

Intertemporal Equivalence Scales: Measuring the Life-Cycle Costs of Children

By

Paul Blacklow

School of Economics
University of Tasmania
GPO Box 252-85
Hobart 7001
Australia

Paul.Blacklow@utas.edu.au

Abstract

This paper provides a preliminary investigation into the lifetime cost of children upon a household's life-time wealth. By comparing the lifetime cost function of a household with children compared to the lifetime cost function of a household without children, an intertemporal equivalence scale can be constructed. By allowing the rate of time preference to vary according to demographics, more specifically with the number of children, the demographic effect on intertemporal allocations can be examined. Solving the model as a function of wealth allows the estimation of the rate of time preference and lifetime equivalence scale in a single cross section of data without the need for panel data on expenditures. The model is estimated for Australian data and finds that households with one child have rates of time preference 4% higher than those without. The estimated life-time cost of children or intertemporal equivalence scale, is highly sensitive to the interest rate.

Introduction

The use of equivalence scales has become common practice in order to make welfare or resource comparisons between households that differ in size and composition. Equivalence scales typically give the 'cost' of children relative to an adult or adult couple in terms of the additional expenditure required to keep the household at the level of welfare it would enjoy without children. Muellbauer (1974) was the first to advocate the estimation of equivalence scales in a utility theoretic framework, through the estimation static demand systems. This procedure has become a popular method of estimating equivalences amongst economists.

The problem is identifying when households differ in composition, but enjoy the same level of welfare. While the static analysis of household expenditure can provide evidence of the way household spending patterns respond to different demographics, it can not identify preferences over demographics, without making assumptions about those preferences, see Pollak and Wales (1979), Blackorby and Donaldson (1991) and Blundell and Lewbell (1991). Banks, Blundell and Preston (1994) show that in an intertemporal framework preferences over demographics can be identified and the true 'cost' of children on expenditure be obtained.

Pashardes (1991) was the first to explicitly examine the cost of children over the life-cycle and notes that households may reduce current consumption when children are not present saving for when children enter the household. Static comparisons of expenditure between demographically different households will be affected by the how willing and able parents are able to save and borrow for their child raising years. Pashardes terms an equivalence scale estimated in a static

framework as an *equivalent expenditure scale* and an *equivalent income scale* as an equivalence scale developed in an intertemporal framework.

Banks, Blundell and Preston (1994) followed with a study on the intertemporal costs of children using pseudo-panel data constructed from the UK's FES from 1969 to 1988. Through simulations from the estimated parameters the authors constructed lifetime scales as the difference in total lifetime sum utility of a household with children and without, but found them too high. By adding an arbitrary linear contribution to lifetime based on the number of children Banks, Blundell and Preston were able to estimate the cost of child born when the household head is 26 years old and leaving 18 years later as a proportion of an adult couple over the life-cycle as being about approximately 16%. An additional child born when the head is 28 years old increases the cost to 40% or 20% for each child. A third child born at 30 raises the total cost of having three children to 75% or 25% per child.

This paper proposes a simple utility maximising intertemporal model of expenditure that can be easily solved as a function of wealth. By allowing the rate of time preference to vary according to demographics, more specifically with the number of children, the demographic effect on intertemporal allocations can be examined. By solving the model as a function of wealth allows the estimation of the rate of time preference and lifetime equivalence scale in a single cross section of data without the need for panel data on expenditures.

Equivalence Scales in Static Demand Systems

The demand systems that are typically used to estimate equivalence scales are static in nature. They seek to maximise a static measure of utility,

$$u = \max_q \{u(\mathbf{q}, z^h) \text{ subject to } x = \mathbf{p}'\mathbf{q}\} \quad (1)$$

with an indirect utility function,

$$u = v(x, \mathbf{p}, z^h) \quad (2)$$

which may be inverted to find the cost or expenditure function,

$$x = c(u, \mathbf{p}, z^h) \quad (3)$$

which is the dual problem to utility maximisation,

$$x = c(u, \mathbf{p}, z^h) = \min_q \{\mathbf{p}'\mathbf{q} \text{ subject to } u(\mathbf{q}, z^h) = u\} \quad (4)$$

or

$$= \min_x \{\mathbf{p}'\mathbf{q} \text{ subject to } v(x, \mathbf{p}, z^h) = u\} \quad (5)$$

The specification of the cost function allows the application of Shephard's Lemma to provide Hicksian demands

$$\mathbf{q}_H(u, \mathbf{p}, z^h) = \frac{\partial c(u, \mathbf{p}, z^h)}{\partial \mathbf{p}} \quad (6)$$

Substituting in the indirect utility function provides Marshallian demand functions: that incorporate demographic variables,

$$q_M(x, \mathbf{p}, z^h) = \frac{\partial c(v(x, \mathbf{p}, z^h), \mathbf{p}, z^h)}{\partial \mathbf{p}} \quad (7)$$

or in budget shares, also termed Engel curves,

$$\begin{aligned} s(x, \mathbf{p}, z^h) &= \frac{\mathbb{1}c(v(x, \mathbf{p}, z^h), \mathbf{p}, z^h)}{\mathbb{1}\mathbf{p}} \frac{\mathbf{p}}{c(v(x, \mathbf{p}, z^h), \mathbf{p}, z^h)} \\ &= \frac{\mathbb{1} \ln c(v(x, \mathbf{p}, z^h), \mathbf{p}, z^h)}{\mathbb{1} \ln \mathbf{p}} \end{aligned} \quad (8)$$

Equivalence scales can be specified as the ratio of expenditure of a household h with certain demographic variables, z^h to the reference household R and setting the scale at unity for the reference period and household,

$$m(z) = \frac{c(u^R, \mathbf{p}, z)}{c(u^R, \mathbf{p}, z^R)}, \quad (9)$$

so that equivalence scale is given by

$$\frac{x^h}{m(z^h)} = \frac{x^R}{m(z^R)} = x^R \quad (10)$$

$$m(z^h) = \frac{x^h}{x^R} \quad (11)$$

An equivalence scale can only be recovered from demand data if the equivalence scale is specified as *independent of base level utility*,

$$m_{IB}^h = m_{IB}^0(\mathbf{p}, z^h) \quad (12)$$

see Lewbell (1989) and Blackorby and Donaldson (1989), such that the cost function is,

$$x^h \circ c(u, \mathbf{p}, \mathbf{z}^h) \circ m_{IB}^0(\mathbf{p}, \mathbf{z}^h) c^R(u, \mathbf{p}). \quad (13)$$

Thus, if an equivalence scale is appropriately specified the parameters describing it may be estimated from demand data, by setting the scale at unity for the reference period and household.

Intertemporal Demand Systems and Equivalence Scales

In order to assess the implication of children on intertemporal allocations it is necessary to establish an intertemporal model that incorporates demographics. Specifying lifetime utility separable between within period utility and a function representing the complete lifetime history of the households demographics, for example whether the household intends to or has a child, gives,

$$U = U\{u_t, \dots, u_T, \mathbf{z}\} \quad (14)$$

More specifically assuming additive separability of within period utility allows,

$$U = \{ \dot{a}_t F_t(u_t, \mathbf{z}_t), \mathbf{z} \} \quad (15)$$

where $u_t = u_t(\mathbf{q}_t, \mathbf{z}_t)$ is within period utility,

\mathbf{q}_t is a vector of goods consumed in period t ,

\mathbf{z}_t is a vector of household characteristics at period t and

The additive separable lifetime utility function allows the problem to be separated into two stages, Banks, Blundell and Preston (1994). The first stage is the intertemporal allocation of expenditure over the life cycle and the second the

allocation of the given level of expenditure to the goods, which is identical to the static demand model. Using the within period indirect utility function (\cdot) , utility at any time in the future is given by,

$$u_t = F_t(v_t(x_t, \mathbf{p}_t, \mathbf{z}_t), \mathbf{z}_t) \quad (16)$$

and lifetime utility can be specified,

$$U = U(\{\dot{\mathbf{a}}_t F_t(v_t(x_t, \mathbf{p}_t, \mathbf{z}_t), \mathbf{z}_t)\}_t | \mathbf{z}) \quad (17)$$

Let $C(U, \mathbf{p}, \mathbf{z})$ denote the sum of the stream of expenditures \mathbf{x}^* that minimise the lifetime cost of reaching lifetime utility U , for a stream of prices \mathbf{p} and demographic history \mathbf{z} ,

$$C(U, \mathbf{p}, \mathbf{z}) = \text{Min}_{\mathbf{x}^*} \{ \dot{\mathbf{a}}_t x_t \text{ subject to } U(\{\dot{\mathbf{a}}_t F_t(v_t(x_t, \mathbf{p}_t, \mathbf{z}_t), \mathbf{z}_t)\}_t | \mathbf{z}) \geq U \} \quad (18)$$

such that $C(U, \mathbf{p}, \mathbf{z}) = \dot{\mathbf{a}}_t x_t^*(U, \mathbf{p}, \mathbf{z})$

Which gives the stream of optimised within period utilities,

$$\mathbf{u}^* = F_t(v_t(x_t^*, \mathbf{p}_t, \mathbf{z}_t), \mathbf{z}_t) \quad (19)$$

The within period cost or expenditure function can be written

$$x_t^* = c(u_t^*, p_t, z_t) = \underset{x_t}{\text{Min}} \left\{ x_t \text{ subject to } F_t(v_t(x_t, p_t, z_t), z_t) \ni u_t^* \right\} \quad (20)$$

Atemporal (Expenditure) Equivalence Scale

If z_t is the vector of demographic variables for a particular household in period t , (for example whether the household has children), and z_t^R is the vector of demographic variables for the reference household in period t , (for example without children) then the within period equivalence scale can be considered,

$$m = \frac{c(u_t^*, p_t, z_t)}{c(u_t^R, p_t, z_t^R)} \quad (21)$$

Intertemporal (Wealth) Equivalence Scale

If z is the vector of demographic variables for a particular household that does not change in time, (for example whether the household is intending or does have children), and z^R is the vector of time constant demographic variables for the reference household, then the intertemporal equivalence scale can be considered as the ratio of the sum of expenditures across the lifetime.

$$M = \frac{C(U^R, p, z)}{C(U^R, p, z^R)} \quad (22)$$

Banks, Blundell and Preston (1994) point out that while information on intertemporal allocations can provide information on the amount to restore $\dot{a}_t F_t(u_t, z_t)$ it can not provide the full lifetime cost and hence intertemporal scale if $U = \{\dot{a}_t F_t(u_t, z_t), z\}$ also depends on z .

Solving The Intertemporal Problem

In this section I establish a utility maximising problem in continuous time similar to the Banks, Blundell and Preston model but ignores the utility effects of demographics on life-time utility.

$$\text{Maximise} \quad U(w_t) = \int_0^{\infty} F_s(v_s(x_s, p_s, z_s), z_s) ds \quad (23)$$

$$\text{subject to} \quad \dot{w}_t = rw_t - x_t - y_t \quad (24)$$

where \dot{w}_t is the change in financial wealth over time

w_t is financial wealth in period t ,

x_t is consumption in period t ,

y_t is labour income in period t ,

r is the constant rate of return on saving,

$v_s(x_s, p_s, z_s)$ is the within period utility function

Note that this model is simple assumes that there are no expectations about future prices or demographics.

Specifying $F_s(\)$ in period s as,

$$F_s(v_s(x_s, p_s, z_s), z_s) = e^{-d(z)s} v_s(x_s, p_s, z_s) \quad (25)$$

Then we may write the Hamilton for any period t as

$$H = e^{-d(z)s} v_s(x_s, p_s, z_s) + I_s(rw_s - x_s - y_s) \quad (26)$$

The standard optimising conditions for the Hamilton are, $H_x = 0$, $H_w = -\dot{I}_s$ and

$H_I = \dot{w}$, which provide the solution for x_s .

$$\underline{H_x = 0}$$

$$\frac{\partial v_s(x_s, p_s, z_s)}{\partial x_s} e^{-d(z)s} - I_s = 0 \quad (H1)$$

Gives
$$I_s = \frac{\partial v_s(x_s, p_s, z_s)}{\partial x_s} e^{d(z)s} \quad (H1')$$

$$\underline{H_w = -\dot{I}_s}$$

$$I_s r = \dot{I}_s \quad (H2)$$

Gives
$$I_s = I_0 e^{-rs} \quad (H2')$$

$$\underline{H_I = \dot{w}}$$

$$\dot{w}_s = rw_s - x_s - y_s \quad (H3)$$

Gives
$$w_s = e^{rs} w_0 + e^{rs} \int_0^s e^{-rt} y(t) dt - e^{rs} \int_0^s e^{-rt} x(t) dt \quad (H3')$$

Dividing by e^{rs}

$$\frac{w_s}{e^{rs}} = w_0 + \int_0^s e^{-rt} y(t) dt - \int_0^s e^{-rt} x(t) dt \quad (27)$$

Letting s approach infinity gives that the discounted sum of expenditure is equal to the sum of initial wealth and discounted sum of expected future income (which can in this paper is considered all labour/government/superannuation income).

$$\begin{aligned} 0 &= w_0 + \int_0^\infty e^{-rt} y(t) dt - \int_0^\infty e^{-rt} x(t) dt \\ w_0 + \int_0^\infty e^{-rt} y(t) dt &= \int_0^\infty e^{-rt} x(t) dt \end{aligned} \quad (28)$$

If assume that expectation of income is constant then $y(t) = y_0$ and

$$\int_0^\infty e^{-rt} y(t) dt = \frac{y_0}{r} \text{ simplifying the above to}$$

$$w_0 + \frac{y_0}{r} = \int_0^\infty e^{-rt} x(t) dt \quad (29)$$

Consider (H1') which gives value of the shadow price of utility, I for any particular future time period s in terms of marginal utility, substituting (H1') into (H2') provides the evolution of marginal utility

$$\begin{aligned} \frac{v_s(x_s, p_s, z_s)}{v_{x_s}} e^{-d(z)s} &= I_0 e^{-rs} \\ \frac{v_s(x_s, p_s, z_s)}{v_{x_s}} &= I_0 e^{(d(z)-r)s} \end{aligned} \quad (30)$$

Since marginal utility in the present $s=0$ is given by,

$$I_0 = \frac{\mathbb{1}v_0(x_0, \mathbf{p}_0, \mathbf{z}_0)}{\mathbb{1}x_0} \quad (31)$$

substituting (31) into (32) gives the evolution of marginal utility

$$\frac{\mathbb{1}v_s(x_s, \mathbf{p}, \mathbf{z})}{\mathbb{1}x_s} = \frac{\mathbb{1}v_0(x_0, \mathbf{p}, \mathbf{z})}{\mathbb{1}x_0} e^{(d(z)-r)s} \quad (32)$$

If the within period utility is specified as the AIDS function

$$v(x, \mathbf{p}, \mathbf{z}) = \frac{\text{Log} \frac{x}{a(\mathbf{p})m(\mathbf{p}, \mathbf{z})}}{b(\mathbf{p})} \quad (33)$$

Differentiating with respect to x_s gives marginal utility in period s

$$\frac{\mathbb{1}v_s(x_s, \mathbf{p}_s, \mathbf{z}_s)}{\mathbb{1}x_s} = \frac{b(\mathbf{p})}{x_s} \quad (34)$$

and in period 0,

$$\frac{\mathbb{1}v_0(x_0, \mathbf{p}, \mathbf{z})}{\mathbb{1}x_0} = \frac{b(\mathbf{p})}{x_0} \quad (35)$$

Inserting (34) and (35) into (32) provides the solution for $x^*(t)$,

$$x(t) = e^{t(r-d(z))} x_0. \quad (36)$$

Inserting (36) into (29) the budget constraint provides a solution for expenditure in terms of wealth,

$$w_0 + \frac{y_0}{r} = \int_0^\infty x_0 e^{t(r-d(z))} e^{-rt} dt \quad (37)$$

yields

$$x_0 = \mathbf{d}(\mathbf{z}) \frac{\mathbf{a}}{\mathbf{e}} w_0 + \frac{y_0 \ddot{\mathbf{0}}}{r \mathbf{\theta}} \quad (38)$$

and

$$x(s) = \mathbf{d}(\mathbf{z}) e^{s(r-d)} \frac{\mathbf{a}}{\mathbf{e}} w_0 + \frac{y_0 \ddot{\mathbf{0}}}{r \mathbf{\theta}} \quad (39)$$

Equation (38) can be estimated from given data on current expenditure, wealth and income and expectations about income allow the recovery of $\mathbf{d}(\mathbf{z})$

Intertemporal Equivalence Scale

As stated before the intertemporal equivalence scale can be specified as the ratio of the sum optimal lifetime expenditures of a particular household to the reference household R in order to obtain the same level of lifetime utility U^R

$$M_{Intertemporal} = \frac{C(U^R, \mathbf{p}, \mathbf{z})}{C(U^R, \mathbf{p}, \mathbf{z}^R)} \quad (40)$$

In order to evaluate this expression we need to make use of the equation for optimal expenditure in any period s in the future and also that since lifetime utility is determined by initial wealth w_0 and thus two households with the same level of starting wealth w_0 will also have the same lifetime utility.

$$\begin{aligned}
M &= \frac{C(U^R, \mathbf{p}, \mathbf{z})}{C(U^R, \mathbf{p}, \mathbf{z}^R)} \\
&= \frac{\int_0^\infty x_s^*(U^R, \mathbf{p}, \mathbf{z}) ds}{\int_0^\infty x_s^*(U^R, \mathbf{p}, \mathbf{z}^R) ds} \\
&= \frac{\int_0^\infty d(\mathbf{z}) e^{s(r-d(\mathbf{z}))} \left[\int_0^\infty w_0 - \int_0^\infty e^{-rt} y(t) dt \right] ds}{\int_0^\infty d(\mathbf{z}^R) e^{s(r-d(\mathbf{z}^R))} \left[\int_0^\infty w_0 - \int_0^\infty e^{-rt} y(t) dt \right] ds} \\
&= \frac{d(\mathbf{z})}{d(\mathbf{z}) - r} \bigg/ \frac{d(\mathbf{z}^R)}{d(\mathbf{z}^R) - r}
\end{aligned} \tag{40}$$

Estimation and Data

The Household Expenditure Survey (HES) confidentialised unit record files (CURFs) from the Australian Bureau of Statistics (ABS), for 1975-76, 1984, 1988-89, and 1993-94 were pooled to form a pooled data set of about 25,649 observations.

The estimation involves regressing optimal expenditure against financial wealth and human capital in the current period across for all h households

$$x_h^* = d(\mathbf{z}_h) \left[\int_0^\infty w_h + \frac{y_h}{r} \right] + u_h \tag{41}$$

where
$$d(\mathbf{z}) = d_0 + \sum_{k=1}^K d_k z_k \tag{42}$$

and d_0 and d_k are parameters to be estimated and z_k are demographic variables that effect intertemporal allocations. To examine the effect of children on intertemporal expenditure and thus construct the intertemporal equivalence scale M , z_1 is specified as the number of children present in the household.

The HES datasets do not contain data on wealth but do contain property income, financial income (income from financial institutions) and capital income (income from investments in capital such as dividends, trusts, debentures). By dividing the income from an asset by the rate of return, an estimate of the level of assets can be obtained. The rate of return on property was assumed to be 5% for all surveys. The rate of return for the latter two of these variables was taken by a weighted sum of the rates or return of the investments that comprised them, with the weights being taken from a supplement to the 1993-94 HES on the proportion of investment types in the two measures.

Table 1 Rates of Return by Year

Year	Nominal Rate of Return on Financial Assets	Nominal Rate of Return on Capital Assets
1975/76	6.71%	9.47%
1984	7.97%	8.87%
1988/89	9.77%	10.04%
1993/94	3.43%	4.48%

The constant interest rate used to obtain human wealth was also chosen to be 5% and this is the figure used the calculation of the equivalence scales.

Estimating $x_h^* = d(z_h) \frac{y_h}{r} + u_h$ where $d(z) = d_0 + d_1 z_1$ by non-linear OLS

provides the following results.

Table 2 Parameter Estimates

	d_0	d_1
Estimate	0.042573	0.001753
SE	0.0001885	0.0001245
t-ratio	225.84	14.09
R^2	0.4352	
\bar{R}^2	0.4352	

The model performs reasonably well for cross section estimation over many households in many different situations that have not been modelled with 44% of the variation in spending explained by the model. More importantly the estimate of the rate of time preference seems reasonable at 4.3% and is significant. The effect of a child on the rate of time preference is significant and raises it by approximately 0.2% for each child. Thus a household with a child spends $\frac{d(z)}{d(z^R)} = \frac{0.04432}{0.04257} = 1.04$ than a household without children.

The intertemporal equivalence scales constructed using

$$M = \frac{d(z)}{d(z) - r} \bigg/ \frac{d(z^R)}{d(z^R) - r} = \frac{d(z)}{d(z^R)} \bigg/ \frac{d(z) - r}{d(z^R) - r}$$

the estimate of $d(z)$ and for values of the interest rate.

Table 3 Intertemporal Scale Estimates M

	$r = 3\%$	$r = 5\%$	$r = 7\%$
Additional lifetime spending for each additional child	0.91	1.36	1.11

The scales are highly dependent upon the interest rate with low interest rates suggesting that households with a child need about 9% less than a household with children, which seems implausible. For higher interest rates the scale seems more realistic. When the interest rate is 5% the same rate as that used to obtain human wealth provides a scales of 1.36 suggesting that a household with a child needs an

additional 36% lifetime expenditure or wealth in order to maintain lifetime expenditure.

By splitting children into those under 5 years and those above 5 may provide insight as to whether households spend less when children are very young saving for when children are older and more expensive to maintain.

Table 4 Intertemporal Scale Estimates with Children Age Differences

	d_0	d_1	d_1
Estimate	0.042558	0.003003	0.001488
SE	0.000189	0.000310	0.000138
t-ratio	225.82	9.67	10.76
R^2	0.4356		
\bar{R}^2	0.4356		

The results suggest that households are more inclined to spend a greater proportion of their wealth when young children are present than when children are older.

Conclusion

This paper has proposed a method for estimating an intertemporal or lifetime equivalence scale without the need for panel data, by solving the optimal intertemporal allocations of expenditures as a function of initial lifetime wealth. Demographic variables affect the intertemporal allocations of expenditure by altering the rate of time preference, which is shown to be the marginal propensity to consume out of wealth. This allows the estimation of an intertemporal equivalence scale, as the ratio of lifetime expenditures of a particular household to the reference household's.

The major limitation of the model is its simple modelling of the intertemporal problem, without allowing for expectations of future prices, demographics (such as family size) or income. The specification of the within period utility as AIDS allows

the recovery of evolution of expenditure with ease but has linear Engel curves and now rich versus poor effects of non-linear models. In fact most of the improvements in the intertemporal utility maximising problems such as liquidity constraints, finite lifetimes and uncertainty can be incorporated into the model and should do in order to provide more accurate intertemporal equivalent scales

References

- Banks, J., Blundell, R. and Preston, I., 1994, 'Measuring the Life-Cycle Consumption Cost of Children', Ch 8 in *The Measurement of Household Welfare*, Blundell, R., Preston, I and Walker, I. (eds), Cambridge University Press, Melbourne.
- Banks, J., Blundell, R. and Preston, I., 1997, 'Life-cycle Expenditure Allocations and the Consumption Costs of Children', *European Economic Review*, vol. 38, no. 2, pp. 1391-1410.
- Blackorby, C. and Donaldson, D., 1988, 'Adult Equivalence Scales and the Economic Implementation of Interpersonal Comparisons of Well-being', *University of British Columbia Discussion Paper*, no. 88-27.
- Blundell, R. and Lewbel, A., 1991, 'The Information Content of Equivalence Scales', *Journal of Econometrics*, Vol. 50, no. 1-2, pp. 49-68.
- Blundell, R., Browning, M. and Meagher, C., 1994, 'Consumer Demand and the Life-Cycle Allocation of Household Expenditures', *Review of Economic Studies*, vol. 61, no. 4, pp. 57-80.
- Browning, M., Deaton, A. and Irish, M., 1985, 'A Profitable Approach to Labour Supply and Commodity Demands over the Life Cycle', *Econometrica*, vol. 53, no. 3, pp. 503-543.
- Browning, M., 1992, 'Children and Household Economic Behaviour', *Journal of Economic Literature*, vol. 30, no. 3, pp. 1434-1475.
- Cooper, R.J. and McLaren, K.R., 1982, 'Modelling Price Expectations in Intertemporal Consumer Demand Systems: Theory and Evidence', *The Review of Economics and Statistics*, vol. , no. , pp. 282-288.
- Cooper, R.J. and McLaren, K.R., 1982, 'Inflationary Expectations in Intertemporal Consumer Demand Systems', *Australian Economic Papers*, vol. , no. , pp. 193-201.
- Fisher, F., 1987, 'Household Equivalence Scales and Intertemporal comparisons', *Review of Economic Studies*, vol. 54, pp. 519-24.
- Keen, M., 1990, 'Welfare Analysis and Intertemporal Substitution', *Journal of Public Economics*, vol. 42, pp. 47-66.
- Muellbauer, J., 1974, 'Household Composition, Engel Curves and Welfare Comparisons between Households', *European Economic Review*, vol. 5, no. 2, pp. 103-122.
- Pashardes, P., 1991, 'Contemporaneous and intertemporal child costs', *Journal of Public Economics*, Vol. 45, pp. 191-213.
- Pollak, R.A., and Wales, T.J., 1979, 'Welfare Comparisons and Equivalence Scales', *American Economic Review*, vol. 69, no. 2, pp. 216-221.