxation and payroll taxation,

(i) have no impact on employment,

(ii) fall entirely upon the effective wage (ie wage less any tax) received by the worker, and

(iii) fall not at all upon the effective wage (ie wage plus any tax) paid by the employee.

These conclusions remains true in the monopoly model: under both an income tax and payroll tax the effective wage received by the worker falls by the amount of the tax, the effective wage paid by the employer is unchanged, and (consequently) employment unchanged.[5]

These conclusion about the incidence of taxes in the labour monopoly model conflict with a widespread presumption that wage rigidity is characterised by the effective wage to worker being invariant to tax. Thus, specifically, the presumption regarding wage rigidity is that if payroll taxes increase, then the wage rate will not fall, effective wages paid by the employer will rise, and consequently employment will fall.[6] But no such presumption is warranted by this model. Effective wages of employees are at the mercy of the tax rate. If payroll taxes rise, wages will fall (and so reduce effective wages of employees), leaving employment unaltered. Thus this model does not

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[5] The maximand under an income tax is,

$$\max_{w} : \frac{L}{\Sigma} w[1-t] \equiv \max_{w} : \frac{L}{\Sigma} w$$

The solution for w and L are unchanged.

The maximand under a payroll tax is,

$$\max : \frac{L(w[1+t])w}{\Sigma} \equiv \max : \frac{L}{\Sigma}\frac{L^{-1}(L)}{1+t} \equiv \max : \frac{L}{\Sigma} L^{-1}(L)$$

The solution for L is unchanged. w is reduced by the amount of the payroll tax,
The identical result means there is no reason to prefer income tax over payroll tax, or visa versa

[6] If income taxes rise, then wages do not rise, effective wages of employees falls, effective wages paid by the employer are unchanged, and so is employment is

recommend cutting payroll taxes to stimulate employment, since wages will just increase by the amount of the reduction.

3.2 Risk neutrality and valuable leisure

The conclusions of the previous section are not greatly revised by allowing for the value of leisure. Suppose that there is some benefit from non-employment; b.

$$EU = pw + [1 - p]b \tag{13}$$

The wage is chosen,

$$\max_{w} \quad EU = \frac{L}{\Sigma} w + [1 - \frac{L}{\Sigma}]b \tag{14}$$

First and Second Order Conditions

The First Order Condition is,

$$e = \frac{1}{1 - b/w} > 1 \tag{15}$$

(15) states the optimal elasticity now exceeds one. [7] The wage bill, and the expected wage, is , therefore, no longer maximised. The wage is increased above its wage bill

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unchanged. If income tax rises, wages will rise and consequently effective wages paid by employers will rise, and employment to fall.

[7] The Second Order Condition requires,

$$\frac{\partial e}{\partial w}[1 - b/w]^2 w > -b/w.$$

∂e/∂w may now be negative. This means that a monopoly equilibrium, with consequent higher wage and unemployment, may occur even if $s \geq 1$.

(and the expected wage) maximising magnitude. This is done on account of the benefit which is now obtained, in the form of leisure, by higher unemployment. To put it another way: unemployment is no longer worth just the wages it buys, as it also buys leisure. Unemployment has become more valuable.

Comparative statics

The first order condition can be written,

$$\frac{s}{p(\frac{L}{K})} = \frac{1}{1-\frac{b}{w(L/K)}} \tag{16}$$

As before a unique effective labour capital ratio is implied by the optimising condition. The comparative statics about K, $\dot{l}$ and $\dot{k}$ remain the same. The wage rigidity remains. The wage floor is lifted by $\dot{l}$. Employment is stimulated by K and $\dot{k}$, and depressed by $\dot{l}$. The one novel comparative: static concerns the impact of an increase in b. An increase in b increases the wage, reduce L, and increases unemployment.[8]

[8] The conclusions concerning taxation are also revised. An increase in tax rates no longer falls wholly on the worker, but is borne in part also by the employer. An increase in tax rates will lead to some increase in the effective wage paid by the employer, and so some reduction in employment. There is now some recommendation of tax-cuts as a policy for reducing unemployment.

3.3 Risk aversion and valueless leisure

The conclusions of section 3.1 are also not greatly revised by allowing for risk aversion. Suppose that we take up the assumption that there is risk aversion, while reverting to our assumption that leisure is valueless.

If we assume a constant relative risk aversion utility function,

$$U = \frac{y^{1-r}}{[1-r]} \qquad r>0 \tag{17}$$

r = co-efficient of relative risk aversion.

then the wage is chosen to maximise,

$$\max_{w} \quad EU = \frac{L}{\Sigma} \frac{w^{1-r}}{[1-r]} \tag{18}$$

The First Order Condition now becomes,

$$e = 1\text{-}r \tag{19}$$

The Second Order Condition remains,

$$\frac{\partial e}{\partial w} > 0 \tag{20}$$

Non-zero relative risk aversion reduces the optimal elasticity to 1-r < 1. This reduction requires a reduction in the wage, and unemployment. **Figure 10** illustrates the reduction in the wage.

The larger r the closer the wage is reduced to its competitive level. The reduction in the wage may be such that full employment is obtained. In this situation the risk of unemployment is not considered worth any attainable increase in the wage.

All the comparative statics remain unaltered.

3.4 Risk aversion and valuable leisure

Allowing both risk aversion and valuable leisure essentially repeats the conclusions of earlier sections. With risk aversion and valuable leisure the union maximises,

$$\max_{w} \quad EU = \frac{L}{\Sigma}\frac{w^{1-r}}{[1-r]} + [1-\frac{L}{\Sigma}]\frac{b^{1-r}}{[1-r]} \tag{21}$$

The First Order Condition is,

$$e = \frac{1-r}{1-(b/w)^{1-r}} \tag{22}$$

The optimal elasticity may now be either greater than OR less than 1.

Comparative-statics

As before, employment remains unit elastic to capital, and the wage remains zero elastic to capital.[9] Employment has a negative elasticity to $\dot{l}$, but it is now less than 1 in absolute value.[10]

An increase in r reduces the elasticity, and so the wage floor, and so increases employment. An increase in b increases the elasticity, and so the wage floor, and so reduces employment.

4. The Work-Maximum Monopoly Union

We now suppose that the union has no control over the wage rate. That is set by competitive forces. But the union does control work practises. More particularly, it can require more labour be employed than is technically necessary. We let μ represent the amount of extra labour that must be employed with 1 unit of necessary labour; is a kind of factor of "feather bedding" or overmanning factor.[11]

The production function is now,

$$\frac{Y}{K} = q(\frac{L/1+m}{K}) \tag{23}$$

$$\text{effective labour} = \text{“work”} = \frac{L}{1+m}$$

[9] Proof: a unit elasticity of L to K preserves e and w, and the satisfaction of the FOC.As before, $\dot{l}$ reduces L, and increases the wage floor.

[10] A negative unit elasticity would preserve the effective labour: capital ratio (and e), and increase the wage, thereby reducing the LHS of the FOC, but leaving the RHS unchanged. Equilibrium requires the effective labour: capital ratio to rise, thereby reducing equilibrium e. Consequently, L would not fall be so much.

The union wishes to choose μ so as to maximize expected utility, as before. But now all labour is employed, by competitive processes, so the probability of employment is one, and the probability of unemployment is zero. So the maximisation problem amounts to maximising w.[12]

In formal terms,

$$\max_{\mu} : w = \frac{1}{1+\mu} q'(\frac{\Sigma / 1+\mu}{K}) \qquad (24)$$

How can μ maximise w? By shifting up the demand curve for labour the maximum amount. An increase in μ provides both a positive and a negative stimulus to the marginal productivity of labour corresponding to any quantity of labour. On one hand, each person works μ less and so produces less: this is the negative stimulus to the marginal product of labour. On the other hand, the reduction in the quantity of effective labour ("work") for a given quantity of labour, increases the marginal product of effective labour ("work"); this is the positive stimulus to the marginal product of labour. In summary, each worker works less during their period of hire, but produces more whenever they are working. The net impact on their total product for their period of hire may be either positive or negative. (**Figures 11** and **12** illustrate). If it nets out as positive then an increase in μ will increase the wage, and μ will be increased. μ will be increased until the net impact falls to zero, and the optimal degree of overmanning had been secured. The First Order Condition is,

$$-q''(\frac{\Sigma / 1+\mu}{K})[\frac{\Sigma / 1+\mu}{K}] = q'(\frac{\Sigma / 1+\mu}{K}) \qquad (25)$$

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[11] Perhaps the easiest way to conceptualise μ is to suppose that each labourer actually works for only 1-μ of the time they are employed.

[12] Notice that there is no interest in overmanning in the hope of reducing unemployment: there is no unemployment.

This FOC can be written more intelligibly,

$$e = 1 \qquad (26)$$

(26) implies μ will be increased until the elasticity of demand for labour is 1. This is makes sense. If μ is at its optimal value then an increase in μ provides zero stimulus to the demand for labour, ie. zero stimulus to the marginal productivity of labour. But this requires that the percentage increase in the marginal product of work following from a given percentage decline in work equals that given percentage decline in work; ie marginal productivity of work is unit elastic to work. But this means that the marginal productivity of labour is unit elastic to labour, ie the wage is unit elastic to labour demand.

The Second Order Condition can be written,

$$\frac{\partial e}{\partial m} > 0 \qquad (27)\,.$$ [13]

[13] *Zero-overmanning outcomes*

The section has assumed that First and Second Order Conditions are satisfied. But the First and Second Order Conditions permit us to infer that a labour monopoly may *not* overman. From the Second Order Condition we may infer that a union monopoly cannot obtain any benefit from overmanning if $\sigma \geq 1$. If $\sigma \geq 1$ then optimal overmanning is zero. Further, even if $\sigma < 1$, a union monopoly would not obtain any benefit from overmanning if the demand for labour is elastic when $\mu = 0$. The First Order Condition could then only be satisfied by $\mu < 0$. But μ cannot < 0, therefore the First Order Condition cannot be satisfied. There is no room to exploit overmanning

Parameterising the elasticity of substitution allows a more compact presentation of the two conditions,

FOC: $$s = p\left(\frac{\Sigma / 1 + m}{K}\right)$$

SOC: $\sigma < 1$.

Comparative Statics

The comparative statics turn on the First Order Condition. That Condition determines a unique magnitude of $\frac{\Sigma / 1 + m}{K}$, the effective labour: capital ratio. That magnitude leaves μ, the amount of overmanning, to be determined by the size of K and Σ.

An increase in K will lead to an equiproportionate decrease in μ.

$$\frac{\partial mK}{\partial Km} = -1 \qquad (28)$$

This reduction in overmanning will mean output increases equiproportionately in response to the increase in K. This reduction in overmanning will also increase the wage, as each worker works more (and the marginal productivity of work is unchanged on account of the unchanged effective labour: capital ratio). The profit rate remains unchanged.

An increase in Σ will lead to an equiproportionate increase in μ.

$$\frac{\partial m\Sigma}{\partial \Sigma m} = 1 \tag{29}$$

This increase in overmanning means all of any increase in labour force is mopped up without any unemployment. This increase in overmanning also means there will be no increase in product. It will also reduce the wage rate. The profit rate remains unchanged.

The impact of technical change can also be analysed. If $\dot{l}$ = rate of labour augmentation and $\dot{k}$ = rate of capital augmentation, we may write

$$s = p\left(\frac{[1+\dot{l}]\frac{\Sigma}{1+m}}{[1+\dot{k}]K}\right) \tag{30}$$

One may conclude,

- $\frac{\partial m}{\partial \dot{l}} = 1$ An increase in labour saving technical progress will be matched (and cancelled) by an equal increase in labour over manning. The wage rate does not change. The profit rate does not change. Product does not change. Labour saving technical progress is a complete non-event.

- $\frac{\partial m}{\partial \dot{k}} = 1$ An increase in capital saving technical progress will be matched (and reinforced) by an equal reduction in labour over manning. The profit rate remains unchanged, but the wage rate goes up by the amount of technical progress.

Comparisons of the minimum wage and maximum work economies

There are obvious parallels between the "work maximum" case and the simplest wage-minimum model explored in section 3.1 (with r = b = 0). Both cases share an identical optimisation, e =1, that implies a number of parities between the wage minimum and productivity maximum models. These are listed in Table 2.

Table 2: the equilibrium of a wage-minimum and a productivity maximum compared

	Wage minimum	Productivity maximum	Difference
Optimising Condition	$s = p(\frac{L}{K})$	$s = p(\frac{\Sigma/1+m}{K})$	Analogous
L	$\Sigma/1+\mu$[14]	Σ	Larger
Effective L/K	$\frac{\Sigma/1+m}{K}$	$\frac{\Sigma/1+m}{K}$	Same
w	$q'(\frac{\Sigma/1+m}{K})$	$\frac{1}{1+m}q'(\frac{\Sigma/1+m}{K})$	Smaller
W	$\frac{\Sigma}{1+m}q'(\frac{\Sigma/1+m}{K})$	$\frac{\Sigma}{1+m}q'(\frac{\Sigma/1+m}{K})$	Same
Profit Rate	$q(\frac{\Sigma/1+m}{K}) - q'(\frac{\Sigma/1+m}{K})\frac{\Sigma/1+m}{K}$	$q(\frac{\Sigma/1+m}{K}) - q'(\frac{\Sigma/1+m}{K})\frac{\Sigma/1+m}{K}$	Same

Table 2 indicates that in the wage minimum model employment is lower and the wage and average productivity of labour is higher. But the wage bill is exactly the same, and the profit rate is exactly the same.[15]

[14] As $\frac{L}{K} = \frac{\Sigma/1+m}{K}$, we may infer that $L = \Sigma/1+m$.

The comparative-statics of the two models show more divergence, as Table 3 shows.

Table 3: the comparative-statics of a wage-minimum and a work maximum compared

	Wage minimum	Work maximum
Σ	L unchanged	L increased
	w unchanged	w reduced
	ρ unchanged	ρ unchanged
	Y unchanged	μ increased
		Y unchanged
K	L increased	L unchanged
	w unchanged	w increased
	ρ unchanged	ρ unchanged
	Y increased	μ reduced
		Y increased
λ	L reduced	L unchanged
	w increased	w unchanged
	ρ unchanged	ρ unchanged
	Y unchanged	μ increased
		Y unchanged
κ	L increased	L unchanged
	w unchanged	w increased
	ρ unchanged	ρ unchanged
	Y increased	μ reduced
		Y increased

[15] In both the wage-fixing and productivity –fixing model the wage bill is higher than in the competitive outcome, and output is lower

What is invariant is that in both cases the labour supply, and labour augmenting technical progress, has no effect on product; while capital accumulation (and capital augmenting technical progress) have a positive effect on product.

Also invariant is the profit rate : none of these shocks affect effect it. So although monopolised labour reduces the profit rate, it perfectly insulates it from all shocks.

minimum wage or maximum work?

Would the monopolist prefer to put a minimum on wages or a maximum on work? The risk neutral monopolist would be indifferent between the two. The wage bill is identical in each case, and that means the expected wage is identical in both cases.

But if risk aversion is introduced (while keeping b = 0) the monopolist would prefer the work maximum model. This is because the expected wage in the work maximum model is the same as in the wage minimum model, but (unlike the wage minimum model) there is no positive probability of unemployment.

On the other hand, if the valuable leisure is introduced (while keeping risk aversion zero) the monopolist would prefer wage minimum model. This is because (as explained before) the wage minimising monopolist can do better than maximising the wage bill; they find it optimising to raise the wage such that the wage bill falls, in order to “buy leisure” in the form of unemployment. The work maximising monopolist cannot obtain this outcome, and is stuck with maximising the wage bill.

5. A monopoly union that fixes minimum wage and maximum work

In truth, the labour monopolist does not have to choose between a wage minimum *or* a work maximum. It may choose to do both.

Suppose a union decides to set both a minimum wage *and* a degree of overmanning

$$\max_{w,m} \quad EU = \frac{L}{\Sigma}\frac{w^{1-r}}{[1-r]} + [1-\frac{L}{\Sigma}]\frac{b^{1-r}}{[1-r]} \qquad (31)$$

There are two First Order Conditions.

$$e = 1 \qquad (32)$$

$$e = \frac{1-r}{1-(b/w)^{1-r}} \qquad (33)$$

These two conditions can be given the following interpretation. (32) says that μ is chosen to push the demand curve for labour outwards to the maximum amount. Given that maximised demand curve, (33) tells how the wage rate is chosen to balance of the benefits of higher expected wage rate, and higher leisure, and against the cost of the disutility of the risk of being unemployed.

We have, or may have, a situation where both overmanning and unemployment occur.

Comparative-statics

(33) determines the wage. We are in a fix wage model, except that the determinants of the fix wage model are now psychic; utility liesure and risk aversion.

(32) determines the effective labour: capital ratio.

The fixed wage is linked to the fixed effective labour: capital ratio by the equality

$$w = \frac{1+\dot{l}}{1+m} q'\left(\frac{\Sigma[1+\dot{l}]\big/1+m}{K}\right) \qquad (34)$$

The comparative statics of this general or mixed model borrow from the minimum wage model and the maximum productivity model

The model resembles the minimum wage model in that increase in capital increases employment, and has no impact on overmanning. An increase in capital does not affect the effective labour: capital ratio (that is tied by (32)), or the wage (that is tied down by (33)). With both the effective labour:capital ratio and the wage unchanged μ is unchanged (see (34)). It is, therefore, employment, L, which must increase (equiproportionately) with K, in order to preserve the effective labour: capital ratio. We are, then, back to the minimum–wage model with K having no impact on overmanning, and all effect on employment.

But the model resembles the maximum work model in that labour saving progress does not increase the wage, or unemployment. An increase in $\dot{l}$ does not increase the effective labour: capital ratio (as that is tied down (32)), and does not affect the wage (as that is tied down by (33)). But with the wage unchanged, we may infer from (34) that μ must increase with $\dot{l}$. We are, then, back to the maximum work model with $\dot{l}$. having no impact on anything (including unemployment), except overmanning.

To summarise: unemployment is a story of excess supply of labour and insufficient capital, and where labour saving progress is completely irrelevant. Overmanning in this economy, by contrast, is purely a story of labour saving technical progress, and where labour supply and capital is irrelevant.

Finally, an increase in b will increase w, and reduce L and μ. In effect, the reduction in the costiliness of unemployment is inducing the monopolist to shift towards higher wages (and higher unemployment) in preference to overmanning.

equilibrium other than unemployment equilibrium

The two FOCs describe an unemployment/overmanning equilibria. They are subject to Second Order Conditions, which may not be satisfied. There are also constraints on L and μ (L≤Σ, and μ ≥ 0) which may mean that the First Order Conditions are not satisfied.

These possibilities spell possibilities other than the unemployment/overmanning equilibria.

1. Full employment and zero overmanning.

If at $L = \Sigma \quad and \quad m = 0$,

$$e > 1. \tag{35}$$

$$e > \frac{1-r}{1-(b/w)^{1-r}} \tag{36}$$

then there is full employment and zero overmanning. As e > 1 any increase in μ will only shift in the demand for labour leftwards, so there is no incentive to overman. And as a

$e > \frac{1-r}{1-(b/w)^{1-r}}$ an increase in w above the full employment level will only reduce expected utility of the worker.

2. overmanning and full employment.

Suppose that at $L = \Sigma$ there is some $m > 0$ such that,

$$e = 1 \tag{37}$$

and

$$e > \frac{1-r}{1-(b/w)^{1-r}} \tag{38}$$

then there is overmanning and full employment.

3. zero overmanning and unemployment.

Suppose at $m = 0$ there is some $L < \Sigma$ such that,

$$e > 1 \tag{39}$$

and

$$e = \frac{1-r}{1-(b/w)^{1-r}} \tag{40}$$

There is no overmanning and unemployment.

How are these three states related to the overmanning/unemployment state?

Suppose one begins at an overmanning/unemployment equilibrium. There is obviously a reduction in Σ sufficient to make L = Σ. The economy has arrived at an overmanning/full employment state. Further in reductions in Σ will now necessitate an increase in w, and reduction in μ in order to preserve e = 1. There will be some reduction in Σ sufficient that the $\mu = 0$. The economy has arrived at a zero overmanning/full employment equilibrium. Thus overmanning may be considered a signal of an excess supply of labour, and unemployment as a signal of acute excess supply of labour.[16]

Suppose, alternatively, that one begins at an zero-overmanning/ full employment equilibrium, but there is now a succession of positive values of $\dot{l}$. This will reduce the effective labour: capital ratio, and so reduce e until $e = \frac{1-r}{1-(b/w)^{1-r}}$. We are now in a situation of zero-overmanning and unemployment. Further positive values in $\dot{l}$ will further increase in the effective labour: capital ratio, and further reduce e until e =1. The economy is in a overmanning: unemployment equilibrium. Thus unemployment may be considered a signal of an excessive labour saving technical progress, and overmanning a signal of acute labour saving technical progress.

To summarise: a labour monopoly's first line of defence against excess supply of labour is overmanning, and its next line is unemployment. By contrast, a labour monopoly's first line of defence against excess labour saving productivity is unemployment, and its next line is overmanning.[17]

6. Conclusions

This paper has examined the equilibrium of an economy with a monopolised labour market. We have concluded that such an economy may both impose a wage floor and restrictive work practises.

[16] A parallel conclusion can be derived concerning capital. Overmanning is a signal of a deficient supply of capital, unemployment is a signal of an acutely deficient supply of capital.

[17] A full employment economy will be transformed into an unemployment economy by a sufficient increase in labour supply, a sufficient reduction in capital, or sufficient improvement in labour saving technical progress. A zero overmanning economy will be transformed into an overmanning economy by a sufficient increase in labour

The analysis of this paper is no more than a foray into the territory in question. In particular, the present paper has ignored: worker ownership of capital, saving, a variable labour supply, and effective demand issues of a monetary economy.[18] And we have maintained throughout the unrealistic assumption that jobs are allocated randomly, so that the probability of moving from an unemployed to an employed state is the same as the probability of moving from an employed to an employed state.

We have also assumed, rather than rationalised, the labour monopoly. A rationalisation is easy, but not invulnerable. On one hand, there is every incentive to monopolise: the expected utility of the worker increases. On the other hand the unemployment and overmanning is pareto inefficient and creates an incentive for capitalists, as a group, to bribe the workers away from the monopoly equilibrium. We are then left with a bilateral monopoly.

We have also imposed a limited strategy space on the monopolist: it can choose to put a minimum on wage and a maximum on work. We have not permitted the monopoly union to the length of the working week. Perhaps more importantly, we have not allowed the labour monopolist to negotiate over employment. Once we do, the maximizing requirements the monopolist will make of employers are simple: employ all the work force and pay them the average product of labour, less epsilon. Capitalists may, of course, league together to fight this: back to bilateral monopoly.

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supply, a sufficient reduction in capital, or sufficient improvement in labour saving technical progress.

[18] For the purposes of exposition we have rested heavily on the supposition of an inverse relation between the e and π, the profit share. But this is not necessary for some of the central results of the paper, such as the wage floor. e = 1 implies, $\frac{q'(L/K)}{q''(L/K)L/K}=1$. Thus L/K, and so the wage, is uniquely determined, regardless of the relationship between e and π.

Appendix of Derivations

Second Order Condition when b = 0 and r = 0.

$$\frac{\partial^2 M}{\partial w^2} = \frac{\partial[L[1-e]]}{\partial w} < 0$$

$$\frac{\partial^2 M}{\partial w^2} = -L\frac{\partial e}{\partial w} + [1-e]\frac{\partial L}{\partial w} < 0$$

$$\frac{\partial^2 M}{\partial w^2} = -L\frac{\partial e}{\partial w} < 0$$

The Elasticity of Substitution and the Elasticity of Demand for Labour

$$\frac{r}{w} = \frac{q - q'(l)\, l}{q y(l)}$$

Therefore,

$$\frac{1}{s} \equiv \frac{\partial r\ / w}{\partial l}\frac{l}{r\ / w} = -\frac{q\, q''\, l}{[q - q'l]q'}$$

By taking advantage of q' = w, l'' = ∂w/∂l, and ρ = q -q'l, we may rewrite this,

$$\frac{1}{s} = -\frac{\partial l\ w}{\partial w\, l} p$$

The standard result that ∂π/∂k < 0 if σ < 1, allows us to infer ∂π/∂w < 0 if σ < 1, and so ∂e/∂w > 0 if σ < 1.

Impact of a higher b on w when r = 0

The First Order Condition

$$e = \frac{1}{1 - b/w}$$

implies

$$e\frac{\partial b}{\partial w} = e - 1 + \frac{\partial e}{\partial w}[w - b]$$

But the Second Order Condition requires,

$$\frac{\partial e}{\partial w} > -\frac{b/w}{(1 - b/w)^2} = -b\frac{e}{w^2}^{2}$$

$$e\frac{\partial b}{\partial w} = e - 1 + \frac{\partial e}{\partial w}[w - b] > e - 1 - b\frac{e^2}{w_2}[w - b] = e - 1 - b\frac{e}{w} = e[1 - \frac{b}{w}] - 1 = 0$$

therefore,

$$\frac{\partial b}{\partial w} > 0$$

Maximum work

$$\max_{m} \quad EU = w = \frac{1}{1+m} q'(\frac{\Sigma/1+m}{K})$$

$$\frac{\partial w}{\partial m} = [\frac{-q'(\frac{\Sigma/1+m}{K})}{1+m} - \frac{q''(\frac{\Sigma/1+m}{K})[\frac{\Sigma/1+m}{K})}{1+m}]\frac{1}{1+m} = 0$$

$$\frac{\partial w}{\partial m} = [\frac{-w}{1+m} - \frac{L\frac{\partial w}{\partial L}}{1+m}]\frac{1}{1+m} = 0$$

$$\frac{\partial w}{\partial m} = [-1 + \frac{1}{e}]w = 0$$

Minimum wage and maximum work

$$\max_{w,m} \quad EU = \frac{L}{\Sigma}\frac{w^{1-r}}{r} + [1 - \frac{L}{\Sigma}]\frac{b^{1-r}}{r}$$

given

$$w = \frac{1}{1+m} q'(\frac{L/1+m}{K})$$

and

$$\frac{L/1+m}{K} = q^{-1}(w[1+m])$$

Normalising K =1, we may write

$$L = q^{-1}(w[1+m])[1+m]$$

and the maximisation problem becomes,

$$\max_{w,m} \quad EU = \frac{q^{-1}(w[1+m])[1+m]}{\Sigma}\frac{w^{1-r}}{r} + [1 - \frac{q^{-1}(w[1+m])[1+m]}{\Sigma}]\frac{b^{1-r}}{r}$$

The First Order Conditions follow.

References

Hayek, F. A. (1980) *1980s unemployment and the unions : essays on the impotent price structure of Britain and monopoly in the labour market*, Hobart Paper 87, London : Institute of Economic Affairs.

Layard, R and Nickell S. (1990) “Is Unemployment Lower if Unions Bargain Over Employment?”, *Quarterly Journal of Economics*, **105**, 773-787

McDonald, I.M. and J.Creedy (1991) "Models of Trade Union Behaviour: A Synthesis" *Economic Record*, **67**, 346-59

McDonald, I.M. and Rober Solow (1981), “Wage Bargaining and Employment”, *American Economic Review*, **71**, 896-908.

Pigou, A. C. (1933) *The Theory of Unemployment*, London : Cass.

Sanfey, Peter J (1995), “Insiders and Outsiders in Union Model”, *Journal Of Economic Surveys*, **9**(3).

Solow, Robert, (1985), “Insiders and Outsiders in Wage Determination”, Scandinavian Journal of Economics, **87**(2), 411-428.