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Abstract: Using high frequency financial data and associated risk decomposition and quantile regression techniques we characterise some stylised facts and relationship(s) between standard betas, diffusion betas and jump betas of individual stocks and portfolios in Japanese market. We then investigate whether the beta in the conventional CAPM is the weighted average of the jump beta and diffusion beta in the jump-diffusion model and how these different betas behave across different banks. Our empirical findings indicate that jump betas are cross-sectionally more dispersed than diffusion and standard betas. We find that the relationship(s) between the three betas are non-linear. We also find that standard betas are influenced more by diffusion betas than the jump betas, although the actual magnitude of the weights differ significantly across the quantile. This relationship holds for both individual stocks and portfolios. Empirical studies have shown that betas vary systematically across large and small firm equities. For large equity portfolios, the jump beta-diffusion beta ratios are lower that the jump beta-diffusion beta ratios of the small equity portfolios. Empirically, we further find that the standard CAPM beta is composed of two-components, i.e. it is the weighted average of the diffusion component and the jump component.

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I. Introduction

In the one factor capital asset pricing model (CAPM), systematic risk, measured by beta, is determined by the asset's covariance with the market over the market variance (Sharpe 1963; Lintner 1965). The traditional way of estimating the asset's constant beta has been by linear regression, typically based on 5 years of monthly data. However, the advent of even more powerful computers and easy access to high frequency data has revived interest into alternative non-parametric approaches to more accurately estimate betas. Compared to traditional parametric methods, a non-parametric approach using high frequency data trivializes calculation and avoids many distortive assumptions necessary for parametric modelling. Studies have shown that the use of high frequency data results in statistically superior beta estimates relative to the traditional regression based procedures. In addition, unlike the constant beta computation, the realized beta computational approach allows a continuous evaluation of the time varying betas and thus provides a simple and robust estimator for measurement of time varying systematic risk. (see Wang et al. (2013)).

From a pricing perspective, the empirical failure of the unconditional Capital Asset Pricing Model (CAPM) has led to three possible approaches to relaxing the overly restrictive CAPM assumptions. The first is to use additional systematic factors, as in Merton (1973), allowing extra-market factors to capture additional systematic risks. The ad-hoc three-factor model of Fama and French (1993) and the four-factor model of Carhart (1997) are some of the widely accepted examples of such multifactor models. The second approach is to relax the static relationship between expected return and risk by allowing time variation in the systematic factors. In that sense, Jagannathan and Wang (1996), Lettau and Ludvigson (2001) and Petkova and Zhang (2005) find that betas of assets with different characteristics move differently over the business cycle and Campbell and Vuolteenaho (2004), Fama and French (1996) and Ferson and Harvey (1999) show that time-variation in betas helps to explain anomalies such as value, industry and size. However, this conditional time-varying framework does not seem to be enough to improve the weak fit of the CAPM, as shown by Lewellen and Nagel (2006).

The third approach is the use of dual or conditional betas whereby the market beta is conditioned on market states i.e. bullish or bearish or positive or negative market returns. Bhardwaj and Brooks (1993), Howton and Peterson (1998) and Pettengill et al. (1995) and among others have investigated the relationship between beta risk and stock market conditions. For example, Pettengill et al. (1995) observe that larger firms experience larges betas in down market conditions than in up market conditions, the reverse being true for smaller firms Fabozzi and Francis (1977) first tested the stability of betas over the “bull” and “bear” markets. Another contribution to the ability to explain the risk-return relationship was made by Campbell and Vuolteenaho (2004). Using an alternative return decomposition method, Campbell and Vuolteenaho (2004) decomposes CAPM betas into discount rate betas and cash flow betas. Following Campbell and Vuolteenaho (2004), Botshekan et al. (2012) construct a return decomposition distinguishing cash flow and discount rate betas in up and down markets. They find that for larger companies, the priced components of risks become more symmetric (both upside and downside market).

In all of the above three approaches, the various beta estimates assume a continuous data generation process, while in fact the empirical papers in high frequency literature support the occurrence and persistence of jumps in the observed data generation process. A large body of literature has evolved to show both theoretically and empirically that jumps explain many of the dynamic features of stylized facts documented in asset prices. Studies on stochastic behaviour of the stock market generally agree that stock returns are generated by a mixed process with a diffusion component and a jump component. If so, the standard CAPM beta is at best a ‘summary proxy’ for the systematic risk of a mixed-process, i.e. a weighted average of the diffusion component and the jump component. It would be prudent to be able to split the standard beta into two component betas so as to capture the two risks separately: one for continuous and small changes (diffusion beta) and the other for discrete and large changes (jump beta). In this light, Todorov and Bollerslev (2010) provide a new theoretical framework for disentangling and estimating the sensitivity towards systematic diffusive and jump risk in the context of factor models. They focus on the decomposition of systematic risk by recognizing the jump occurrence at aggregate market level and show that diffusion and jump betas with respect to aggregate market portfolio differ significantly and substantially. Furthermore, the use of high frequency data ensures that both betas are also time-varying.

The key insight in this paper is that, though the continuous returns and jump returns are orthogonal by the Todorov and Bollerslev (2010) decomposition, the three realised betas (i.e. standard, diffusion and jump betas) are not restricted nor expected to be orthogonal. In fact, a simple correlation test indicates some dependencies. The rich cross-sectional and time–series heterogeneity in our estimates of monthly betas enable us to study the how standard beta, diffusion beta and jump betas vary both across quantiles and over time. To explore the cross-sectional relationships of the betas over quantiles, we adopt a quantile regressions (QR) approach. By so doing, it is possible to model the relationship between standard betas and diffusion and jump betas not just for the mean of the conditional distribution, but also at various quantiles. While the classical linear regression only describes the conditional mean, the quantile regression method allows us to estimate the effects of diffusion beta and jump beta on standard beta (e.g. Koenker and Hallock (2001)).

Our empirical investigations are based on high-frequency stock data of the 50 Japanese banks included in the Nikkei 225 index over the 2002-2012 sample period. We begin by estimating two separate betas; the diffusion and jump betas as well as a standard CAPM beta for each of the individual stocks on a monthly basis over the whole sample period. We rely on 5-minute intraday sampling frequency for the beta estimation, as a way to guard against the market microstructure complications that arise at the highest intraday sampling frequency. We regress the standard beta against the diffusion and jump beta and we find that the quantile regression relations between standard beta and diffusion and jump beta varies widely depending on the quantile level of standard beta, where the quantile ranges from zero to one.

We find that on average the standard beta is weighted more by the diffusion beta component then the jump beta component. The relationship holds across the quintiles. However, the actual magnitude of the weights differ across the quintiles. In general, the weights are jointly lower for low standard betas until the pick around the 50th-75th quintiles with value dropping down again post 75th quantile.

Sorting stocks based on the size, we find that large banks have high betas and small banks have low betas. The results holds for all the three betas; indicating that larger Japanese banks are more sensitive to both market movements than smaller institutions, regardless of whether they occur through a jump or not. However, the ratios across the betas differ substantially. The ratios of large equity to small equity standard beta is 2.81 than the ratios of large equity diffusion beta over small equity diffusion beta is 5.81. On the other hand, the ratios of large equity to small equity jump beta is 1.16. Over and above this, a unique feature of small equity portfolio, is the jump-diffusion beta ratio, where the jump beta disproportionately is larger than its associated diffusion beta, indicating another layer of a possible size effect.

This study also makes a comparison between the jump-diffusion model and the conventional CAPM. At the 50th quantile, the hypothesis is that standard beta is the weighted average of jump beta and diffusion beta cannot be rejected at 10% significance level. All other quantiles have rejection at 1% significance level. Empirical findings from this study agree with the model is that the systematic risk of an asset is the weighted average of both diffusion and jump risk.

The rest of the paper is organised as follows. In Section II, we present our theoretical framework. Section III presents the methodology used in this study. Section IV describes the data. The empirical analysis are present in Section IV. Section V concludes the paper.

II. Theoretical Framework

A. Capital Asset Pricing Model

The standard capital asset pricing model (CAPM) is formulated as follows:

$$r_{i,t} = \alpha_i + \beta_{it} r_{m,t} + \varepsilon_{i,t} \quad (1)$$

where $r_{i,t}$ is the monthly excess stock return on stock *i*, and $r_{m,t}$ is the aggregate market returns at time *t*; α_i is the constant term for the asset *i ;* the error term $\varepsilon_{i,t}$ is the idiosyncratic risk of stock *i*, which is uncorrelated with r_m or the idiosyncratic risk of any other stock under CAPM assumptions. The slope coefficient, $\beta_{i,t}$, in Eq. (1), commonly known as the Standard Beta, is the systematic risk of asset *i*, and measures the responsiveness of the changes in stock's prices to changes in market prices. According to the CAPM, the equilibrium expected return on all risky assets are a function of the covaraiance with the market portfolio.

The Standard Beta, in CAPM is defined as,

$$\beta_{i,t} = \frac{Cov(r_{i,t} r_m, t)}{Var(r_m, t)} \quad (2)$$

The CAPM model basically depends on stock and market returns, which in turn, depends the underlying prices of individual stocks. It is now widely agreed in the literature that financial return volatilities and correlations are time-varying and returns follow the sum of a diffusion process and a jump process.†

We consider that the log-price (p_t) process of an asset at time *t* follows a continuous-time jump-diffusion process defined by the stochastic differential equation as follows:

$$dp_t = \mu_t dt + \sigma_t dW_t + k_t dq_t \quad (3)$$

where μ_t is the instantaneous drift of price process and σ_t is the diffusion process; W_t is standard Brownian motion. These first two terms correspond to the diffusion part of the total variation process. The diffusion part which is responsible for the usual day-to-day price movement. The changes in stock prices may be due to variation in capitalization rates, a

† See, for example, Press (1967), Merton (1976), and Ball and Torous (1983) and among others.

temporary imbalance between supply and demand, or the receipt of information which only marginally affects stock prices. The final term, $k_t dq_t$ refers to the jump component of the total process, where q_t is a counting process such that $dq_t = 1$ indicates a jump at time *t* and $dq_t = 0$ otherwise and k_t is the size of jump at time *t* if a jump occurred. The jump part which is due to the receipt of any important information that causes a more than marginal change (i.e. abnormal change) in the price of stock. The arrival of this kind on information is random. The number of information is assumed to be distributed according to a poisson process.

If the return of stocks should be divided into jump part and diffusion part certainly the risk associated with returns of securities should be decomposed into two parts, too. The CAPM states that beta, a diffusion risk, is systematic and non-diversifiable. So is the jump risk when taking both diffusion process and jump process into account. The presence of jump variations in both individual assets and aggregate market affect co-variance estimations and consequently the estimations of Realized Beta and systematic risk. Thus it would be prudent to disentangle the jump component and the diffusion component in prices because they are basically two quite different sources of risk; see, e.g. Bates (2000), Eraker (2004), Pan (2002) and Todorov (2009).

B. Decomposing Systematic Risk: Diffusion and Jump components

Our framework motivating the different betas and the separate pricing of diffusion and jump market price risk and relies on the theory originally developed by Todorov and Bollerslev (2010) for decomposing market returns into two components: one associated with diffusion price movement and another associated with jumps. Hence, in the presence of both components, equation (1) becomes:

$$r_{i,t} = \alpha_i + \beta^c_{i,t} r^c_{m,t} + \beta^j_{i,t} r^j_{m,t} + \varepsilon_{i,t} \tag{4}$$

where $r_{i,t}$ is the monthly excess stock return on stock i, $\alpha_i$is its drift term and the total market risk ($r_{m,t}$) is modelled as a combination of a diffusion ($r^c_{m,t}$) and jump component $\left(r^j_{m,t}\right)$. β^c_{it} and β^j_{it} denotes the responsiveness of each stock's movement to the diffusion and jump components of market risk and ε_i denotes the idiosyncratic term which is also made up of a continuous and jump component. This decomposition is interesting because standard factor models of risk implicitly assume that an asset's systematic risk is uncorrelated with jumps in the market (i.e. that the asset's beta does not change on days when the market experiences a jump). Equation (1) does not distinguish between the diffusion and jump components of total return, but does decompose total returns into systematic ($\beta_{i,t} r_{m,t}$) and nonsystematic ($\alpha_i +$

$\varepsilon_{i,t}$) components. Any market jump is embedded in $r_{m,t}$, while any nonsystematic jump unique to firm *i* is included in the error tem. When the systematic risks exposure of a firm to both diffusion and jump price movements are identical, i.e. $\beta_{i,t}^{c} = \beta_{i,t}^{j}$, then, the two-factor market model collapses to the usual one-factor market model, which relates the stock return $r_{i,t}$ to the total market return $r_{m,t} = r_{m,t}^{c} + r_{m,t}^{j}$. The restriction that $\beta_{i,t}^{c} = \beta_{i,t}^{j}$ implies that the asset responds the same to market diffusion and jump price movements, or intuitively that the asset and the market co-move in the same manner during “normal” times and periods of “abrupt” market moves. If, on the other hand, $\beta_{i,t}^{c}$ and $\beta_{i,t}^{j}$ differ, empirical evidence for which is provided below, the cross—sectional variation in the diffusion and jump betas may be used to identify their separate pricing. The literature suggests that the two betas are not the same, i.e. the reactiveness of an asset return of the two components of systematic risk can be different, denoted by $\beta_{i,t}^{c}$ and $\beta_{i,t}^{j}$ respectively.

We have shown that market returns contain two components, both of which display substantial volatility and which are not highly correlated -with each other. This raises the possibility that different types of stocks may have different betas with two components of the market. Chen (1996) shows that under the same assumption of CAPM, except the normality of asset returns, the jump-diffusion model takes two different types of beta when pricing the underlying asset. One is diffusion beta, which measures the systematic risk when no jumps occurs. The other is the jump beta, which measures the systematic risk when jumps take place in the market. In a similar form as that of CAPM, the jump-diffusion two beta model is as follows:

$$r_{i,t} = \alpha_i + r_{m,t}[(1-\emptyset)\beta_{i,t}^{c} + \emptyset\beta_{i,t}^{j}] + \varepsilon_{i,t} \tag{5}$$

The left hand side of (5) is the monthly stock return on asset *i*. The right hand side of (5) is weighted average of two betas: the diffusion beta, which gets a weight of $(1-\emptyset)$ and the jump beta, which gets a weight of $\emptyset$. β_{it}^{c} is the diffusion beta as defined by $\beta_{i,t}^{c} = \frac{Cov(r_{i,t} r_{m,t}^{c})}{Var(r_{m,t}^{c})}$; β_{it}^{j} is the jump beta as defined by $\beta_{i,t}^{j} = \frac{Cov\left(r_{i,t} r_{m,t}^{j}\right)}{Var\left(r_{m,t}^{j}\right)}$. If there is no jumps in the market, $\lambda = 0$ which implies $\emptyset = 0$, equation (5) collapses to the conventional CAPM,

$$r_{i,t} = \alpha_i + r_{m,t}[\beta_{i,t}^{c}] + \varepsilon_{i,t} \tag{5a}$$

On the other hand, if asset returns are generated by a pure jump process, $\sigma^2(r_m) = 0$ which implies $\emptyset = 1$, then equation (5) reduces to pure jump CAPM,

$$r_{i,t} = \alpha_i + r_{m,t}[\beta_{i,t}^{j}] + \varepsilon_{i,t} \tag{5b}$$

Equation $(5a)$ and $(5b)$ are two special cases of equation (5), the jump-diffusion two-beta asset pricing model.‡

The two-way decomposition beta allows us to ask how individual equity prices respond to diffusion and jump market moves.

III. Methodology

In this paper we study the relationship between Standard Beta, Diffusion Beta and Jump Beta across Japanese banks.

A. Realized Beta

Standard Betas are not directly observable. The traditional way of addressing the estimation problem betas has relied on using rolling linear regressions, typically requiring 5 years of monthly data to satisfy sample size requirements.§ However, the advent of readily available high frequency data in recent years, have now made it possible to compute Realized Betas over varying frequencies that can be used as proxies for Standard Betas.

Realized Beta is defined as the ratio of realized covariance of stock and market to the realized market variance. Andersen et al. (2005) argue that Realized Beta is a more accurate measurement of the Standard Beta because it employs more information than the traditional regression on monthly returns.

The estimate of Realized Beta for individual stock, $\hat{\beta}_{i,t}^{s}$ is defined as:

$$\hat{\beta}_{i,t}^{s} = \frac{RCOV_{i,t,s}^{s}}{RV_{m,t,s}^{s}} = \frac{\sum_{s=1}^{n} r_{i,t,s}\, r_{m,t,s}}{\sum_{l=1}^{n} \left(r_{m,t,s}\right)^{2}} \tag{6}$$

Despite the numerous advantages of Realized Beta, it is important to note that equation (3) still defines the Standard Beta in a one-factor CAPM model.

The same readily high frequency data that made possible the computation of the Realized Betas has also made possible the disentanglement of these Realized Betas into Diffusion Betas and

‡ See, Chen (1996) for more details.
§ see, e.g., the classical work by Fama and MacBeth (1973).

Jump Betas, thus effectively giving rise to a two-factor CAPM model for pricing assets which follow not only a diffusion process but also a jump process.

B. Diffusion and Jump betas

The calculation of jump beta is motivated by the fact that the price process of any asset is a combination of Brownian semi-martingale plus jumps [**]The decomposition of the return for the market into separate diffusion and jump components that formally underly $\beta^{c}_{i,t}$ and $\beta^{j}_{i,t}$ in equations (4) are, of course, not directly observable. Instead, we assume that prices are observed at discrete time grids of length 1/M over the active trading day$[0, T]$.

Empirical studies rely on discretely sampled returns; discrete-time intraday returns on trading day t are denoted as

$$r_{t,s} = p_{t,s} - p_{t,s-1}, \qquad s = 1, \ldots, n\,; t = 1, \ldots, T \tag{7}$$

where $p_{t,s}$ refers to the s th intra-day log-price for day *t*; *T* is the total number of days in the sample and *M* is the sampling frequency.

We start by assuming that the intraday stock price processes for the aggregate market index, denoted by $dp_{m,t}$, and the *i*th stock, denoted by $dp_{i,t}$, follow general diffusion-time processes. To allow for the presence of jumps in the price process Todorov and Bollerslev (2010) consider the following specification for stock *i* and aggregate market *m*. Now, the log price process evolves as follows[††]:

For the market,

$$r_{m,t,s} \equiv dp_{m,t} = \alpha_{m,t}dt + \sigma_{m,t}dW_{m,t} + dJ_{m,t} \tag{8}$$

and for the stock,

$$r_{i,t,s} \equiv dp_{i,t} = \alpha_{i,t}dt + \beta^{c}_{i,t}\sigma_{m,t}dW_{m,t} + \beta^{j}_{i,t}dJ_{m,t} + \sigma_{i,t}dW_{i,t} + dJ_{i,t}, \qquad i = 1, \ldots, \mathrm{N} \tag{9}$$

[**] Before the Diffusion and Jump Betas can be computed, we have to determine the jump and non-jump days. We use the statistics prosed by Barndorff-Nielsen and Shephard (2006), hereafter, BNS, to detect jumps in the Nikkei 225 index. These estimators are provided in the Appendix.

[††] The notation here is simplified relative to that in Todorov and Bollerslev (2010) see their article for more details.

Where, $W_{m,t}$ and $W_{i,t}$ are standard Brownian motions for the market and asset i; $\alpha_{m,t}$ and $\alpha_{i,t}$ denote the diffusive volatility of the aggregate market and stock i, respectively; and $J_{m,t}$ and $J_{i,t}$ refer to the pure jump Levy processes in the aggregate market and stock i, respectively. $\beta^{c}_{i,t}$ and $\beta^{j}_{i,t}$ measure the responsiveness of an individual stock to the diffusion and jump component of market risk. In this framework,$[\beta^{c}_{i,t}, \beta^{j}_{i,t}]$ is assumed constant throughout each day but can change from day to day.

In order to disentangle the $\beta^{c}_{i,t}$ and $\beta^{j}_{i,t}$, Todorov and Bollerslev (2010) propose a non-parametric beta estimation technique using multipower covariation/variation between the returns of individual stocks and the market portfolio for given diffusion and jump components respectively. They show that $\beta^{c}_{i,t}$ and $\beta^{j}_{i,t}$ can be theoretically identified. .

To begin, consider the estimation of diffusion betas. Suppose that neither the market or nor stock i, jumps, so that $J_{m,t} \equiv 0$ and $J_{i,t} \equiv 0$ almost surely. For simplicity, suppose also that the drift terms in equations in (5) and (6) are both equal to zero, so that,

$$r_{i,t,s} = \beta^{c}_{i,t} r_{m,t,s} + \widetilde{r_{i,t,s}}\,, \quad \text{where } \widetilde{r_{i,t,J}} \equiv \int_{t-1}^{t} \sigma_s^2\, ds\,,$$

for any $s \in [t-1, t]$. In this situation, the ratio of the intra-day covariance between an asset and the market, and the market with itself will estimate diffusion beta using high-frequency intraday returns. The diffusion beta is given by

$$\beta^{c}_{i,t} = \frac{\sum_{s=1}^{n} r_{i,t,s}\, r_{m,t,s}}{\sum_{l=1}^{n} \left(r_{m,t,s}\right)^2} \tag{10}$$

In general, of course, the market may have jump over the $[t-1, t]$ time-interval and the drift terms are not identically equal to zero. Meanwhile, it follows readily by standard argument that for $n \to \infty$, the impact of the drift terms are asymptotically negligible. However, to allow for the possible occurrence of jumps, the simple estimator defined above needs to be modified by removing the jump components. In particular, following Todorov and Bollerslev (2010), we consider their ratio statistics for the discretely sampled data series which consistently estimate the diffusion beta for $n \to \infty$, under very general conditions. These are:

$$\hat{\beta}_{i,t}^{c} = \frac{\sum_{s=1}^{n} r_{i,t,s}\, r_{m,t,s}\, \mathbb{I}_{\{|r_{t,s}|\leq\theta\}}}{\sum_{s=1}^{n} \left(r_{m,t,s}\right)^{2} \mathbb{I}_{\{|r_{t,s}|\leq\theta\}}} \quad , \quad i = 1, \ldots\ldots N. \tag{11}$$

Where,$\mathbb{I}_{\{|r_{t,s}|\leq\theta\}}$ is the indicator function,

$$\mathbb{I} = \begin{cases} 1 & if \; \{|r_{t,s}|\leq\theta\} \\ 0 & Otherwise \end{cases} \tag{12}$$

based on the truncation level, θ, for diffusion component.

Now, we consider the estimation of jump beta. The actually observed high-frequency returns, of course, contain both diffusive and jump risk components. However, by raising the high-frequency returns to powers of orders greater than two, the diffusion components become negligible, so that the systematic jump dominate asymptotically for $n \to \infty$.‡‡ As formally shown in Todorov and Bollerslev (2010), the following estimator is indeed consistent for jump beta when there is at least one significant jump in the market portfolio for the given estimation window for $n \to \infty$.

$$\hat{\beta}_{i,t}^{j} = \operatorname{sign}\left\{\sum_{s=1}^{n} \operatorname{sign}\{r_{i,t,s}\, r_{m,t,s}\}\left|r_{i,t,s}\, r_{m,t,s}\right|^{\tau}\right\} \times \left(\frac{\left|\sum_{s=1}^{n} \operatorname{sign}\{r_{i,t,s}\, r_{m,t,s}\}\left|r_{i,t,s}\, r_{m,t,s}\right|^{\tau}\right|}{\sum_{s=1}^{n} \left(r_{m,t,s}\right)^{2\tau}}\right)^{\frac{1}{\tau}}, \tag{13}$$

Here, the power τ is restricted to be ≥ 2 so that the diffusion price movements do not matter asymptotically. The sign in equation (20) is taken to recover the signs of jump betas that are eliminated when taking absolute values.

Following Todorov and Bollerslev (2010) and Alexeev et al. (2017) we set the parameter values for θ, ϖ, and α estimate the $\hat{\beta}_{i,t}^{c}$ and $\hat{\beta}_{i,t}^{j}$ on both monthly and daily basis. For estimating the $\hat{\beta}_{i,t}^{c}$ and $\hat{\beta}_{i,t}^{j}$, the truncation threshold,$\theta = \alpha\Delta_{n}^{\varpi}$, uses $\varpi = 0.49$ and $\alpha \geq 0$, suggesting that the threshold values may very across stocks and across different estimation window. The threshold

‡‡ The basic idea of relying on higher orders powers of returns to isolate the jump component of the price has previously been used in many other situations, both parametrically and nonparametrically; see e.g., Barndorff-Nielsen and Shephard (2003) and Aıt-Sahalia (2004).

for the diffusion price movement, $\theta = \alpha_i^c = \sqrt[3]{BV_i^{[0,T]}}$ for $\hat{\beta}_{i,t}^c$ suggesting that the diffusion components discards only three standard deviation away from mean, and thus unlikely to be associated with diffusion price movements, where, $BV_i^{[0,T]}$ is the bi-power variation of the i-th stock at time $[0, T]$; the value of $\tau = 2$ for equation (13).

III. Sample and Data

Samples of publicly-traded Japanese Banks from January 2001 through December 2012 are analysed. We examine this period because it includes the different business cycles. However, certain banks are excluded from this analysis if they are not available for testing period. Hence, our final sample consists of 50 of the 63 commercial banks traded on the Tokyo Stock Exchange (TSE) for the period January 2001 through December 2012, a total of 3053 trading days. All the high-frequency data are extracted from the Thompson Reuters Tick history (TRTH) database available via the SIRCA. We used the Nikkei 225 index as a proxy of the market portfolio. Following the standard high-frequency literature, we use a sampling frequency of 5 minutes for all data. The choice of five-minute sampling frequency reflects a trade-off between using all available high-frequency data and avoiding the impact of market microstructure effects, such as infrequent trading or nonsynchronous trading. Unlike the more commonly investigated US and European markets, daily trading on the TSE is interrupted by a lunch break, trading between 09:00 am - 11:00 am and 12:30 pm- 3:00 pm local time. We sample prices from 9:05 am-11:00 pm and 12.35 pm-3.00 pm, with overnight and over-lunch returns excluded from the data set. Missing data at 5-minute intervals is filled with the previous price; when no actual trade occurs during a time interval, it is logical to assume that a stock price carries the same price of the previous particular time interval. Hansen and Lunde (2006) showed that the previous tick method is a sensible way to sample prices in calendar time. Thus we have 53 intra-day observations for 2866 active trading days over a 12 year period (144 months).

IV. Empirical Results

A. Betas

Our main empirical results are based on monthly standard, diffusion and jump beta estimates for each of the stocks in the sample. We rely on fixed intraday sampling frequency of 5 minutes in our estimation of the standard, diffusion and jump betas, with the returns spanning 9.05am to 3.00pm. We compute the means and standard deviations of the time varying betas for period 2003- 2012 and three sub periods (pre-crisis period, crisis period and post-crisis period) and present the results in Table 1. The statistics show that the jump beta has a higher mean of 0.912 and volatility of 0.626, relative to the 0.501 and 0.280; and 0.324 and 0.309 estimated for standard betas and diffusion betas respectively for the sample period. The difference in means of diffusion beta and jump beta (0.65) are significant based on the pooled variance t-tests. When we split the period into three sub periods: pre-crisis, crisis and post-crisis period, we see a clear contrast in the means and standard deviation between three betas. The standard, diffusion and jump betas are higher and more volatile in crisis period compared to pre-crisis and post-crisis period.

Table 1: Summary Statistics for Standard, Diffusion and Jump Betas

The table summarizes of the time varying betas estimated using the Jump-Diffusion CAPM model. The statistics include mean and standard deviations (in parentheses) are summarized by the full sample periods and three sub-periods. We include the pooled variance t-test of the difference between the two sample means for the Standard Beta, Continuous Beta and Jump Beta. The t-statistics are given in parentheses. * denotes significance at 10 % level; ** denotes significance at 5 % level, and *** denotes significance at 1 % level

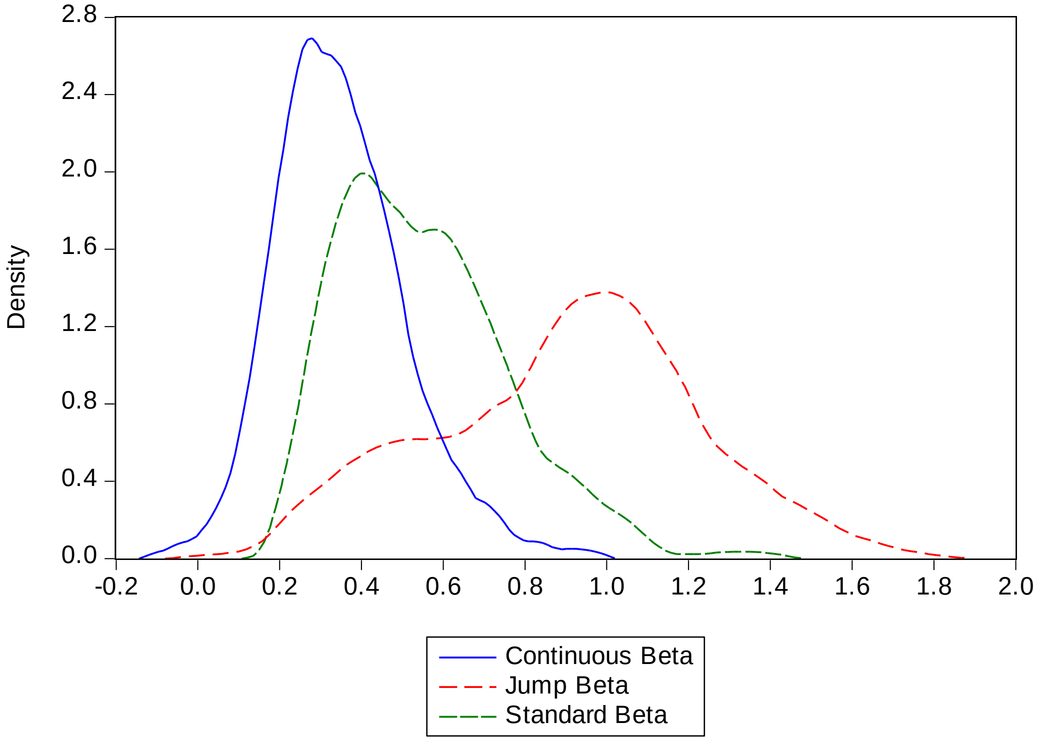
	Standard Beta	Diffusion Beta	Jump Beta
Full-sample Period			
Mean	0.501	0.280	0.912
Std.Dev	0.324	0.309	0.626
t-test of difference		-0.649***	
Pre-crisis Period			
Mean	0.390	0.223	0.759
Std.Dev	0.276	0.276	0.572
t-test of difference		-0.557***	
Crisis Period			
Mean	0.702	0.452	1.095
Std.Dev	0.342	0.321	0.746
t-test of difference		-0.647***	
Post-crisis Period			
Mean	0.548	0.248	1.042
Std.Dev	0.306	0.308	0.552
t-test of difference		-0.819***	

To get a sense of what the relationship across the different betas looks like, Figure 1 plots the kernel density estimates of the unconditional distributions of the three different betas averaged across time and stocks. The jump betas tend to be somewhat higher on average and also more right skewed than the diffusion and standard betas. At the same time, the figure also suggests that the diffusion betas are the least dispersed of the three betas across time and stocks. Part of the dispersion in the betas could be attributed to estimation errors.§§

In order to visualize the temporal and cross-sectional variation in different betas, Figure 2 shows that the time series of equally weighted portfolio betas, based on monthly quintile sorts for each of the three different betas and all of the individual stocks in the sample. The figure suggests that the variation in the standard beta and diffusion beta sorted portfolios in Panel A and B are clearly fairly close as would be expected. The plots for the jump beta quintile portfolios in Panel C, are distinctly different and more dispersed than the standard and diffusion betas quintile portfolios. Jump beta is significantly different from diffusion and standard beta. Motivated by these above findings and in order to shed light on this issue and to address the significant heterogeneity observed across Japanese banking sectors, we depart from the previous literature and employ the quantile regression analysis to estimate the relationship between standard, diffusion and jump betas.

Figure 1: Distributions of Betas

The figure displays kernel density estimates of the unconditional distributions of the three different betas averaged across firms and time.

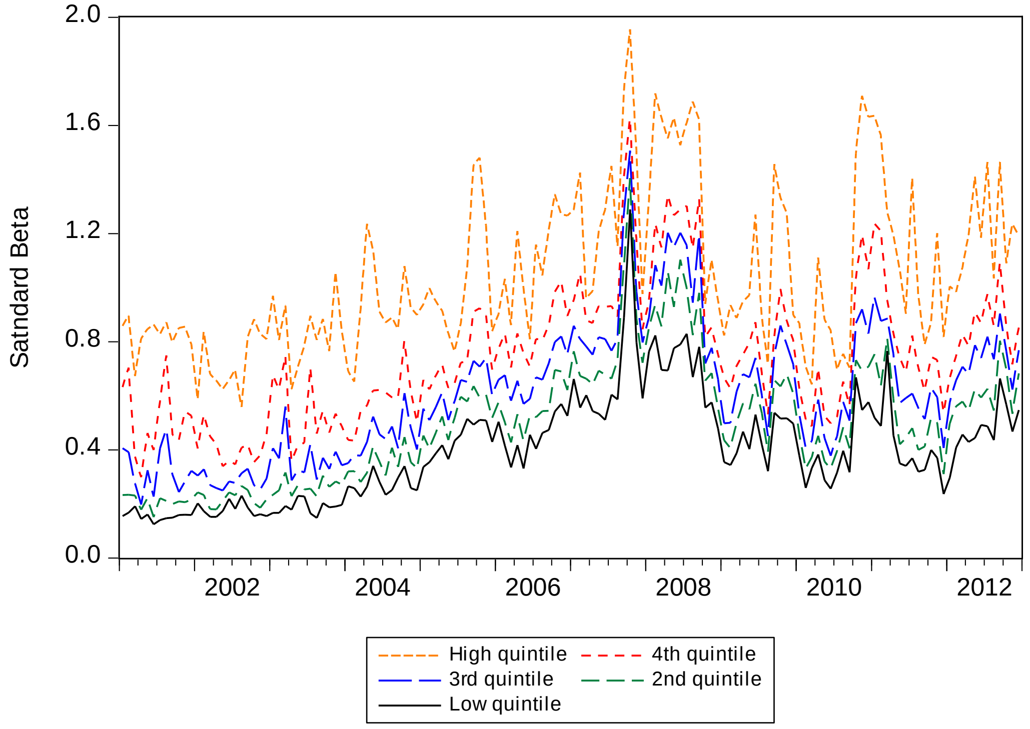


§§ Based on the expressions derived in Todorov and Bollerslev (2010), Bollerslev et al. (2015) report that the asymptotic standard errors for diffusion and jump betas averaged across all of the stocks and months in the sample equal 0.06 and 0.12, respectively, compared with 0.14 for the conventional OLS- based standard errors for the standard beta estimates

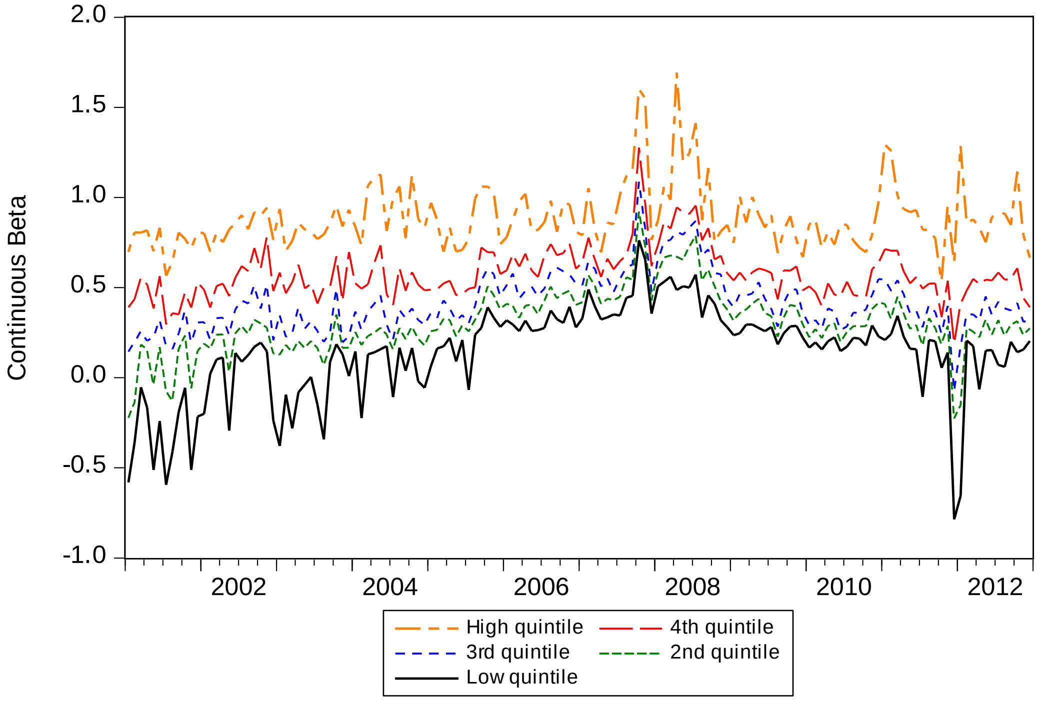
Figure 2: Time series plots of betas

The figure displays the time series of betas for equally weighted beta-sorted quintiles portfolios. Panel A shows the result for the standard beta sorted portfolios, Panel B the diffusion beta sorted portfolios and Panel C the jump beta sorted portfolios.

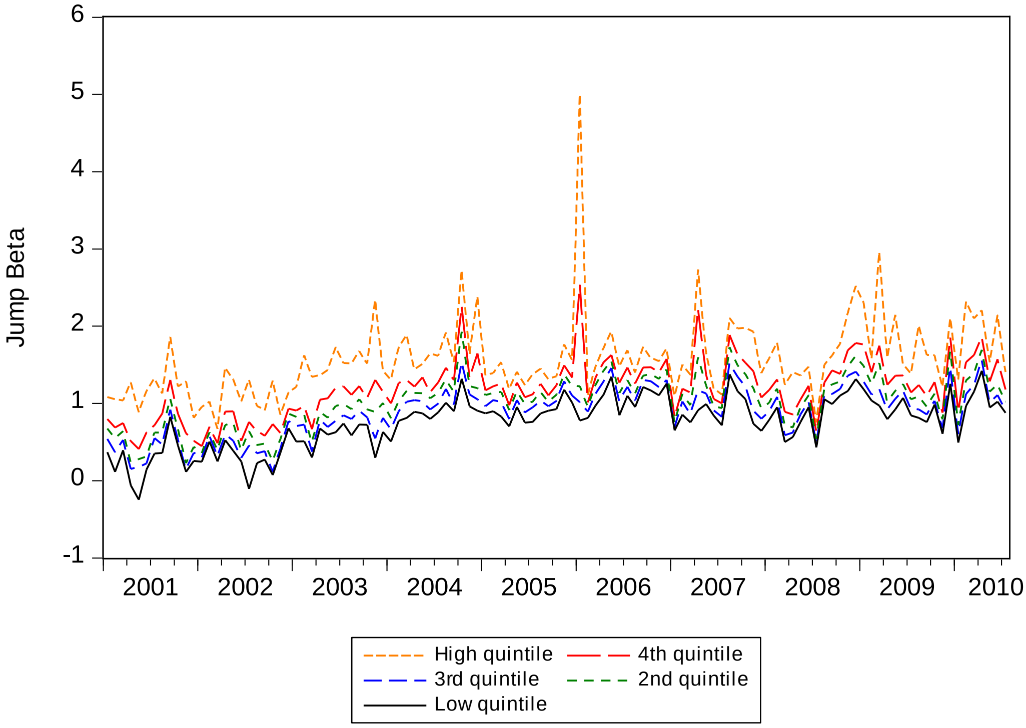
Panel (A): Standard Beta



Panel (B): Diffusion Beta



Panel (C): Jump Beta



Our estimates shows that there is interesting variation across assets and across time in the two components of the market beta. Our main findings is that stocks have higher jump betas than their diffusion betas.

B. Quantile regression model

An ordinary least squares determines the average relation between the dependent and a set of relevant explanatory variable. It focus on the estimation of the conditional mean, whereas quantile regression (QR) model allows us to estimate the relationship between a dependent and independent variables at any specific quantiles. In particular, quantile regression relaxes one of the fundamental conditions of the OLS and permits the estimation of various quantile functions, helping to examine the tail behaviour of that distribution. Moreover, it is well know that quantile regression is robust to heteroskedasticity, skewness and leptokurtosis, which are the features of financial data (Koenker and Xiao 2006). Thus, quantile regression methodology provides a better picture in testing how the relationship between diffusion and jump betas vary across quantiles of the conditional distribution.

The quantile regression approach has been widely used in many areas of applied economics and econometrics such as the investigation of wage structure (Buchinsky 1994) earnings mobility (Trede 1998; Eide and Showalter 1999), and educational quality issues (Eide and Showalter 1998; Levin 2001). There is also growing interest in employing quantile regression methods in the financial literature. Applications in this field include work on Value at Risk (Taylor 1999; Chernozhukov and Umantsev 2001; Engle and Manganelli 2004), option pricing (Morillo 2000), and the analysis of the cross section of stock market returns (Barnes and Hughes, 2002), return distributions (Allen et al. 2013), mutual fund investment styles (Bassett Jr and Chen 2002), the investigation of hedge fund strategies (Meligkotsidou et al. 2009), the return- volume relation in the stock market (Chuang et al. 2009), and the diversification and firm performance relation (Lee and Li 2012). Following this line of thought, a QR technique developed by Koenker and Bassett Jr (1978) is used in this study to examine the relationship between the standard beta, diffusion beta and jump beta.

The quantile regression takes the following form

$$y_i = x'_i b^\tau + \varepsilon_i^\tau \tag{14}$$

where y_i is the dependent variable of interest and x_i the vector of predictor variables. The parameter vector b^τ is associated with the τ-quantile while ε_i^τ is the error term, allowed to have a different distribution across quantiles. Note that the local effect of x_i on the τ-quantile is assumed to be linear. The slope coefficient vector b^τ differs across quantiles and the estimator for $b^\tau$is obtained from

$$min \sum_{i:\varepsilon_i^\tau > 0} \tau \times |\varepsilon_i^\tau| + \sum_{i:\varepsilon_i^\tau < 0} (1-\tau) \times |\varepsilon_i^\tau|$$

$$= \sum_{i:y_i - x'_i\beta^\tau \geq 0} \tau \times |y_i - x'_i b^\tau| + \sum_{i:y_i - x'_i\beta^\tau < 0} (1-\tau) \times |y_i - x'_i b^\tau| \tag{15}$$

The quantile function is estimated by minimizing a weighted sum of absolute residuals, where the weights are functions of the quantiles of interest. The coefficient estimates are computed by using linear programing methods (for more details, see Koenker (2005)). For $\tau = 0.5$, i.e., the conditional median of x, the problem collapses to the well known least absolute deviation (LAD) estimation. The value of b can be obtained using linear programming algorithms and standard errors can be bootstrapped. We conduct the minimization procedure at quantiles of $\tau = 0.05, 0.25, 0.50, 0.75, 0.95$ and thus obtain a full picture of the relationship between dependent and independent variables across the whole distribution of the former, not just for its mean value.

C. Quantile Regression Analysis

As a preliminary exercise, we first explore what OLS regressions to say about this relations of three beta across Japanese banks. Table 2 presents the results from OLS regressions to explain the cross-sectional and time series variation in the standard betas as a function of the variation in the two other betas, diffusion and jump betas. Model (1) in Table 2 shows that the diffusion beta exhibits the highest explanatory power for standard beta, with an average adjusted R-squared of 0.64. To get an impression on the contribution of jump betas, we include model (2). The jump beta explain 48% variation in standard beta. When we add the diffusion beta and jump beta as in model (3), we see that altogether, 80 % of the variation in standard beta may be accounted for by the high frequency betas, with diffusion beta having by far largest and most significant effect. It is also noted that the OLS regression results is consistent with our earlier results in Figures 1 and 2.

Table 2: The relationship between Standard, Diffusion and Jump betas across Japanese Banks

This table presents the pooled OLS regression results between Standard beta, Diffusion beta and Jump beta across different banks. **Standard errors** are displayed in parentheses below the **coefficients**. The asterisks *, **, and *** indicate the significance at the 10%, 5%, and 1% level, respectively.

Dependent Variable=Standard Beta			
	(1)	**(2)**	**(3)**
Diffusion Beta	0.874***		0.678***
	(0.029)		(0.027)
Jump Beta		0.362***	0.229***
		(0.022)	(0.011)
Constant	0.257***	0.164***	0.107***
	(0.013)	(0.015)	(0.008)
R-squared	0.64	0.48	0.80

However, it should be noted that the OLS estimator focus only on the central tendency of distributions. Therefore, they do not allow us to examine the relationship between three betas in non-central regions. A Quantile regression offers more information than OLS regression does as it looks at whether coefficient changes significantly across quantiles. To help further gauge this relations, the QR analysis used in this paper to investigate how the standard, diffusion and jump betas are related to each other at their various quantiles. [***]

[***] We proceed to examine the relations relationship between standard beta, diffusion beta and jump beta across Japanese bank using the following quantile regression model:

$$Q(\tau)_{\beta^s}\ (\beta^s_{i,t}) = a_0(\tau) + b_1(\tau)\beta^c_{i,t} + b_2(\tau)\beta^j_{i,t} + \varepsilon_{i,t}$$

The quantile regression procedures yields a series of quantile coefficients, one for each sample quantile. We may thus test whether standard beta respond differently to changes in the regression depending on whether the bank is in the left tail of distribution (low risk bank) or in the right tail of the distribution (high risk bank). In Table 3, we present the parameter estimates for selected quantiles ranging from 0.05 to 0.95. A closer look at the individual conditional quantiles reveals that the relation between standard, diffusion and jump betas changes in magnitude across the distribution quantiles. For example, while the response rate for diffusion beta and jump beta at the 5^{th} quantile are, respectively, 0.55 and 0.16, at the median they are 0.71 and 0.28, and at the 95^{th} quantile they are 0.68 and 0.22. All coefficients are strongly statistically different from zero. Additionally, our results show that the conditional mean approach is also misleading in terms of goodness-of-fit. While the R-squared of 0.80 of the conditional mean would suggest that the convaraites are relatively successful at explaining the variation in standard beta, the quantile regressions show that while this is true for high-risk firms (e.g., the pseudo R-squared at the 75^{th} quantile is 0.60), for low-risks firms the empirical variables have much less explanatory power (e.g., the pseudo R-squared at the 5^{th} quantile is 0.48). This indicates that high risk firms are more sensitive to diffusion risks than the jump risks compared to low risk firms.

In order to check the significance of the differences with regard to the coefficients of diffusion diffusion beta and jump beta across different quatalies, this study employs a bootstrap procedure extended to construct a joint distribution to test various pairs of quantiles (Chuang et al. 2009). Table 4 presents the F-test results for the null hypothesis of equal slopes across quantiles to formally test whether the slopes of explanatory variables change across quantiles. These results indicate that the coefficients are significantly different from each other between all quintiles. Further, we observe that there are significant differences between the coefficient of 5^{th} quantile and 95^{th} quantile, supporting the notion that at low and high of standard betas within Japanese banking sector the relationships between standard, diffusion and jumps betas differ significantly. More importantly our results indicate that the relationship may be far more complicated than what can be described using least-squares regression. Indeed, the relationships between standard betas, diffusion betas and jump betas for Japanese banking

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The variable of primary interest is the coefficient of diffusion and jump betas on the standard betas. The slopes of the regressors are estimated at five different quantiles τ −the 5^{th}, 25^{th}, 50^{th}, 75^{th}, 95^{th}- using the same set of explanatory variables for each quantile.

stock may be non-linear across quantiles and the relationships at tail quantiles may be quite different from those at middle quantiles and at the mean.

Table 3: The relationship between Standard beta, Diffusion beta and Jump beta different quantiles

This table presents the regression results between Standard beta, Diffsion beta and Jump beta across different quantiles. **Standard errors** are displayed in parentheses below the **coefficients**. Standard errors are obtained by bootstrapping with 100 replications. The asterisks *, **, and *** indicate the significance at the 10%, 5%, and 1% level, respectively.

Dependent Variable= Standard Beta					
	5th quant	**25th quant**	**50th quant**	**75th quant**	**95th quant**
Diffusion Beta	0.555***	0.689***	0.709***	0.684***	0.677***
	(0.025)	(0.012)	(0.010)	(0.012)	(0.028)
Jump Beta	0.157***	0.245***	0.281***	0.291***	0.222***
	(0.006)	(0.004)	(0.006)	(0.010)	(0.018)
Constant	-7.77e-16	-3.28e-15	0.0410***	0.120***	0.376***
	(0.003)	(0.000)	(0.005)	(0.008)	(0.018)
Pseudo R-squared	0.48	0.58	0.61	0.60	0.53

Table 4: Post estimation linear hypothesis testing.

The table presents F-test for testing whether coefficients between different quintile are equal. Quantiles have been estimated by simultaneous regression analysis. Standard errors were obtained by bootstrapping with 100 replications. The asterisks *, **, and *** indicate the significance at the 10%, 5%, and 1% level, respectively.

H0: Test whether continuous beta and Jump beta coefficients are qual across different quantiles	
H0: Q5=Q25	F(2, 5401) =214.87***
	Prob > F = 0.0000
H0: Q25=Q50	F(2, 5401) = 48.98***
	Prob > F = 0.0000
H0: Q50=Q75	F(2, 5401) = 3.18**
	Prob > F = 0.0417
H0: Q75=Q95	F(2, 5401) = 10.92***
	Prob > F = 0.0000
H0: Q05=Q95	F(2, 5401) = 22.42***
	Prob > F = 0.0000
H0: Q25=Q75	F(2, 5401) = 23.40***
	Prob > F = 0.0000

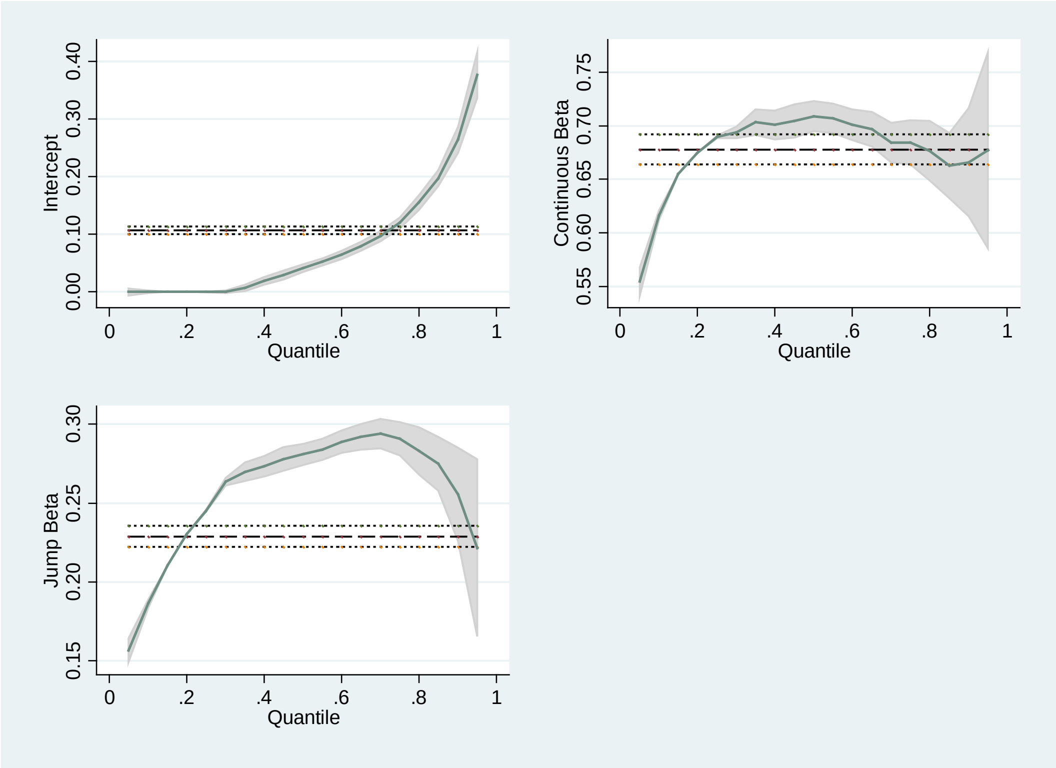
Figure 3 graphically shows how the beta values vary across quantiles. The figure depicts point estimates of the slope of explanatory variable along with a 95% pointwise confidence band. The vertical axis measures the magnitude of the coefficient, and the horizontal axis measures the quantiles. The horizontal axis lists quantiles running from 0.05 through 0.95.

If assumptions for the standard linear regression model hold, the quantile slope estimates should fluctuate randomly around a constant level, with only the intercept parameters systematically increasing with τ. However, none of the slople estimates of the varaibles could be described as random fluctuations here. In fact, the quantile slope estimates of the variables such as diffusion beta jump beta followed a systematic pattern with low values in the left tail and high values in the right tail. These two variables were significant in the tail parts of the distribution, but little impact in the middle. It is apparent that the slope of regression changes across the quantiles and is clearly not constant, as presumed in OLS. The results indicate that on average the jump betas for a quantile are higher than the corresponding diffusion betas. However, companies with low quantiles standard betas are less sensitive to market jumps as compared to companies with high quantiles standard betas.

Figure 4 shows the scatter plots of the monthly standard betas versus diffusion betas and monthly standard beta versus jump betas for quantile regressions for quantiles= 0.05, 0.25, 0.50, 0.75 and 0.95 respectively. The scatter plot in panel A of figure 4 suggests heteroskedasticity in the dataset, given that the dispersion of results seems to somewhat smaller in the middle of the distribution. The estimates lines for the 5^{th}, 50^{th}, 95^{th} quantiles shown in the panel A, indicate that for firms that are relatively risky in terms of standard beta- in other words, firms to the right of the distribution – the diffusion beta to the 5^{th} and 95^{th} quantiles are not very different. But unlike the case of panel B, the gap between the 5^{th} and 95^{th} quantiles is higher on the right side of the graph; in other words, among those firms- the jump beta to the 5^{th} and 95^{th} quantiles are quite different. It indicates that when the distribution reaches extreme, the diffusion betas and jump betas behave differently from that in or around median observation.

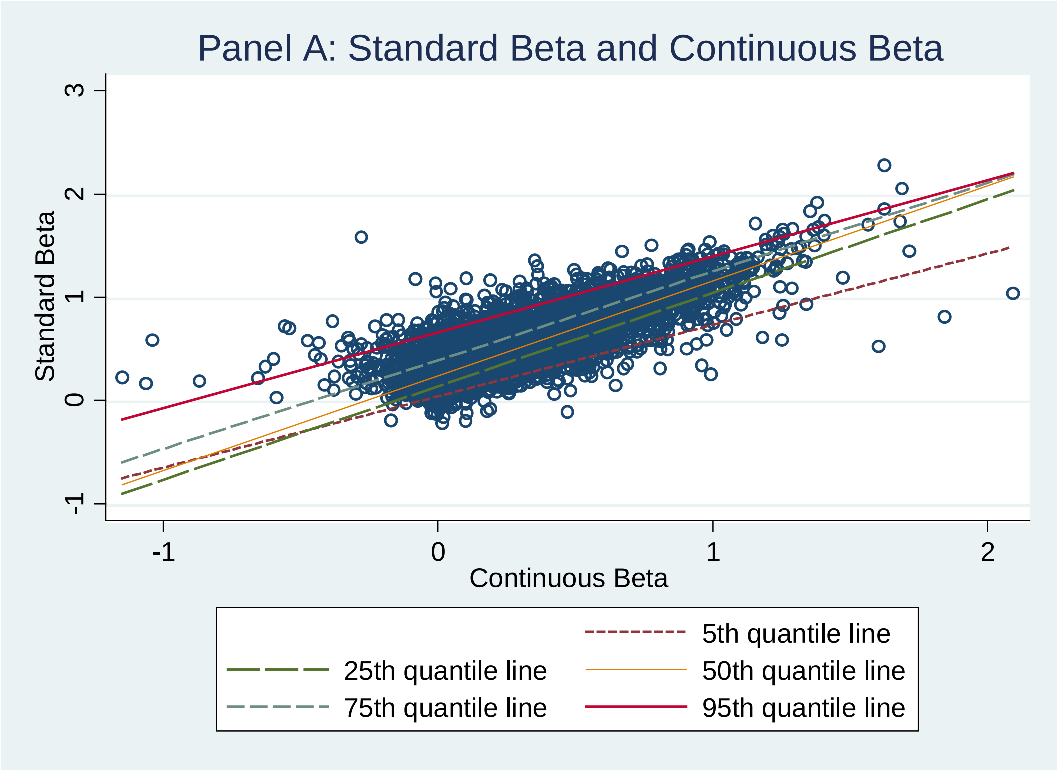
Figure 3: Quantile plot of estimated slopes and 95% confidence interval

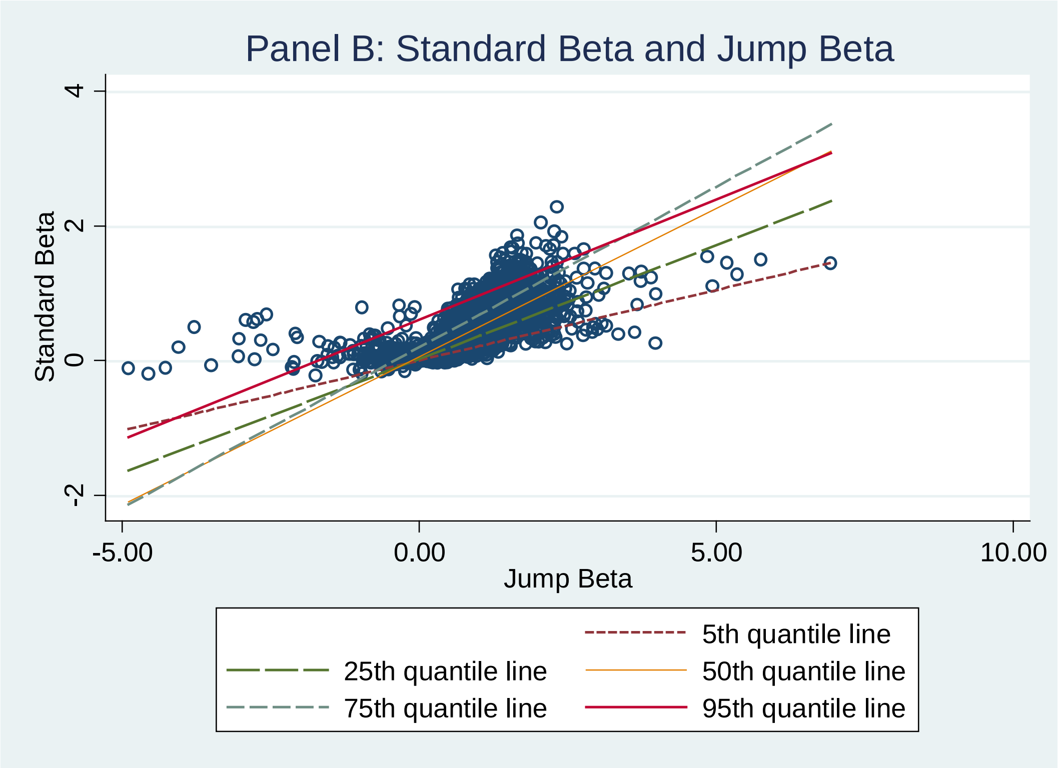
The solid line gives the coefficients of diffusion beta estimates from the quintile regression, with the shaded grey area depicting a 95% confidence interval. The dashed line gives the OLS estimate of mean effect, with two dotted lines again representing a 95% confidence interval for this coefficient.



The general conclusion that can be drawn in that there exists a wide disparities in behaviour between high risk firms and low risk firms that may be receiving diffusion and jump shocks and that such behaviour differs for high risk firms as opposed to low risk firms. The quantile regression technique provides considerable insight that cannot be obtained by using standard regression techniques. The differences in information content of the betas also manifest in different relations with underlying diffusion and discontinues price variation.

Figure 4: Scatterplot of Betas across different quantiles





D. Size-sorted portfolios

It is often implicitly assumed that small and large banks behave differently. To control further for possible size effects, we test the relationship between standard beta, diffusion beta and jump beta using 5 subsamples constructed by sorting the data with respect to size. Tables 5 and 6 report the results for portfolios sorted on stock size and rebalanced each year. The banks are grouped in five benchmark portfolios ranked by size and based on market capitalization at the end of each year *t*. Portfolio 1 includes the smallest banks in the group and portfolio 5 includes largest banks in the sample. Table 4 shows a clear effect of size on the estimated coefficient for the jump-diffusion model. The diffusion beta coefficient is lower for the largest quintiles and is statistically significant. For the jump beta, the decrease for large-cap companies is much less strong, though also statistically significant. We apply a quantile regression methodology in Table 6 to estimate the relationship between different betas and we obtain the same results as those from Table 5.

Comparing the relative magnitude of the different coefficients, we see that for small companies the jump components are the dominant ingredients. For large companies, however, it is predominantly the diffusion component. The results lead us to conclude that the jump risk is much more relevant for small companies than diffusion risk.

Table5: The relationship between Standard beta, Diffusion beta and Jump beta across for size-sorted stock portfolios

This table presents the pooled OLS regression results between Standard beta, Diffusion beta and Jump beta across different banks. **Standard errors** are displayed in parentheses below the **coefficients**. The asterisks *, **, and *** indicate the significance at the 10%, 5%, and 1% level, respectively

Dependent Variable= Standard Beta					
	Small	**2**	**3**	**4**	**Big**
Diffusion Beta	0.600***	0.685***	0.740***	0.469***	0.573***
	(0.043)	(0.066)	(0.047)	(0.063)	(0.026)
Jump Beta	0.192***	0.199***	0.215***	0.271***	0.203***
	(0.015)	(0.018)	(0.021)	(0.020)	(0.016)
Constant	0.103***	0.112***	0.107***	0.161***	0.242***
	(0.013)	(0.012)	(0.013)	(0.026)	(0.019)
R-squared	0.67	0.71	0.79	0.70	0.73

Table 6: The relationship between Standard beta, Diffusion beta and Jump beta across different quantiles for size-sorted stock portfolios

This table presents the regression results between Standard beta, Diffusion beta and Jump beta across different quantiles. **Standard errors** are displayed in parentheses below the **coefficients**. Standard errors are obtained by bootstrapping with 100 replications. The asterisks *, **, and *** indicate the significance at the 10%, 5%, and 1% level, respectively.

Dependent Variable= Standard Beta					
	5th quant	**25th quant**	**50th quant**	**75th quant**	**95th quant**
	Small				
Diffusion Beta	0.329***	0.510***	0.632***	0.638***	0.665***
	(0.049)	(0.042)	(0.049)	(0.032)	(0.052)
Jump Beta	0.170***	0.195***	0.224***	0.225***	0.185***
	(0.012)	(0.010)	(0.013)	(0.019)	(0.036)
Constant	-0.019**	0.029***	0.064***	0.133***	0.308***
	(0.009)	(0.006)	(0.009)	(0.015)	(0.036)
Pesudo R-squared	0.39	0.40	0.42	0.44	0.45
	2				
Diffusion Beta	0.498***	0.624***	0.638***	0.717***	0.656***
	(0.048)	(0.053)	(0.042)	(0.035)	(0.044)
Jump Beta	0.142***	0.205***	0.248***	0.259***	0.192***
	(0.015)	(0.012)	(0.010)	(0.019)	(0.032)
Constant	0.006	0.030***	0.063***	0.122***	0.354***
	(0.005)	(0.005)	(0.006)	(0.014)	(0.029)
Pesudo R-squared	0.40	0.47	0.50	0.49	0.47
	3				
Diffusion Beta	0.558***	0.690***	0.762***	0.736***	0.760***
	(0.064)	(0.045)	(0.031)	(0.030)	(0.047)
Jump Beta	0.157***	0.223***	0.244***	0.262***	0.209***
	(0.017)	(0.017)	(0.012)	(0.020)	(0.026)
Constant	-0.006	0.022**	0.071***	0.141***	0.357***
	(0.008)	(0.011)	(0.008)	(0.015)	(0.021)
Pesudo R-squared	0.45	0.51	0.56	0.57	0.56
	4				
Diffusion Beta	0.265***	0.463***	0.540***	0.524***	0.512***
	(0.060)	(0.040)	(0.027)	(0.026)	(0.053)
Jump Beta	0.213***	0.276***	0.316***	0.339***	0.292***
	(0.025)	(0.016)	(0.016)	(0.026)	(0.034)
Constant	0.040***	0.058***	0.081***	0.163***	0.397***
	(0.014)	(0.011)	(0.013)	(0.024)	(0.041)
Pseudo R-squared	0.38	0.47	0.48	0.49	0.49
	Big				
Diffusion Beta	0.630***	0.591***	0.590***	0.570***	0.540***
	(0.043)	(0.033)	(0.022)	(0.031)	(0.048)
Jump Beta	0.170***	0.257***	0.260***	0.225***	0.170***

	(0.027)	(0.028)	(0.021)	(0.024)	(0.025)
Constant	-3.33e-16	0.081**	0.158***	0.296***	0.563***
	(0.002)	(0.033)	(0.021)	(0.028)	(0.038)
Pseudo R-squared	0.54	0.52	0.50	0.48	0.45

E. Size Effect on Betas

The effect of size on bank systematic risk is debatable; where whilst Demsetz and Strahan (1997) find that large banks tend to diversify their business more efficiently and are less prone to bankruptcy, Saunders et al. (1990) and Anderson and Fraser (2000) find that bank systematic risk increases with bank size as large banks could be more sensitive to general market movements than small banks. Therefore, it is important to recognize that there is an inherent association between size and different betas. We test if the time varying betas are related to the market capitalisation or size of the portfolios. Table 7 presents the mean and standard deviations of the standard, diffusion and jump betas for the small and large portfolios. We report the t-statistics for the test of the hypothesis that the difference between small and large is zero. We find that in all cases there is negative and statistically different between the betas of small and large banks indicating that large banks react more severely than small banks. The results support that larger Japanese banks are more sensitive to market movements than smaller institutions, regardless of whether they occur through a jump or not.

A notable point is that although betas large firms are larger than the small firms, for large equity portfolio, the jump-diffusion beta ratios is lower than the jump-diffusion beta ratios of the small equity portfolio. The means for small equities, the influence on jump beta is proportionately much larger in compared to large equity portfolio. This is further corroborated by the larger magnitude of the constants for large portfolios than small portfolios (see Table 5 and 6). Small portfolios equities are more sensitive to large surprises than the large portfolio equities. The explanation for this phenomenon is that small bank equities are riskier than large bank equities because less information is available about the former than about the latter. Therefore, small bank portfolios react more severely to surprises than do the large bank portfolios. Reinganum and Smith (1983) have pointed out that for the differential information explanation to hold, the additional risk caused by the relative lack of information must not be idiosyncratic. That is, the lack of information must be a source risk that cannot be diversified away.

Table 7: Characteristics of time varying betas

The table summarizes of the time varying betas estimated using the Jump-Diffusion CAPM model. The statistics include mean and standard deviations (in parentheses) are summarized by the full sample periods and three sub-periods. We report the time varying betas for two size-sorted equity portfolios (large size equity beta portfolio, and small size equity beta portfolio). We include the pooled variance t-test of the difference between the two sample means for the Standard Beta, Continuous Beta and Jump Beta and also the size-sorted equity portfolio. The t-statistics are given in parentheses. * denotes significance at 10 % level; ** denotes significance at 5 % level, and *** denotes significance at 1 % level

	Large equity portfolio			Small equity portfolio		
	Standard Beta	Continuous Beta	Jump Beta	Standard Beta	Continuous Beta	Jump Beta
Full-sample Period						
Mean	0.814	0.576	1.165	0.290	0.099	0.707
Std.Dev	0.282	0.319	0.630	0.203	0.173	0.595
t-test of difference	-0.524***	-0.478***	-0.457***			
Pre-crisis Period						
Mean	0.720	0.528	1.080	0.159	0.036	0.443
Std.Dev	0.252	0.300	0.508	0.109	0.101	0.513
t-test of difference	-0.561***	-0.492***	-0.637***			
Crisis Period						
Mean	0.988	0.752	1.251	0.438	0.226	0.868
Std.Dev	0.266	0.254	0.955	0.230	0.211	0.712
t-test of difference	-0.550***	-0.526***	-0.382***			
Post-crisis Period						
Mean	0.888	0.527	1.306	0.316	0.073	0.830
Std.Dev	0.267	0.363	0.449	0.181	0.152	0.516
t-test of difference	-0.572***	-0.454***	-0.476***			

IV. Difference between the Jump-Diffusion Model and the CAPM

How distinct is the jump-diffusion model from the conventional CAPM? Tests on model specification are in favour of jump-diffusion model. However, it is necessary to test whether jump-diffusion model is related to the CAPM. Since the jump diffusion model can be written as

$$r_{i,t} = r_{m,t}[(1-\emptyset)\beta_{i,t}^{c} + \emptyset\beta_{i,t}^{j}] \quad (16)$$

With equation (16), such a test should be based on whether the beta in the conventional CAPM is the weighted average of the jump beta and diffusion beta in the jump diffusion model. The hypothesis is

$$H_0: \beta_{i,t}^{S} = [(1-\emptyset)\beta_{i,t}^{c} + \emptyset\beta_{i,t}^{j}] \quad (17)$$

The hypothesis can be tested with the following regression model

$$\beta_{i,t}^{S} = c_0 + c_1\beta_{i,t}^{c} + c_2\beta_{i,t}^{j} + \in_{i,t} \quad (18)$$

The testable hypothesis is

$$c_1 + c_2 = 1 \quad (19)$$

In Table 8, we report the results of F-test for the test of whether the systematic risk is a weighted average of diffusion and jump betas. Panel A and B of Table 8, the F-tests do not reject the hypothesis in the conditional median distribution that the conventional CAPM is a weighted average of diffusion and jump betas. Empirical findings from this results agree with the model in that on average the systematic risk on an asset is the weighted average of both jump and diffusion betas.

Table 8: Testing Distinction between the Jump-Diffusion Model and the CAPM

The table presents F-test for testing whether the beta in the conventional CAPM is the weighted average of the jump beta and diffusion beta in the jump-Diffusion model. The asterisks *, **, and *** indicate the significance at the 10%, 5%, and 1% level, respectively.

H0: Test whether the beta in conventional CAPM is the average of continuous beta and jump beta in the jump-diffusion model						
H0: C1+C2=1						
Panel A: Individual Stocks						
	OLS	**5th quant**	**25th quant**	**50th quant**	**75th quant**	**95th quant**
F-stat	18.63***	172.21***	47.85***	1.71	8.5**	18.07***
P-value	0.000	0.000	0.000	0.192	0.004	0.000
Panel B: Portfolios						
	OLS	**5th quant**	**25th quant**	**50th quant**	**75th quant**	**95th quant**
Small	17.77***	91.04***	29.56***	8.82***	21.77***	8.45***
	0.001	0.000	0.000	0.003	0.000	0.004
2	3.69	67.95***	11.08***	7.82***	0.78	18.94***
	0.065	0.000	0.001	0.005	0.379	0.000
3	1.55	23.2***	6.63***	0.05	0	0.82
	0.225	0.000	0.010	0.827	0.960	0.364
4	14.8***	63.99***	56.38***	38.84***	27.01***	8.59***
	0.001	0.000	0.000	0.000	0.000	0.004
High	114.88***	74.26***	20.38***	49.41***	64.65***	37.35***
	0.000	0.000	0.000	0.000	0.000	0.000

V. Conclusion

In this paper, we used high-frequency data and a novel method of decomposing a security's systematic risk into two components: the diffusion beta and the jump beta components to empirically test for any relationship between the standard betas, diffusion betas and jump betas and to characterise how these different betas behave across different Japanese banks. We employ the quantile regression technique to investigate the observed non-linear relationship.

Via the decomposition of the standard CAPM beta into two sub-components, we show that we can increase our understanding of the behaviour and relationship between the ensuring three systematic risks i.e. standard beta, diffusion beta and jump beta. Using high-frequency data of the Japanese banks from 2001-2012, we find that the relationship between standard, diffusion and jump betas is different across the quantiles. More precisely, we find that the standard beta, as expected, is weighted more by the diffusion component than the jump component, though the actual magnitude of the weights differ significantly across the quantiles. The relationship holds for both individual stocks and portfolios.

Past empirical studies have shown that standard betas vary systematically across firm size. A close look at our results indicates that, on average, large banks have larger betas whereas small banks have smaller betas i.e. larger Japanese banks are more sensitive to both types of market movements than smaller institutions, regardless of whether these movements are continuous or jumps. However, in our study the smaller bank portfolios exhibit larger jump-diffusion beta ratios than the larger bank portfolios, suggesting that the jump betas are disproportionately larger than the corresponding diffusion betas in the small portfolios, indicating an additional size effect. The results suggest that the jump-diffusion beta asymmetry could be more severe for smaller banks than larger banks in Japan.

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Appendix:

A1. Jump Test

We apply the nonparametric jump-detection methods prosed by Barndorff-Nielsen and Shephard (2006), hereafter, BNS, to detect jumps in the Nikkei 225 index. BNS propose two general measures based on realized power variations to test for jumps and to estimate the contribution of jumps to total variation- realized variance (RV) and bi-power variation (BV). Realized, or historical, Variance of a sequence of prices p_t can be derived from the returns. Realized variance (RV) is defined as the sum of squared intraday-returns,

$$RV_t = \sum_{s=1}^{n} r_{t,s}^2 , \qquad t = 1, \ldots, T \tag{A.1}$$

where n is the sampling total sample (usually daily/monthly) and $r_{t,s}$ is the intraday logarithmic return. Note that equation (7) uses only returns from within each trading day (intraday returns), discarding any overnight returns (intraday-returns). As a result, any jumps resulted from overnight returns are excluded from realized variance. When M goes to zero, Realized Variance converges to integrated variance plus the jumps (Barndorff-Nielsen and Shephard 2004; Andersen and Bollerslev 1998). We can re-write this as:

$$RV_t \xrightarrow{p} \int_{t-1}^{t} \sigma_s^2 \, ds + \sum_{S=q_{t-1}}^{q_t} k_s^2 , \qquad t = 1, \ldots, T \tag{A.2}$$

Where, M = sampling frequency, σ_s^2 is the time-diffusion intergrade variance function and k_s^2 is the squared discrete jump term. It is clear that Realized Variance is not a robust measure of the variance σ_s^2 in the presence of jumps.

Therefore, to improve the robustness of variance estimation in the presence of jumps, BNS proposes the bi-power variation (BV) is given by

$$BV_t = \mu_1^{-2} \frac{n}{n-1} \sum_{j=2}^{n} |r_{t,s}||r_{t,s-1}| , \quad t = 1, \ldots, T \tag{A.3}$$

where $\mu_1 = \sqrt{2/\pi}$. (Barndorff-Nielsen and Shephard 2004), show that BV consistently estimates the diffusion true or integrated variance (i.e. jump free) when the sampling frequency goes to zero. Intuitively, in the presence of any jump, one of the two consecutive returns is bound to be larger. The product of the smaller return and the larger returns, however, will be small and thus neutralize the effect of the jumps. Therefore,

$$BV_t \rightarrow \int_{t-1}^{t} \sigma_s^2 \, ds, \qquad for\ M \rightarrow 0 \tag{A.4}$$

Combining equations (4) and(6), for $M \to 0$

$$RV_t - BV_t \to \sum_{s=q_{t-1}}^{q_t} k_s^2, \quad t = 1, \dots, T \tag{A.5}$$

Thus, the difference between the RV_t and BV_t consistently estimates the jump contribution to the total variation.

Following Huang and Tauchen (2005), we define the jump ratio statistic

$$RJ_t = \frac{RV_t - BV_t}{RV_t}, \tag{A.6}$$

which converges to a standard normal distribution when scaled by its asymptotic variance in the absence of jumps. That is

$$ZJ_t = \frac{RJ_t}{\sqrt{\left[\left(\frac{\pi}{2}\right)^2 + \pi - 5\right]\frac{1}{M}\max\left(1, \frac{DV_t}{BV_t^2}\right)}} \xrightarrow{d} N(0,1) \tag{A.7}$$

where DV_t is the quad-power variation robust to jumps as shows in Barndorff-Nielsen and Shephard (2004) and Andersen et al. (2007). The quad-power varaition is defined as

$$DV_t = n\mu_1^{-4}\left(\frac{n}{n-3}\right)\sum_{j=4}^{n}|r_{t,s-3}||r_{t,s-2}||r_{t,s-1}||r_{t,s}|, \quad t = 1, \dots, T \tag{A.8}$$

The ZJ_t statistic in equation (13) can be applied to test the null hypothesis that there is no jump in the return process during a trading day, *t*. Huang and Tauchen (2005) show that this test has very good size and power properties and is quite accurate for detecting jumps. Significant jumps are identified by the realizations of ZJ_t in excess of the 99.9% critical value$\emptyset_\alpha$.

$$J_{t,\alpha} = I[Z > \emptyset_\alpha].[RV_t - BV_t] \tag{A.9}$$

where I refers to the indicator function equal to one if a jump occurs and zero otherwise.