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Ramsey Prices and Qualities

Hugh Sibly

(University of Tasmania)

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RAMSEY PRICES AND QUALITIES

Hugh Sibly
School of Economics
University of Tasmania
GPO Box 252-85
Hobart Tas 7001

Email: hugh.sibyl@utas.edu.au
Ph: (03) 62 262825
Fax: (03) 62 267587
(IDD code +61+3+)

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ABSTRACT

This paper considers a monopolist whose customers differ in their perception of the quality of a good. The monopolist is constrained to supply the same quality of good to different market segments. The price and quality per item (qpi) are described under the assumption the monopolist (a) maximises social welfare subject to a profit constraint, (b) profit maximises and (c) maximises the welfare of a sub-group of its customers. In each case, price and qpi is described under third degree price discrimination and uniform pricing. The chosen qpi is compared to cost minimising qpi.

Ramsey Prices and Qualities

Customers often differ in their perception of the quality of a good. If a firm is able to segment the market according to differences in perception, then it can benefit from price discrimination. The literature has given considerable attention to the case in which different quality levels are supplied to each segment of the market.[1] However, producing different quality levels of a particular good is costly. Thus, even when the heterogeneity in customer perception provides a significant benefit to offering differing quality levels, this may prove too costly to be profitable.[2]

This paper considers a monopolist that is constrained to supply different market segments with the same quality of good. Prominent examples are many public utilities and government business enterprises, including parks, electricity, public transport, water supply, and telecommunications. Therefore this paper considers a monopolist is assumed to act to maximise social welfare subject to satisfying a profit constraint. Many profit maximising firms are also constrained to offer a single level of quality. For example restaurants are usually constrained to offer a similar service level to all customers. Therefore both the efficient and profit maximising choice of quality of a monopoly are also derived as special cases of the analysis in this paper.

When price is the only choice variable the need to satisfy the profit constraint determines the price level. However when quality is also endogenous, the firm may choose from a menu of price and quality combinations that satisfies the profit constraint. Regulation of public utilities (or government business enterprises) occurs within a political environment, with different groups of applying varying degrees of political pressure on the government, regulator or management of the firm. The heterogeneity of tastes for qpi amongst customers opens up the possibility that there may be conflict amongst customers over the appropriate trade off between price and qpi. In particular

[1] For example Mussa and Rosen (1978), Maskin and Riley (1984).

[2] In the case of public utilities, political considerations may require provision of uniform quality, even if the market can be segmented in quality at modest cost.

institutions one type of customer may come to dominate the decisions the regulated firm. Therefore this paper also models the impact on quality choice when this occurs.

To capture the impact of the different perceptions of quality amongst consumers, a distinction is made between the customer's perception of a firm's qpi (perceived qpi) and efforts the firm makes to achieve a level of qpi (technical qpi). Each customer's perceived qpi is related to technical qpi, although the nature of this relationship may vary from customer to customer. As the main analysis of this paper concerns the firm's decisions, it is conducted in terms of technical qpi. For brevity, technical qpi is often referred to simply as qpi. The firms' technical qpi is assumed to be common knowledge.

The utility function of consumers that underpins this paper's analysis is different, yet arguably more plausible, than that assumed in the existing related literature. Consumer's utility is assumed to be a function of the uoq consumed: ie the product of perceived qpi and noi (quantity) consumed. In this case consumers are said to have no autonomous taste for qpi. (Sibly 2003a). This representation of preferences is often realistic. Many, if not most, goods have the property that consumer benefit is related to the overall pleasure (units of quality) gained from consumption rather than the quality per item alone or number of items alone. For example, the utility gained from visiting an art gallery is the product of the enjoyment per visits and the number of visits. In any event, even though not all goods may have this property, it is important to understand pricing and qpi choice for those that do.

If customers have no autonomous taste for qpi, it is natural to think of consumers choosing their optimal uoq given the price of quality set by the firm (Sibly 2003a). When, additionally, the perception of qpi is common to all consumers, the 'Levhari – Peles partition' holds. That is, for any market structure, (i) the uoq are chosen according to the usual 'equating at the margin' rules and (ii) qpi is chosen to minimize the cost of producing the relevant number of uoq (Levhari and Peles, 1973, Kihlstrom and Levhari, 1977, Sibly 2003b).[3]

[3] There exists a cost minimizing level of qpi because an increase in qpi has two effects on cost. One is the direct cost of producing increased qpi. The second effect is that the

The 'Levhari – Peles partition' should be seen as a benchmark result. This paper considers the impact on it of assuming heterogeneity in perceptions of qpi. A trivial extension of previous results indicates that 'Levhari – Peles partition' will hold if the social welfare (or profit) maximising firm can sell different quality of goods to different market segments. As noted above, this paper is concerned with the consequences of the firm being constrained to selling uniform quality. The choice of qpi is described when the firm can segment the market (in setting price but not qpi) and when it cannot segment the market. It is shown that the 'Levhari – Peles partition' holds when the social welfare (or profit) maximising firm can conduct third degree price discrimination. When the firm cannot segment the market in price setting, the 'Levhari – Peles partition' only holds when (i) the firm is producing efficiently or (ii) all customers have identical elasticities of demand.

If the firm's decisions are dominated by one type of customer, it is shown that the 'Levhari – Peles partition' holds provided the firm can use third degree price discrimination. Where the market cannot be segmented in price setting, the firm moves the choice of qpi away from the cost minimising level in favour of the dominant customer type.

It is instructive to compare the specification of consumer preferences with that in related literature. Spence (1975) and Sheshinksi (1976) implicitly assume a representative consumer, whose demand is an arbitrary function of the number of items (noi) consumed and (technical) qpi. Even with homogenous consumer preferences, it is difficult to draw general conclusions and make analytic progress with such a general specification of utility. This paper restricts preferences so that each customer is characterised by no autonomous taste for qpi. However most previous work on heterogeneity assumes, following Mussa and Rosen (1978), that the indirect utility function is given by $U = \theta q - P + I$, where θ is a taste parameter, q is qpi and P is price per item and I is utility from consumption of all other goods. While this form of the utility function provides

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quantity required to produce a given number of uoq is reduced, thereby reducing cost. Cost is minimized at the qpi where the former effect just outweighs the latter effect.

considerable analytical convenience, it is highly restrictive. Each consumer buys only one item, and the willingness to pay for that item varies with taste (and income). For the majority of goods, quantity demanded will vary with the quality of the good.[4] This is particularly true of items sold by government run or regulated organizations.

There are a number of other distinctions between this paper and the previous literature. Previous studies that adopt no autonomous taste for qpi preferences treat preferences as homogenous. For this reason the analysis is conducted in terms of the choice of noi and qpi. However, as there is assumed a heterogeneity in perceptions of qpi, it is more straightforward to examine the firms decisions in terms of price per item and qpi.. In practice, firms are more likely to choosing price per item rather than noi.

The literature on quality choice is closely related to that on durability choice (see Waldman 2003 for a recent survey). The primary difference is that the former is a static model, while the latter models the dynamics of product durability. The static quality models are therefore best suited to describing the choice of qpi for non-durable goods or services. However the model of quality per choice can act as a guide to durability choice, particularly when the details of the dynamic adjustment of durability are not important. Consequently the literatures have some common results. The most prominent is Swan invariance, which is said to occur when a monopoly produces the efficient level of qpi/durability (Swan 1970).[5] Swan invariance will hold when customers have homogenous perceptions of qpi and cost exhibits 'constant returns to scale'. It is demonstrated that Swan invariance does not, in general, hold if perceptions of qpi are instead assumed heterogeneous. However in the special case in which all customers have unit elastic demand, Swan invariance does hold.

Section 1 of the paper describes consumer demand and the firm's cost function. Section 2 presents the model of the social welfare (and profit maximising) monopolist.

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[4] In reality, unit consumer demand often arises because of moral hazard problems or transaction costs, rather than due to preferences. For example, a person may only 0.2 units of a car (because it is not needed all the time) but is constrained purchase either 0 or 1 cars because of these complications.

[5] Kihlstrom and Levhari (1977) show that Swan invariance follows from and a cost function that is iso-elastic in noi. More generally, Swan invariance implies qpi is independent of shifts to firms' residual demand, in particular resulting from a change in the level of competition (Sibly 2003b).

The cases in which the firm can use third degree price discrimination and uniform pricing are considered separately. Section 3 presents the model of the firm whose decision making is dominated by one type of customer. Again the cases in which the firm can use third degree price discrimination and uniform pricing are considered separately. Section 4 concludes the paper.

1. The Model

1.1 Consumers

Assume two types of consumers, i=1,2, with utility $U^i(q^iX^i, \underline{\mathbf{x}}^i)$, where X^i is the number of items purchased by type i consumers, $\underline{\mathbf{x}}^i$ is a vector of the consumption of other goods by type i consumers and q^i is the qpi of the good as perceived by type i consumers. Type i consumers have a common perception of qpi of the good amongst themselves and qpi is exogenous from the point of view of each consumer. The consumer's budget constraint is:

$$P^iX^i + \underline{\mathbf{p}}.\underline{\mathbf{x}}^i = I \qquad (1)$$

where P^i is the price of per item charge to type i customers, and $\underline{\mathbf{p}}$ it the vector of prices of other goods. Define x^i, type i's demand for uoq, by $x^i \equiv q^iX^i$. Define p^i, the price of quality facing type i consumers, by $p^i = P^i/q^i$. Then the type i's optimisation problem becomes:

$$\text{Max } U^i(x^i, \underline{\mathbf{x}}^i) \text{ subject to } p^ix^i + \underline{\mathbf{p}}.\underline{\mathbf{x}}^i = I \qquad (2)$$

Using standard consumer theory the demand for uoq and demand for noi is given by:

$$x^i = x^i(p^i, \underline{\mathbf{p}}, I) \Leftrightarrow X^i(P^i, q^i, \underline{\mathbf{p}}, I) = x^i(P^i/q^i, \underline{\mathbf{p}}, I)/q^i \qquad (3)$$

As both $\underline{\mathbf{p}}$ and I are exogenous to the analysis below, for brevity subsequent reference to them is suppressed. Type i's elasticity of demand is given by:

$$\varepsilon_x^i \equiv -p^i x^{i\prime}(p^i)/x^i = -(P^i/X^i)(\partial X^i/\partial P^i) \qquad (4)$$

Define total demand for uoq, x and total demand for noi, X(P,q) as:

$$x(p^1,p^2) = x^1(p^1) + x^2(p^2) \quad \Leftrightarrow \quad X(P,q) = X^1(P^1,q^1) + X^1(P^1,q^1) \qquad (5)$$

The industrial organization literature generally formulates its analysis using consumer surplus and consumer benefit rather than consumer utility. For consistency with this approach, and also for ease of analysis, this approach is adopted in this paper. : Integrating consumer i's uoq demand curve yields their consumer surplus[6], $v^i(p^i)$, where $v^i(p^i) = \int_{p^i}^{\infty} x^i(r)dr$. Note that (3) may be inverted to yield $p^i(x^i)$. In this case the total benefit from production can be expressed as:

$$B(x^1,x^2) = v^1(p^1(x^1)) + v^2(p^2(x^2)) + p^1(x^1).x^1 + p^2(x^2).x^2 \qquad (6)$$

Thus the benefit from production is a function of both the quantity and distribution of uoq produced.

1.2 Heterogeneity in Consumer perceptions of qpi

Perceived qpi is related to technical qpi, which is an objective measure of the quality level of the good. Allowing for a distinction between the two concepts of quality admits the possibility that the two groups have different perceptions of quality. In addition, it allows the possibility that consumers exhibit diminishing returns to technical

[6] The surplus provides a good approximation of welfare when income effects are negligible, in particular when PX^i is a small fraction of the consumer's income (see Tirole, 1988, p. 11).

qpi in their perception of qpi.[7] Let $q^i(y)$ be the perceived qpi of type i, which is achieved by supplying technical qpi y. As the analysis of firm's production decision is conducted in terms of technical qpi, for brevity it is referred to below as simply qpi. Let $\varepsilon^i_{qy} \equiv yq^{i\prime}(y)/q^i(y)$, i=1,2 be the elasticity of i's perceived qpi with respect to (technical) qpi. Without loss of generality assume that, when perceptions of qpi is heterogenous, type 1 consumers are more sensitive to quality than type 2s, ie $\varepsilon^1_{qy} > {}^2_{qy}$. Perceptions are said to be homogenous when $\varepsilon^1_{qy} = {}^2_{qy}$.

1.3 Technology

The firm's technology is summarised by the cost function C(X,y), ie the total cost of production is a function of the noi produced and the qpi. It is assumed that marginal cost, $C_1(X,y)$, and the marginal cost of qpi, $C_2(X,y)$, are non-decreasing, ie $C_1 \geq 0$ and $C_2 \geq 0$. By (5) the cost function can be expressed in the following way:

$$c(x^1,x^2,y) \equiv C(x^1/q^1(y)+x^2/q^2(y),y) \qquad (7)$$

It is useful to interpret $c(x^1,x^2,y)$ as the cost of providing x^1 and x^2 to the respective customers as a function of qpi. The minimum cost of creating x^1 and x^2 uoq $y^*(x^1,x^2)$, is determined by the following optimisation problem:

$$y^*(x^1,x^2) = \underset{y}{\operatorname{argmin}}\ c(x^1,x^2,y) \qquad (8)$$

Note that, by (6), $y^*(x^1,x^2)$ can be interpreted as the qpi which minimises the cost of providing benefit $B(x^1,x^2)$. The first order condition of (8) is:

$$c_3(x^1,x^2,y) = -[X^1(q^{1\prime}(y)/q^1(y))+X^2(q^{2\prime}(y)/q^2(y))]C_1(X,y) + C_2(X,y)= 0 \qquad (9)$$

or:

$$\varepsilon_{Cy} = \varepsilon_{qy} . \varepsilon_{CX} \qquad (10)$$

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[7] By judicious definition of perceived qpi, a wide class of consumer preferences are admitted to the analysis. For example, if benefit is $y^3(1+X\ln y)$, the approach of this paper is followed by setting q=lny

where $\varepsilon_{qy} = [(X^1/X)\varepsilon^1_{qy} + (X^2/X)\varepsilon^2_{qy}]$ is the weighted sum of the elasticities of perceived qpi with respect to qpi, $\varepsilon_{CX}=XC_1(X,y)/C(X,y)$ is the elasticity of cost with respect to noi and $\varepsilon_{Cy}=XC_2(X,y)/C(X,y)$ is the elasticity of cost with respect to qpi. Figure 1 depicts the marginal cost of quality, $c_3(x^1,x^2,y)$, as a function of qpi for a given allocation of uoq. The marginal cost of qpi is upward sloping as it is assumed that $c_{33}>0$. The cost minimising qpi, $y^*(x^1,x^2)$, occurs when c_3 cuts the horizontal axis. As can be seen from figure 1, if $c_3<(>)0$ qpi is less (more) than $y^*(x^1,x^2)$.

The implications of heterogeneity in customers' perception of qpi for the cost minimising qpi are indicated by (10). If customers have homogeneous perceptions, the cost minimising qpi is simply a function of the noi produced. However when there is heterogeneity of perceptions then the cost minimising qpi depends on both the noi and the distribution of noi across types. To further clarify this point, consider the 'constant returns to scale' cost function:

$$c(X,y) = X\psi(y) \tag{11}$$

where $\psi'(y) > 0$. If (11) holds, the cost minimising qpi satisfies:

$$\varepsilon_{\psi y} = \varepsilon_{qy} \tag{12}$$

where $\varepsilon_{\psi y} = y\psi'(y)/\psi(y)$.When consumers have homogeneous perceptions ε_{qy} is a function of y alone. In this case (12) implies a requirement of Swan invariance: that the cost minimising qpi is independent of the noi produced. However, when consumers have heterogeneous perceptions ε_{qy} is a function of both y and the X^1/X, the distribution of noi across consumers. In this case Swan invariance cannot hold unless the distribution of noi is not affected by a relaxation of the profit constraint. It is seen below that this requires excessively restrictive assumptions.

2. Constrained Efficient (Ramsey) Price and Quality

This section considers the social welfare maximising firm that is required to satisfy a profit constraint. The determination of the efficient price per item and qpi is determined under, first, the assumption the firm may use third degree price discrimination and, second, must offer a uniform price per item to each customer.

2.1 Third Degree Price Discrimination

Under third degree price discrimination the firm can identify members of each group of customers. A distinct price per item, P^i, is charged to members of group. The profit of the firm, $\pi(p^1,p^2,y)$, is:

$$\pi(p^1,p^2,y) = p^1x^1(p^1) + p^2x^2(p^2) - c(x^1(p^1),x^2(p^2),y) \qquad (13)$$

The surplus from production of the good is given by:

$$S(p^1,p^2,y)= v^1(p^1) + v^2(p^2) + \pi(p^1,p^2,y) \qquad (14)$$

It is assumed that the minimum profit the firm must earn is $\bar{\pi}$. The constrained efficient (CE) prices of quality and qpi under third degree price discrimination is given by maximising the surplus, (14), subject to the profit constraint, ie:

$$\max_{p^1\, p^2\, y} \; S(p^1,p^2,y) \text{ s.t. } \pi(p^1,p^2,y) \geq \bar{\pi} \qquad (15)$$

The Lagrangian for this optimisation problem is:

$$L(p^1,p^2,y) = v^1(p^1)+v^2(p^2)+(1+\lambda)[\, p^1x^1(p^1)+p^2x^2(p^2)-c(x^1(p^1),x^2(p^2),y)]-\lambda\bar{\pi} \qquad (16)$$

where λ is the Lagrange multiplier. Note that, by the Kuhn-Tucker conditions, $\lambda>0$ when the profit constraint is binding and $\lambda=0$ when it is not. The first order condition $\partial L/\partial p^i=0$ yields the condition for the CE prices:

$$\frac{p^i-c_i(x^1,x^2,y)}{p^i} = \left(\frac{\lambda}{1+\lambda}\right)\frac{1}{\varepsilon_x^i} \Leftrightarrow \frac{P^i-C_1(X,y)}{P^i} = \left(\frac{\lambda}{1+\lambda}\right)\frac{1}{\varepsilon_x^i} \tag{17}$$

This is the familiar condition for Ramsey prices. The following proposition follows directly from the first order condition $\partial L/\partial y=0$

> Proposition 1: Under third degree price discrimination the social welfare maximising firm and profit maximising firm exhibits the Levhari-Peles partition. That is, uoq for each customer type is chosen by the Ramsey condition (17) and qpi is chosen to minimise cost of supplying the uoq to each type. Specifically qpi is given by (8).

The first order conditions of the constrained optimisation problem (15) yields, as a special case, both the monopoly and unconstrained efficient outcomes. When the profit constraint is not binding ($\lambda=0$), equation (17) shows that (unconstrained) efficiency requires both types to pay the same price per item, which is equal marginal cost. However, because both types value quality differently, they pay different prices of quality. The monopoly level of p^i is given by (17) when $\lambda=\infty$. As is usual with price discrimination, the type with the higher elasticity of demand will pay a lower price per item. If the type with higher elasticity of demand also places a low value on quality per item, then this group will face a higher price of quality than the other group. However, if this is not the case, the relative size of the price of quality between the groups is ambiguous.

Proposition 1 shows the firm chooses the cost minimising qpi irrespective of the magnitude of the profit constraint. In general the relaxation of the profit constraint changes both X and the distribution of X. By (10) this leads to a change in qpi. As a result Swan invariance will not hold. Under the constant returns to scale (11) invariance can only hold if there is no change in the distribution of X. Clearly, by inspection of (17) and the specification of consumer demand, this can only occur under very specific circumstances. One case is when consumers' demand has unit elasticity:

$$x^i = A^i/p^i \Rightarrow X^1/X^2 = A^1/A^2 \tag{18}$$

where A^i is a constant. Thus, by (12) qpi is given by:

$$\varepsilon_{\psi y}(y) = [A^1\varepsilon^1_{qy}(y)+A^2\varepsilon^2_{qy}(y)]/(A^1+A^2) \tag{19}$$

Thus qpi is independent of $\overline{\pi}$ (or equivalently λ) and Swan invariance holds.

2.2. Common price per item

Suppose the firm cannot segment the market, and must apply a common price per item to each customer. In this case the surplus may be written:

$$S(P,y) = V(P,y) + \pi(P,y) \tag{20}$$

where $V(P,y) = v^1(P/q^1)+v^2(P/q^2)$ and:

$$\pi(P,y) = P[x^1(P/q^1)/q^1+x^2(P/q^2)/q^2]–C(x^1(P/q^1)/q^1+x^2(P/q^2)/q^2,y) \tag{21}$$

The constrained efficient price per item and qpi satisfy:

$$\max_{P\ y} S(P,y) \text{ s.t. } \pi(P,y) \geq \overline{\pi} \tag{22}$$

Equation (22) defines the social optimisation problem with a common price per item. The Lagrangian for it is:

$$L(p,y,\lambda) = V(P,y) + \pi(P,y) + \lambda(\pi(P,y)-\overline{\pi})$$

$= v^1(P/q^1)+v^2(P/q^2)-\lambda\bar{\pi}$

$+(1+\lambda)[(P[x^1(P/q^1)/q^1+x^2(P/q^2)/q^2]–C(x^1(P/q^1)/q^1+x^2(P/q^2)/q^2,y)\}$ (23)

The profit constraint is binding $\lambda>0$ and is not binding if $\lambda=0$. Observe from (23) that if the profit constraint is binding, the optimisation problem is equivalent to maximising $V(P,y)$ subject to $\pi(P,y)=\bar{\pi}$. In this case a direct intuitive description of the solution to the social optimisation problem (22) can be shown in figure 2. The profit constraint requires P and y to lie on the curve $\bar{\pi}$. The highest social indifference curve that can be reached is therefore given by $\bar{v}$. The CE combination of P and y is therefore given by the point CE in figure 2, at which $\bar{v}$ is tangent to $\bar{\pi}$.

The first order condition $\partial L/\partial P=0$ yields the familiar condition for the CE prices:

$$\frac{P-C_1}{P}=\left(\frac{-\lambda}{1+\lambda}\right)\frac{1}{\varepsilon_x} \qquad (24)$$

where $\varepsilon_x=[X^1\varepsilon_x^2+X^2\varepsilon_x^1]/X$ is elasticity of the total demand curve. When the profit constraint is not binding, $\lambda=0$, and (24) yields $P=C_1$ (price equals marginal cost). When $\lambda>0$ the profit constraint is binding, and $P>C_1$ (price is greater than marginal cost). In particular, when $\lambda=\infty$ the firm sets profit maximising price for a given y. As $\lambda\in[0,\infty]$, for a given y, P must lie between the profit maximising level (shown by the curve $\pi_1=0$ in figure 2) and marginal cost (shown by the curve $P=C_1$).

Using the first order conditions $\partial L/\partial P=0$ and $\partial L/\partial y=0$ to eliminate λ yields:

$$\frac{V_1(P,y)}{V_2(P,y)}=\frac{\pi_1(P,y)}{\pi_2(P,y)} \qquad (25)$$

The RHS of (25) is the slope of the social indifference curve and the LHS is the slope of the isoprofit curve. Thus (25) represents the contract curve: the locus of points at which the iso-profit curve and the social indifference curves are tangent. The slope of the social indifference curve is:

$$\frac{V_1(P,y)}{V_2(P,y)} = \frac{P\varepsilon_{qy}}{y} > 0 \qquad (26)$$

As the social indifference curve is upward sloping, the contract curve must pass through the upward sloping segment of the isoprofit curve. This is the bold segment of the isoprofit curve indicted in figure 2.[8]

The contract curve is depicted in figure 2 as the curve CC as running between the point π^* and E. The point π^* is the profit maximising combination of P and y. The point E is the (unconstrained) efficient combination of P and y. It occurs at the point at which the marginal cost curve ($P=C_1$) cuts the contract curve. In figure 2, the point E lies outside the boundary of the profit constraint, $\overline{\pi}$. The CE combination of price per item and qpi satisfies both the (binding) profit constraint and the contract curve. In figure 2, it is indicated by the point CE, at which the curve $\overline{\pi}$ cuts the curve CC. Note that it is possible that the point E lies within the boundary of the profit constraint. In this case the profit constraint is not binding, and the CE combination of P and y is given by the point E, which is also the efficient combination of P and y.

The contract curve traces out the path of CE price per item and qpi as a binding profit constraint is relaxed. If the contract is downward sloping, as depicted in figure 2, then a relaxation of the profit constraint will lead to a lower price per item and increased qpi. However the contract curve need not be downward sloping as depicted in figure 2. For example, the contract curve could be upward sloping and also lie to the 'south-west' of π^*. In this case a relaxation of the profit constraint would lead to a fall in both price and qpi. Alternatively, the contract curve could be positively sloped and lie to the 'north-east' of π^*. In this case a relaxation of the profit constraint could lead to both an increased price per item and qpi. In this case the CE and efficient prices would actually be greater

[8] Of course the other upward sloping segment is not considered as it lies over lower indifference curves.

than the profit maximising price. Intuitively, there would be a high benefit to qpi, but its marginal cost is relatively high.

The contract curve, (25), can be expressed as:

$$c_3(x^1,x^2y) = g(x^1,x^2,y) \tag{27}$$

where $g(x^1,x^2,y) \equiv (P-C_1)X^1X^2(\varepsilon^1_{qy}-\varepsilon^2_{qy})(\varepsilon^1_x-\varepsilon^2_x)/X$ is the constrained marginal revenue of qpi. Consequently:

<u>Proposition 2</u>: Suppose the firm sets a common price per item for all customers. The social welfare maximising firm exhibits the Levhari-Peles partition if:

(i) customers have homogeneous perceptions of qpi or,

(ii) customers have equal elasticities of demand or,

(iii) the profit constraint is not binding

If these conditions do not hold social welfare maximising firm's qpi is greater (less) than the cost minimising qpi when type 1's demand is more elastic (inelastic) than type 2's demand.

As is usual under Ramsey pricing, the firm can minimise the efficiency loss arising from the requirement to satisfy the profit constraint by lowering the relative the price of quality to the group with the most elastic demand. If type1 has the most elastic demand, one way this can be achieved is by rasing qpi. By assumption, when perceptions of qpi are heterogeneous type 1 customers are more sensitive to quality changes. An increase in qpi lowers 1's price of quality by relative to that of 2s. This provides an incentive to increase qpi beyond the cost minimising level. Specifically the constrained marginal revenue of qpi is positive as depicted in figure 1. As a result the CE qpi is $y^e(x^1,x^2)$ which is above $y^*(x^1,x^2)$, the cost minimising qpi.

The profit maximising firm is described by the special case of (24) when $\lambda\rightarrow\infty$, and also by (27). Hence:

<u>Proposition 3</u>: Suppose the firm sets a common price per item for all customers. The profit maximising firm exhibits the Levhari-Peles partition if:

(i) customers have homogeneous perceptions of qpi or,

(ii) customers have equal elasticities of demand.

If these conditions do not hold social welfare maximising firm's qpi is greater (less) than the cost minimising qpi when type 1's demand is more elastic (inelastic) than type 2's demand.

The intuitive explanation of Proposition 3 is similar to that of Proposition 2. The choice of qpi by the profit maximising firm is designed to increase the price of quality paid by the customers with the more inelastic demand.

Propositions 2 and 3 shows the Levhari-Peles partition will hold when demand is given by (3.5) (as different types have a common elasticity of demand). Thus Swan invariance holds when the firm must set a single price per item and demand is given by (3.5).

3. One dominant customer type

Most regulated firms operate in a political environment. The decisions of the firm are either influenced or controlled by organised vested interests. In this case one group of customers may be able to organise sufficiently to dominate the decision making process of the firm. In this section it is assumed that one type of customers dominates the decision making process. While this may be an extreme assumption, it is important to understand its implications in order to understand the structure of the regulated firm's decision-making process.

3.1 Third Degree Price Discrimination

Suppose, for instance, that it is the type 1 customers that dominates the decision making process. Further suppose the firm can practice third degree price discrimination. In this case, given the firm must satisfy the profit constraint, it chooses according to:

$$\max_{p^1\, p^2\, y} \; v^1(p^1) \;\text{ subject to } \pi(p^1,p^2,y) \geq \overline{\pi} \qquad (28)$$

The Lagrangian for this optimisation problem is:

$$L(p^1,p^2,y) = v^1(p^1) +\lambda[p^1x^1(p^1)+p^2x^2(p^2)–c(x^1(p^1),x^2(p^2),y)- \bar{\pi}] \qquad (29)$$

The first order condition $\partial L/\partial p^2=0$ yields the condition for the price charged to type 2s:

$$\frac{p^2-c_2(x^1,x^2,y)}{p^2} = \frac{1}{\varepsilon_x^2} \Leftrightarrow \frac{P^2-C_1(X,y)}{P^2} = \frac{1}{\varepsilon_x^2} \qquad (30)$$

Thus type 2's are charged their monopoly price. The first order condition $\partial L/\partial p^1=0$ yields:

$$\lambda = x^1/\pi_1 \qquad (31)$$

It can be noted that $\lambda \neq 0$, provided (as is reasonable to assume) that $x^1>0$ and $|\pi_1|<\infty$. Thus the profit constraint is always binding. That is, type 1's price of quality is the lowest which enables the profit constraint to be satisfied. The following proposition follows directly from the first order condition $\partial L/\partial y=0$

> <u>Proposition 4</u>: Under third degree price discrimination, the type 1 customer dominated firm exhibits the Levhari-Peles partition. That is, qpi is chosen to minimise cost of supplying the uoq to each type. Specifically qpi is given by (8).

As was the case with both the profit maximising and social welfare maximising firms, third degree price discrimination gives the type 1 customer dominated firm additional flexibility in the distribution of uoq. It is therefore able to pursue its objectives and also choose qpi to minimise the cost of producing the desired level of uoq.

3.2 Common Price per item

In this section a firm that is not able to segment the market according to type is considered. Clearly such firms cannot price according to type. In this case the type 1 dominated firm chooses price per item and qpi according to:

$$\max v^1(P/q^1) \text{ subject to } \pi(P,y) \geq \overline{\pi} \tag{32}$$

The Lagranian for this optimisation problem is:

$$L(P,y) = v^1(P/q^1) + \lambda[\pi(P,y) - \overline{\pi}] \tag{33}$$

where:

$$\pi(P,y) = (P/q^1)x^1(P/q^1) + P/q^2x^2(P/q^2) - C(x^1(P/q^1)/q^1+x^2(P/q^2)/q^2,y) \tag{34}$$

The first order condition $L_1(P,y)=0$ yields:

$$\lambda = X^1/\pi_1 \tag{35}$$

It can be noted that $\lambda \neq 0$, provided (as is reasonable to assume) that $X^1>0$ and $|\pi_1|<\infty$. Thus the profit constraint is always binding. Furthermore, the requirement that $\lambda>0$ ensures that $\pi_1>0$, that is, price (for a given y) is below the monopoly level. The first order conditions $L_1(P,y)=L_2(P,y)=0$ yield:

$$\frac{\partial v^1/\partial P}{\partial v^1/\partial y} = \frac{\pi_1(P,y)}{\pi_2(P,y)} \tag{36}$$

where the RHS is the slope of 1's indifference curve. Thus (36) represents type 1's contract curve: the locus of points at which 1's indifferent curves are tangent to the iso-profit curve. The constrained optimal price per item and uoq combination occur at the point where the contract curve, (36), coincides with the profit constraint.

Observe that the slope of type 1's indifference curves are given by:

$$\frac{\partial v^1/\partial P}{\partial v^1/\partial y} = \frac{P\varepsilon^1_{qy}}{y} > 0 \qquad (37)$$

Thus type 1's indifference curves are upward sloping. Comparison of (37) with (26) indicates that, when perceptions of qpi are heterogenous ($\varepsilon^1_{qy} > \varepsilon_{qy}$), at each point type 1's indifference curves are steeper than the social indifference curves. Thus, as depicted in figure 2, type 1's contract curve, CC^1, must lie above CC, the social contract curve. The constrained optimal price per item and uoq combination occur at the point 1*, where CC^1 intersects $\bar{\pi}$, the profit constraint. At the optimum type 1's receive consumer surplus $\hat{v}^1$. Intuitively, type 1 maximises their consumer surplus by choosing the price-qpi combination, 1*, at which the indifference curve, $\hat{v}^1$, is tangent to $\bar{\pi}$, the iso-profit curve.

By a similar argument to above, it can be shown type 2's indifference curves are less steep than the social indifference curves. Figure 2 shows the indifference curve of type 1, $\bar{v}^1$, and that of type 2, $\bar{v}^2$, that pass through the (constrained) efficient allocation CE*. Figure 2 indicates how type 1 achieves a higher consumer surplus $\hat{v}^1$ at allocation 1* than $\bar{v}^1$ which is achieved at the allocation CE*. Type 1s choose a higher price per item and qpi than at CE*.[9] Intuitively, because type 1s receive a higher benefit from qpi than average, they lower their price of quality by raising qpi beyond the level at CE*. Price per item is raised by a lesser amount than the increase in type 1’s perceived qpi. Consequently type 1’s price of quality falls. However, the price per item rises more than type 2s perceived qpi, hence their price of quality falls. Hence type 1s lower their price of quality at the expense of raising type 2's price of quality. Alternatively the constrained efficient allocation trades off the price of quality of both types, and yields a lower price of quality to each type than they could achieve by unilateral control.

To identify the relationship between type 1's choice of qpi and the cost minimising level, write the contract curve (37) as:

[9] Assuming the iso profit curves are convex.

$$c_3(x^1,x^2y) = (\varepsilon^1_{qy}-\varepsilon^2_{qy})[1 - (P-C_1)\varepsilon^2_x/P]PX^2 \qquad (38)$$

Hence:

Proposition 5: Consider a type 1 customer dominated firm that must set a common price per item to all customers. When perceptions of qpi are homogeneous the Levhari-Peles partition holds. When, however, perceptions of qpi are heterogeneous, the firm chooses a qpi which is greater than the cost minimising qpi if price is less than type 2's monopoly price, ie $(P-C_1)/P < 1/\varepsilon^2_x$.

Intuitively, Proposition 5 holds because an increase in qpi holding the uoq of each type (x^i) fixed must be accompanied by a decrease in the noi (X^i) of both types. As type 1 is the more sensitive to changes in qpi, X^1 is decreased by a greater amount than X^2. The decreasing in noi will lower revenue. Assuming price is less than the monopoly level, an increase in price per item is required to satisfy the profit constraint. When price per item is less than type 2's monopoly price, an increase in price increasing the qpi beyond the cost minimising level allows type 1 customers to shift the burden of satisfying the profit constraint to type 2 customers.

4. Conclusion

This paper considers the quality choice of a monopolist when customers have heterogeneous perceptions of qpi. (The analysis shows that groups materially differ in their perceptions of qpi when they differ in their elasticitites of perceived qpi with respect to technical qpi.) The analysis is conducted under the assumption that customers have no autonomous taste for qpi. The monopolist cannot segment the market in quality, but may be able to segment it in price. The social welfare and profit maximising qpi are described under the assumptions that (i) the monopolist can third degree price discriminate and (ii) the monopolist must set a uniform price per item to all customers. The qpi of a firm whose choices are dominated by one group of customers is also found under both these assumptions.

The qpi choice is characterised by whether the 'Levhari-Peles partition' holds. This result acts as a benchmark for qpi choice. It indicates whether the qpi is chosen to minimise the cost of producing a given uoq (and thus utility). It is shown that the Levhari-Peles partition continues to hold when either the social welfare or profit maximising firm can use third degree price discrimination. However when constrained to set a uniform price per item, the social welfare maximising firm only satisfies the Levhari-Peles partition when either the profit constraint is not binding or each group of customers has the same elasticity of demand. A firm that is dominated by one group will not satisfy the 'Levhari-Peles partition', and will distort qpi choice away from the cost minimising level in favour of the preferences of the dominant group.

The other important characterisation of qpi choice is whether Swan invariance holds. A requirement for Swan invariance to hold is that the cost minimising qpi is independent of the noi produced. This is the case when perceptions are homogeneous and the cost function exhibits constant returns to scale. However it is shown that when preferences are heterogeneous, the cost minimising level of qpi depends on the distribution of noi across groups. Thus only when the distribution of noi across groups does not change with noi, such as when demand is unit elastic, does Swan invariance hold.

Two extensions to the analysis presented in this paper suggest themselves. First, the choice of the profit constraint may not be exogenous, but the result of political process. The political process will often be represented by bargaining between the various parties involved, for example between the regulator (or government) and the firm. Often regulators take the form of consumer advocates, in which case the regulators preference may be proxied by consumer preferences. The analysis in this paper is relevant to such a scenario, as the result of bargains would lie on the contract curve. Bargaining would simply determine the profit level.

This paper modelled a firm that is captured by one section of its customers. However government business enterprises are often thought to act as revenue maximising. In this event the revenue maximising choice or price and qpi may result in the profit constraint being violated. Using the approach of this paper, it is relatively straightforward to show that the he profit-constrained revenue-maximising firm satisfies the 'Levhari-Peles partition' when it can use third degree price discrimination. However the 'Levhari-Peles partition' is not, in general, satisfied when the firms must set a uniform price per item to all customers.

The analysis in this paper is made tractable through the adopting assumption that consumers have no autonomous taste for qpi. Although it has not be widely used in the literature, this is a realistic assumption for many goods, and a useful benchmark for others. Using it, many other extensions or modifications to the analysis presented in this paper are possible.

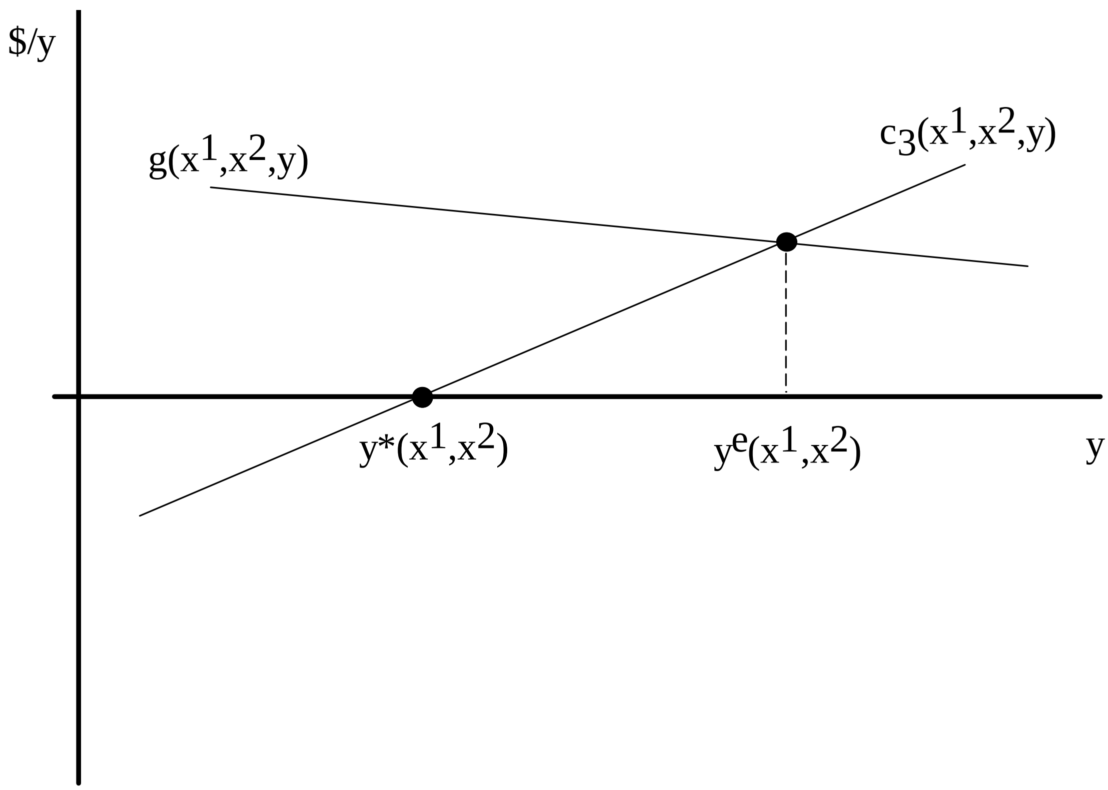

Figure 1: CE qpi for a given allocation of uoq.

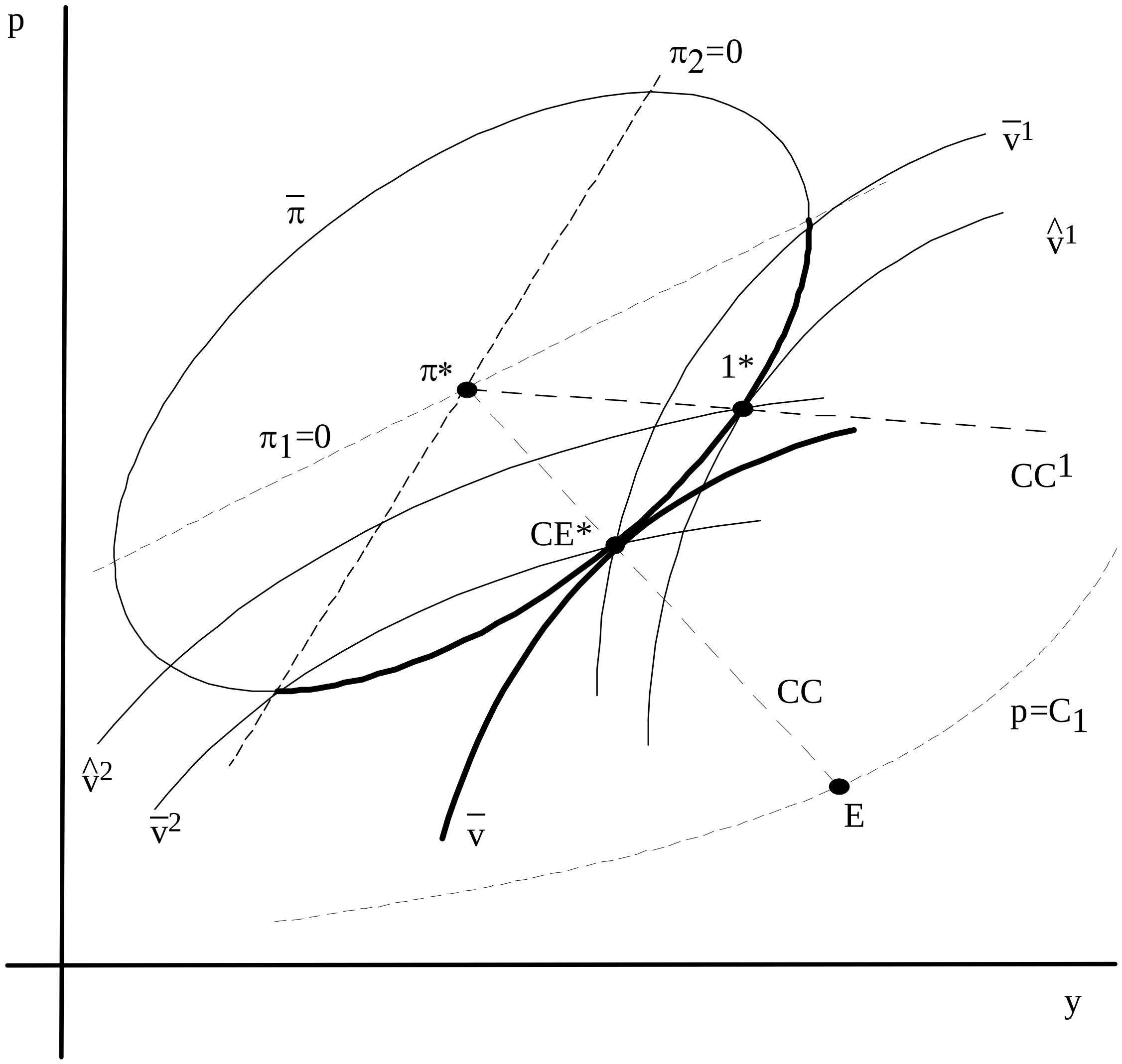

Figure 2: The choice of price per item
and qpi

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