Forecasting natural gas prices using highly flexible time-varying parameter models

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Abstract

The growing disintegration between the natural gas and oil prices, together with shale revolution and market financialization, lead to continued fundamental changes in the natural gas markets. To capture these structural changes, this paper considers a wide set of highly flexible time-varying parameter models to evaluate the out-of-sample forecasting performance of the natural gas spot prices across the US, European and Japanese markets. The results show that for both Japan and EU markets, the best forecasting performance is found when the model allows for drastic changes in the conditional mean and gradual changes in the conditional volatility. For the US market, however, no model performs systematically better than the simple autoregressive model. Full sample estimation results further confirm that allowing t-distributed error is important in modelling the natural gas prices, especially for EU markets.

JEL-codes: C32, E32, Q43

Keywords: Natural gas price; Structural breaks; Forecasting; Time-varying parameter; Markov switching; Stochastic volatility.

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1   Introduction

For many years, the price of natural gas is generally considered to be indexed to the international price of crude oil, often referred to oil indexation. Even today, oil indexation has a dominant role in the Asian natural gas market. However, numerous recent studies have revealed that oil indexation has started to lose its foundation and the two prices have been decoupled (Stern, 2014; Shi and Variam, 2017; Batten et al., 2017; Zhang and Ji, 2018; Zhang et al., 2018a). The instability in the crude oil-natural gas price linkage reflects dramatic changes in the global energy market. Technological development, such as the extraction of shale gas in the US, along with the increasing financializaion in natural gas markets, are seen as key factors that alter the world energy landscape (Creti et al., 2015; Caporin and Fontini, 2017; Zhang et al., 2018a; Wang et al., 2019).
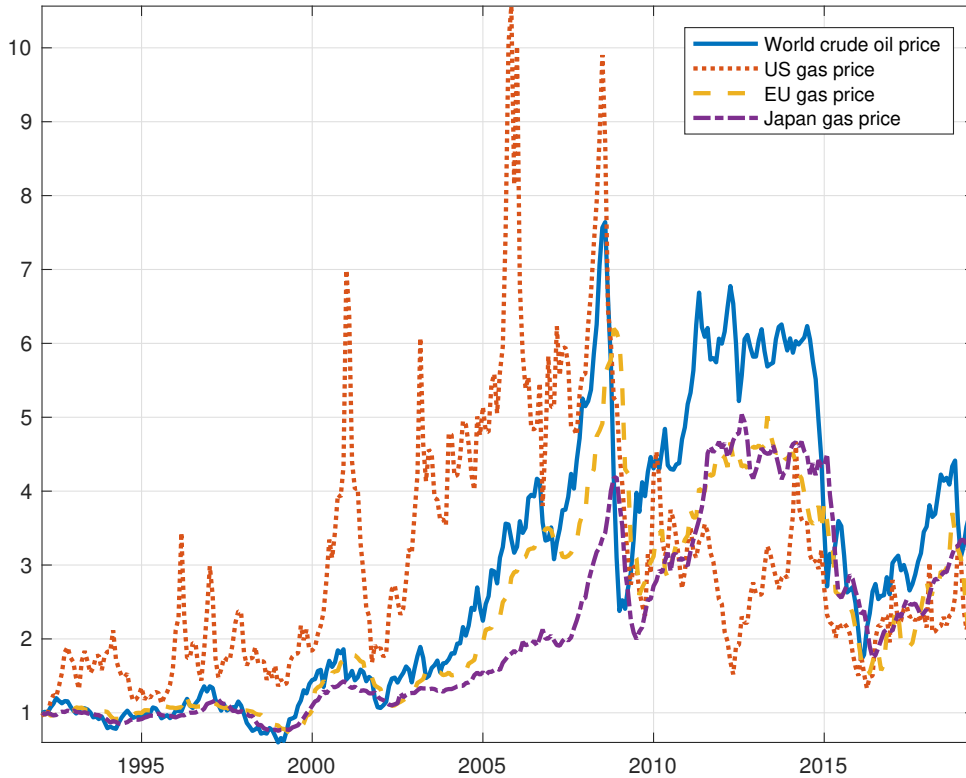

Figure 1: Crude oil and natural gas prices

Note: The figure plots the nominal spot price of crude oil and US, EU, Japan natural gas spot prices from 1992:1 to 2018:5, normalize to 1 in 1992:1. Source: World Bank Commodity Price Data (The Pink Sheet).

As clearly shown in Figure 1, the price of crude oil and natural gas have greatly de-

viated. This emergent phenomenon suggests that oil and gas would not share the same fundamentals and thus calls for more studies that explicitly treat the price of natural gas independently from the price of crude oil. The figure also shows that natural gas prices in three major markets, namely the US, EU and Japanese market, have widely diverged. Unlike crude oil, which has a global market, the natural gas markets are geographically segmented and the pricing mechanisms across different regions are significantly distinguished. More precisely, US gas prices are set through gas-on-gas competition at the Henry Hub, whereas gas prices in Asia are mostly based on oil indexation. The European market has been in the process of liberalization and thus gas prices in continental European are mixed-hub prices as well as oil indexation. From a modelling perspective, the strong regional characteristics of these markets also require a more sophisticated model that is capable of capturing the distinct behaviour of natural gas prices in these regions.

The aim of this paper is to evaluate the out-of-sample forecasting performance for natural gas spot prices across the US, European and Japanese natural gas market. In particular, we evaluate the predictive abilities to a set of highly flexible time series models that enable to capture the distinct characteristics of gas prices in these three regions. We contribute to the current literature by augmenting the standard autoregressive model with different features. Briefly, we consider three wide classes. The first class of models is the time-varying parameters with stochastic volatilities (TVPSV) models. The main feature of these models is to be able to capture structural instability in time series but allows the model parameters to be changing gradually over time (Koop and Potter, 2007; Wang et al., 2017; Wiggins and Etienne, 2017; Balcilar and Ozdemir, 2019). The second class of models is the Markov switching (MS) models. Different from the first class that assumes the model parameters to change gradually, parameters in the MS models are allowed to change drastically (Fong and See, 2002; Kosater and Mosler, 2006; Alizadeh et al., 2008; Vo, 2009; Sanzo, 2018). While these two classes of models are both able to capture the characteristic of structural instability which may exist in the natural gas markets, they restrict only one type of time-variation and model uncertainty. In order words, conventional model with its parameters changing over time only allows all parameters to vary either gradually or drastically. We therefore develop a class of hybrid time-varying parameters models that enable flexible time-varying characteristic in capturing different type of dynamics in model parameters. To be specific, these proposed hybrid models allow for a combination of gradual changes in mean and abrupt changes in variance, and vice versa. We then evaluate the forecasting performance of these models for the three natural gas markets from one to nine months ahead.

Our analysis is related to previous studies that attempt to accommodate fundamental

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changes in the natural gas markets. Apergis et al. (2015), for example, examine the cointergration between city-gate and residential prices and find structutural breaks relating to deregulation in the early 1990s. In line with these findings, Hou and Nguyen (2018) employ a Bayesian model comparision method and determine four regimes existing in the US natural gas market. Regime shifts in the US market are often associated with the periods of important regulatory reforms in the US natural gas market. Recently, the US shale gas revolution is detected as a major factor causing the structural changes (Wakamatsu and Aruga, 2013; Arora and Lieskovsky, 2014). Indeed, Wakamatsu and Aruga (2013) find that the US shale gas revolution not only has an impact on the domestic market but also changes on the relationship between the US and Japanese markets. The market linkage existing between the two markets disappeared since the shale gas revolution. Structural breaks and time variance manners are also found in the European market Bastianin et al. (2019).

To capture a more nuanced picture of the structural shifts, recent studies have also attempted to include different sources of time variation and model uncertainty to the natural gas markets. Accordingly, to accomodate structural breaks in both the coefficients and volatility, Hailemariam and Smyth (2019) apply a structural heterogeneous autoregressive VAR model. Along the same vein, Wiggins and Etienne (2017) utilize a VAR model that allows for both time-varying parameters and stochastic volatility, while Hou and Nguyen (2018) assume regime switching existing in some periods in the market. Despite this fact, no detailed analysis of the forecasting performances these similar models have been applied in the natural gas markets. Most of studies being conducted for natural gas price forecasting rely heavily on the artificial neural network approaches (MacAvoy and Moshkin, 2000; Buchananan et al., 2001; Nguyen and Nabney, 2010; Abrishami and Varahrami, 2011; Azadeh et al., 2012; Busse et al., 2012; Salehnia et al., 2013; Mishra and Smyth, 2016; Čeperić et al., 2017; Su et al., 2019). Therefore, the existence of structural instability, which is documented in more recent studies, is likely to be overlooked in these studies. Ignoring this important feature is often recognized as a leading cause of forecast failure (Carriero et al., 2019).

In general, our results show that time-varying parameter models perform better than static models in terms of out-of-sample forecasting. Moreover, allowing different time varying dynamics of the model parameters is influential on the natural gas price forecast. In particular, for both Japan and EU, the best forecasting performance is found when allowing for drastic changes in the conditional mean and gradual changes in the conditional volatility. For the US, however, no model performs systematically better than the simple autoregressive model. In addition, allowing t-distributed error is important in modelling the natural gas prices across the three markets, especially for EU markets.

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The rest of this paper is organized as follows. In section 2, a class of time series models with different time varying dynamics are presented. In section 3, a description of the data and prior is given. Section 4 presents the forecasting metrics and results. In section 5, the full sample estimation results of the best performing model are discussed and Section 6 concludes the paper.

2 Models

In this section we present various models with different types of time-varying parameters. Several studies in the forecasting literature have shown that simple univariate time series model typically outperforms a multivariate or far more complicated model in forecast perspective (Atkeson et al., 2001; Stock and Watson, 2007; Chan, 2013). Furthermore, assuming an autoregressive conditional mean has been widely used as a benchmark and shown to be a competent setup in forecasting literature (Clark and Ravazzolo, 2015). In this paper, we consider the following autoregressive model and its various extension

$$y_t = \beta_0 + \beta_1 y_{t-1} + \ldots + \beta_p y_{t-p} + \epsilon_t^y, \quad \epsilon_t^y \sim \mathcal{N}(0, \lambda_t \sigma^2), \tag{1}$$

where $\mathcal{N}(\cdot, \cdot)$ denotes the Gaussian distribution and λ_t is a scale-mixing parameter which follows a inverse-gamma distribution $\mathcal{IG}(\frac{\nu}{2}, \frac{\nu}{2})$. It can be shown that the marginal distribution of ϵ_t follows a t-distribution Geweke (1993). Note that by assuming $\lambda_t = \ldots = \lambda_T = 1$, it recovers the standard AR model with normal innovation. It is convenient to rewrite the AR as

$$y_t = \mathbf{x}_t \boldsymbol{\beta} + \epsilon_t^y, \quad \mathcal{N}(0, \lambda_t \sigma^2),$$

where $\mathbf{x}_t = (1, y_{t-1}, \ldots, y_{t-p})$ and $\boldsymbol{\beta} = (\beta_0, \beta_1, \ldots, \beta_p)'$. Many studies have shown that allowing model parameters to be changing over time can be beneficial for improving forecast accuracy. Two classes of models are often used to capture the parameters instability. The first class of models is the time-varying parameters with stochastic volatility models. The second set of models is the Markov switching models. Both classes of models are widely used in modelling macroeconomic and financial variables. Recently, these models have also been successfully applied in modelling energy variables for improving the in-sample fit and out-of-sample forecasting.

2.1 Time-Varying Parameter with Stochastic Volatility Models

In this section, we discuss the time-varying parameters with stochastic volatility (TVPSV) models, which can be specified as

$$y_t = \mathbf{x}_t \boldsymbol{\beta}_t + \epsilon_t^y, \quad \epsilon_t^y \sim \mathcal{N}(0, \lambda_t e^{h_t}), \tag{2}$$

$$\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1} + \boldsymbol{\epsilon}_t^\beta, \quad \boldsymbol{\epsilon}_t^\beta \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_\beta), \tag{3}$$

$$h_t = h_{t-1} + \epsilon_t^h, \quad \epsilon_t^h \sim \mathcal{N}(0, \sigma_h^2), \tag{4}$$

where $\boldsymbol{\Sigma}_\beta = \mathrm{diag}(\sigma_{\beta,0}^2, \ldots, \sigma_{\beta,p}^2)$, and the initial condition of the state parameters are assumed to be $\boldsymbol{\beta}_1 \sim \mathcal{N}(\boldsymbol{\beta}_0, \mathbf{V}_\beta)$ and $h_1 \sim \mathcal{N}(0, V_h)$.

It is well-konwn in the literature that the TVPSV model implicitly assumes that the change of the model parameters in subsequent period of time is restricted in a small magnitude. This can be seen from the equations (3) and (4) in which the variances of the innovation, $\sigma_{\beta,0}^2, \ldots, \sigma_{\beta,p}^2$ and σ_h^2 are often assumed to be small. In other words, the TVPSV model can be interpreted as a structural break model where breaks occur in each period of time with small magnitudes.

2.2 Markov Switching Models

We discuss a class of Markov Switching models (MS) in this section. A general framework of Markov Switching models we consider in this paper can expressed as

$$y_t = \mathbf{x}_t \boldsymbol{\beta}_{s_t} + \epsilon_t^y, \quad \epsilon_t^y \sim \mathcal{N}(0, \lambda_t \sigma_{z_t}^2), \tag{5}$$

$$\Pr(s_t = j | s_{t-1} = i) = p_{ij}^s, \tag{6}$$

$$\Pr(z_t = l | z_{t-1} = k) = p_{kl}^z, \tag{7}$$

where $i, j = 1, \ldots, M_s$ and $k, l = 1, \ldots, M_z$. The M_s and M_z are the numbers of regimes in the AR coefficients and volatilities respectively, which are determined prior to estimation. The regime indicator s_t and z_t are assumed to follow independent Markov processes that govern the dynamics of the conditional mean coefficient and the variance at time t. The main difference between the TVPSV and the MS models is that only a small number of structural breaks are assumed to occur in the sample and the magnitude of change in the model parameters when a break occurs is not restricted.

2.3 Hybrid Models

Both TVPSV and MS models have different implicit assumptions on the number of break and the magnitude of change in the model parameters when a break occurs. Researcher

often select one of these models in their empirical studies based on the data feature or theoretical consideration. In this section, we propose two hybrid models in which the evolutions of the AR coefficients and volatilities possess different types of time-varying dynamics. To be specific, the first hybrid model we consider is given by

$$y_t = \mathbf{x}_t \boldsymbol{\beta}_t + \epsilon_t^y, \quad \epsilon_t^y \sim \mathcal{N}(0, \lambda_t \sigma_{z_t}^2), \tag{8}$$

$$\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1} + \boldsymbol{\epsilon}_t^\beta, \quad \boldsymbol{\epsilon}_t^\beta \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_\beta), \tag{9}$$

$$\Pr(z_t = j | z_{t-1} = i) = p_{ij}^z, \quad i, j = 1, \ldots, M_z. \tag{10}$$

We refer this model as ARt-TVPMS. The second hybrid model proposed in this paper is

$$y_t = \mathbf{x}_t \boldsymbol{\beta}_{s_t} + \epsilon_t^y, \quad \epsilon_t^y \sim \mathcal{N}(0, \lambda_t e^{h_t}), \tag{11}$$

$$h_t = h_{t-1} + \epsilon_t^h, \quad \epsilon_t^h \sim \mathcal{N}(0, \sigma_h^2), \tag{12}$$

$$\Pr(s_t = j | s_{t-1} = i) = p_{ij}^s, \quad i, j = 1, \ldots, M_s. \tag{13}$$

We refer this model as ARt-MSSV. It can be seen that the time-variation in the AR coefficients and the volatilities for the ARt-TVPMS and the ARt-MSSV have different implicit assumptions. The main purpose of this paper is to investigate if allowing various types of time-varying dynamics in the model parameters improve the natural gas forecasting performance.

All models are estimated using Markov chain Monte Carlo method (MCMC). The posterior sampler for models with constant parameters is standard. For models with time-varying parameters with stochastic volatility, we sample the log volatilities using the auxiliary mixture sampler of Kim et al. (1998). The time-varying AR coefficients are sampled using the precision sampler developed in Chan and Jeliazkov (2009). For models with parameters driven by Markov switching processes, the regime indicators are sampled by applying the method proposed by Chib (1996). Lastly, the degree of freedom parameter ν for t-distributed models can be sampled by an independence-chain Metropolis-Hastings step proposed by Chan et al. (2014). The estimation results in our empirical studies are all based on 10000 posterior samples obtained after a burn-in period of 5000.

To have a complete investigation of the contribution of various time-varying dynamics on natural gas forecast, we consider a variety of the TVPSV and MS models. In addition, we also consider models with Gaussian distributed error term to shed light on whether allowing heavy-tailed distributed error term produces forecasting gains. We summarize all the models in our out-of-sample forecasting exercise in Table 1.

Table 1: A list of models

Benchmark model	
AR	ARt with $\lambda_1 = \ldots = \lambda_T = 1;$
Time-Varying Parameter with Stochastic Volatility Models	
ARt	defined in equation (1).
AR-SV	ARt-SV with $\lambda_1 = \ldots = \lambda_T = 1.$
ARt-SV	ARt-TVPSV with $\boldsymbol{\beta}_1 = \ldots = \boldsymbol{\beta}_T.$
AR-TVP	ARt-TVP with $\lambda_1 = \ldots = \lambda_T = 1.$
ARt-TVP	ARt-TVPSV with $h_1 = \ldots = h_T.$
AR-TVPSV	ARt-TVPSV with $\lambda_1 = \ldots = \lambda_T = 1.$
ARt-TVPSV	defined in equations (2) - (4).
Markov Switching Models	
AR-MS	ARt-MS with $\lambda_1 = \ldots = \lambda_T = 1.$
ARt-MS	ARt-DMS with $s_t = z_t.$
AR-DMS	ARt-DMS with $\lambda_1 = \ldots = \lambda_T = 1.$
ARt-DMS	defined in equation (5) - (7).
AR-MSC	ARt-MSC with $\lambda_1 = \ldots = \lambda_T = 1.$
ARt-MSC	ARt-DMS with $s_1 = \ldots = s_T.$
AR-CMS	ARt-CMS with $\lambda_1 = \ldots = \lambda_T = 1.$
ARt-CMS	ARt-DMS with $z_1 = \ldots = z_T.$
Hybrid Models	
AR-TVPMS	ARt-TVPMS with $\lambda_1 = \ldots = \lambda_T = 1.$
ARt-TVPMS	defined in equations (8) - (10).
AR-MSSV	ARt-MSSV with $\lambda_1 = \ldots = \lambda_T = 1.$
ARt-MSSV	defined in equations (11) - (13).

3 Data and Prior

3.1 Data

Natural gas prices used in this paper were collected from the World Bank commodity price data, or the Pink Sheet, as in Zhang et al. (2018b) and Zhang and Ji (2018). For the US, the Pink Sheet reports the monthly data from 1976:06 but for the EU and Japan market the data is in yearly frequency until 1992:01, as shown in Figure 2. For this reason, the sample period considered in this paper starts January 1992 and ends in May 2019. According to the Pink Sheet, the price of natural gas is defined differently for

each market. The US gas price is the spot prices trading in the Henry Hub Louisiana. The European gas price is the average import border price and a spot price component (excluding UK between June 2000 and March 2010). For Japan, the Pink Sheet reports the imported LNG prices with the most recent two-month averages.
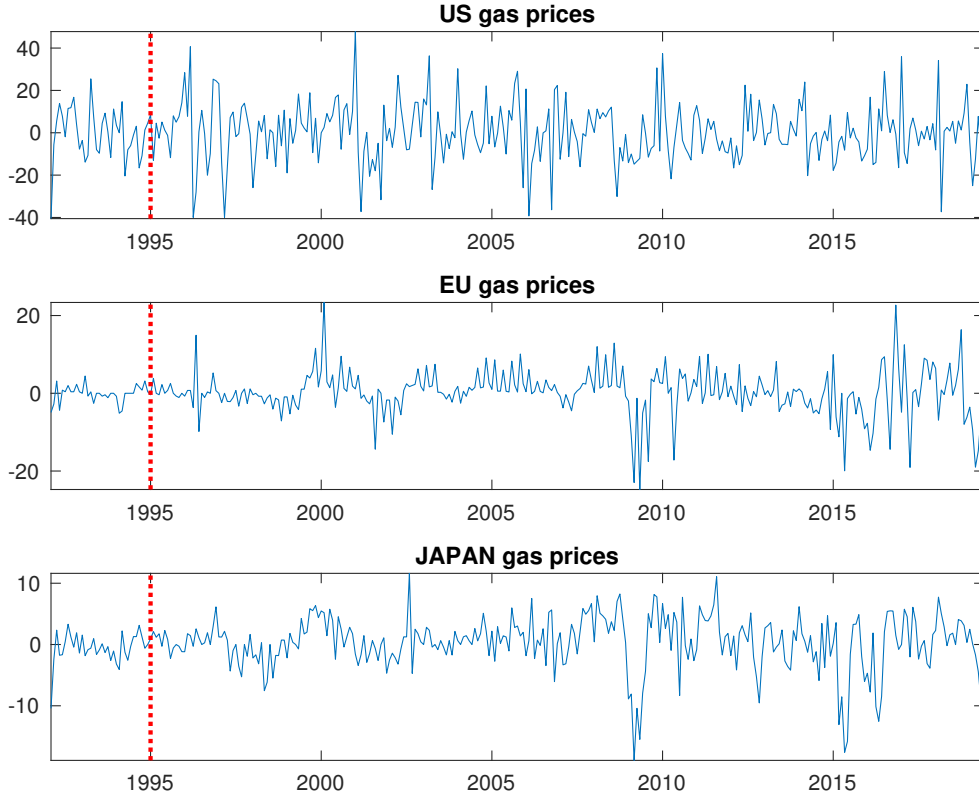

Figure 2: Natural gas prices

Note: The figure plots the monthly percentage nominal gas prices considered in this study. The dashed vertical (red) line indicates the starting date of our forecasting evaluation period, from 1995:01 to 2019:05. The full sample spans from 1992:01 to 2019:05.

3.2 Prior

To complete the model specification, we give the details of the prior on the model parameters in this section. We assume same priors for parameters common across models. To be specific, the priors for the AR coefficients and the variance for models with constant parameters are given by

$$\boldsymbol{\beta} \sim \mathcal{N}(\boldsymbol{\beta}_0, \mathbf{V}_\beta), \quad \sigma^2 \sim \mathcal{IG}(\nu_y, S_y).$$

The variances of the disturbance terms of the time-varying parameters and the log volatilies are assumed to be independently distributed as

$$\sigma_h^2 \sim \mathcal{IG}(\nu_h, S_h), \quad \sigma_{\beta,i}^2 \sim \mathcal{IG}(\nu_i, S_i), \text{ for } i = 0, 1, \ldots, p.$$

For the regime transition probability, we assume

$$(p_{i1}^s, \ldots, p_{iM_s}^s) \sim \mathcal{D}(\alpha_{i1}^s, \ldots, \alpha_{iM_s}^s), \quad \text{for } i = 1, \ldots, M_s,$$
$$(p_{j1}^z, \ldots, p_{jM_z}^z) \sim \mathcal{D}(\alpha_{j1}^z, \ldots, \alpha_{jM_z}^z), \quad \text{for } j = 1, \ldots, M_z,$$

where $\mathcal{D}(\alpha_1, \ldots, \alpha_M)$ denotes the Dirichlet distribution with concentration parameters $(\alpha_1, \ldots, \alpha_M)$. Lastly, we assume the following independent prior for model parameters of the MS model:

$$\boldsymbol{\beta}_i \sim \mathcal{N}(\boldsymbol{\beta}_{MS}, \mathbf{V}_{MS}), \quad \text{for } i = 1, \ldots, M_s,$$
$$\sigma_j^2 \sim \mathcal{IG}(\nu_{MS}, S_{MS}), \quad \text{for } j = 1, \ldots, M_z.$$

In particular, we set the hyper-parameters to be $\boldsymbol{\beta}_0 = \boldsymbol{\beta}_{MS} = \mathbf{0}$ and $\mathbf{V}_\beta = \mathbf{V}_{MS} = \mathbf{I}_{p+1}$. For the degree of freedom parameters, we set $\nu_y = \nu_h = 10$ and $\nu_i = 10$ for $i = 0, 1, \ldots, p$. The scale parameters are set to result in the prior means of σ^2, σ_h^2 and $\sigma_{\beta,i}^2$ to be 1, 0.1^2 and 0.01^2 respectively. For the concentration parameters, we set

$$(\alpha_{i1}^s, \ldots, \alpha_{iM_s}^s)' = \mathbf{1}_{M_s} + 100\mathbf{e}_i^{M_s}, \quad \text{for } i = 1, \ldots, M_s,$$
$$(\alpha_{j1}^z, \ldots, \alpha_{jM_z}^z)' = \mathbf{1}_{M_z} + 100\mathbf{e}_j^{M_z}, \quad \text{for } j = 1, \ldots, M_z,$$

where $\mathbf{1}_M$ denotes a $M \times 1$ vector with its entries equal to 1, and $\mathbf{e}_l^M$ denote the l-th column of the identity matrix with dimension M. Lastly, the degree of freedom parameters for the t-distributed models ν is assumed to follow a exponential prior with mean 5.

4 Forecasting

This section performs a recursive out-of-sample forecasting exercise to evaluate the forecasting performance of the models listed in Table 1. We evaluate the iterated m-step-ahead forecasts of each model with $m = 1, 3, 6, 9$. The full sample used for each country covers time period from 1992M1 to 2019M5 and our forecasting evaluation period starts from 1995M1 to 2019M5.

4.1 Forecasting Metrics

Let y_{t+m}^o be the observed value of y_{t+m}. We compute the m-step-ahead predictive posterior median $\hat{y}_{t+m}$ as the point forecast for a given model. The mean absolute forecast

error (MAFE) is used to measure the accuracy of the point forecast of a model, which is defined as

$$\text{MAFE} = \frac{1}{T - m - t_0 + 1} \sum_{t=t_0}^{T-m} \left| y_{t+m}^o - \hat{y}_{t+m} \right|.$$

For evaluating the performance of density forecasts, we use the average log-predictive likelihoods (ALPL) which is defined as

$$\text{ALPL} = \frac{1}{T - m - t_0 + 1} \sum_{t=t_0}^{T-m} \log p_{t+m}(y_{t+m} = y_{t+m}^o | y_1^o, \ldots, y_t^o),$$

where p_{t+m} denotes m-step-ahead predictive density function. The log-predictive likelihood is often used to compare forecast performance of models in the Bayesian framework since there is a close connection between the predictive likelihood and the marginal likelihood. More discussions about the log-predictive likelihoods can be found in Geweke and Amisano (2011).

To facilitate comparison, we report the relative scores to the AR benchmark model. To be specific, we report the ratios of MAFEs of a given model to the benchmark. Hence values less than unity indicate better point forecast performance than the benchmark. For density forecasts, we report the difference of ALPLs of a given models to the benchmark. Thus positive values indicate better density forecast performance than the benchmark.

4.2 Forecasting Results

The forecasting results are presented in Table 2 - Table 4. The results for the AR benchmark is the raw scores. The relative scores are reported for other models.

Our results show that no models can consistently outperforms the AR benchmark in term of point and density forecasts for the US natural gas price forecasting. Although models allowing t-distributed disturbances and stochastic volatility tend to slightly improve the density forecast accuracy over their Gaussian counterparts, the improvements do not seem to be significant. For example, the largest difference of the ALPL among all forecasting horizon between ARt-SV and the AR benchmark is only 0.08. It is interesting that models with time-varying parameters produce little gain in forecast accuracy. In general the MS models perform relatively poor compared with TVPSV models, which suggests that the time-varying dynamic of the model parameters in modelling US gas price market is likely to be changing gradually instead of drastically.

For forecasting the natural gas prices of EU, model allowing its parameters to be changing over time is likely to generate more accurate forecasts, especially for density forecast. Moreover, the forecasting performance can be further improved by assuming an

t-distributed innovations. It is also evident from our results that the types of time-varying dynamics for modelling the conditional mean and variance are crucial for EU natural gas prices forecast. For example, the AR-SV consistently produces better density forecasts than the AR-CMS among all forecast horizons, which indicates that it is beneficial in improving forecasting performance by assuming the variations in the variances to change gradually instead of drastically. However, the overall best model is the ARt-MSSV, highlighting the empirical relevance of allowing different time-varying dynamics to govern the conditional means and variances.

In the perspective of forecasting the Japan natural gas prices, models with different time-varying dynamics tend to compare more favourably to the AR benchmark. In particular, the ARt-SV and ARt-MSSV perform relatively better than other models in density forecast. For the point forecast, no model consistently produce competent performance. Allowing heavy-tailed distributed disturbances have been again shown to be a solid practical choice. This can be seen that models with heavy-tailed disturbances often perform better than their counterparts with Guassian innovations. It is interesting that the time-varying dynamics governing the AR coefficient does not seem to play an important role in improving forecasting accuracy for Japan natural gas prices.

Table 2: Forecast performance relative to the standard AR for US.

	relative MAFE				relative ALPL			
	m=1	m=3	m=6	m=9	m=1	m=3	m=6	m=9
AR	15.31	26.64	33.80	35.96	−4.46	−4.98	−5.25	−5.35
AR-SV	1.00	0.99	0.99	0.99	0.03	0.02	0.04	0.07
ARt	1.00	0.99	0.97	0.96	−0.06	−0.03	−0.01	0.02
ARt-SV	1.00	0.99	0.98	0.98	0.03	0.02	0.05	0.08
AR-TVP	1.01	1.04	1.07	1.10	0.00	−0.02	−0.04	−0.06
ARt-TVP	1.01	1.06	1.09	1.12	−0.06	−0.07	−0.10	−0.11
AR-TVPSV	1.01	1.03	1.06	1.09	0.02	−0.01	−0.01	0.00
ARt-TVPSV	1.01	1.03	1.06	1.09	0.02	0.00	0.00	0.00
AR-MS	1.04	1.01	1.00	1.00	−0.03	−0.02	0.01	0.01
ARt-MS	1.01	1.05	1.20	1.61	−0.07	−0.05	−0.05	−0.02
AR-DMS	1.05	1.02	1.04	1.07	−0.01	−0.02	0.00	0.01
ARt-DMS	1.05	1.10	1.31	1.87	−0.09	−0.09	−0.12	−0.13
AR-MSC	1.05	1.02	1.02	1.03	−0.01	−0.02	−0.03	−0.02
ARt-MSC	1.08	1.05	1.03	1.05	−0.05	−0.06	−0.11	−0.12
AR-CMS	1.00	1.00	1.00	1.00	−0.01	−0.01	0.01	0.00
ARt-CMS	1.00	0.99	0.98	0.97	−0.07	−0.04	−0.01	0.01
AR-TVPMS	1.01	1.04	1.07	1.10	−0.01	−0.02	−0.03	−0.06
ARt-TVPMS	1.01	1.06	1.09	1.12	−0.07	−0.08	−0.10	−0.12
AR-MSSV	1.00	0.98	0.99	1.00	0.02	0.02	0.04	0.06
ARt-MSSV	1.01	0.99	1.00	1.01	0.03	0.01	0.01	0.02

Note: Results for AR benchmark are actual values and the results for the remaining models are reported relative to the benchmark. Values in gray cells indicate the best relative MAFE and ALPL.

Table 3: Forecast performance relative to the standard AR for EU.

	relative MAFE				relative ALPL			
	m=1	m=3	m=6	m=9	m=1	m=3	m=6	m=9
AR	5.43	11.34	19.02	23.97	−3.61	−4.18	−4.90	−5.57
AR-SV	0.98	1.01	1.02	1.01	0.18	0.08	0.14	0.38
ARt	0.97	1.03	1.03	1.04	0.20	−0.11	0.09	0.55
ARt-SV	0.97	1.03	1.04	1.03	0.30	−0.01	0.15	0.59
AR-TVP	1.04	1.04	1.08	1.13	−0.01	−0.01	−0.04	−0.02
ARt-TVP	0.99	1.05	1.10	1.13	0.19	−0.08	0.11	0.52
AR-TVPSV	1.01	1.02	1.06	1.11	0.17	0.05	0.10	0.31
ARt-TVPSV	0.99	1.04	1.08	1.11	0.28	−0.01	0.16	0.55
AR-MS	1.05	1.11	1.40	3.06	0.09	0.06	0.19	0.55
ARt-MS	1.00	1.04	1.05	1.05	0.20	−0.10	0.10	0.54
AR-DMS	1.09	1.17	1.50	3.25	0.11	0.03	0.15	0.50
ARt-DMS	1.01	1.05	1.06	1.06	0.20	−0.09	0.11	0.57
AR-MSC	1.20	1.35	2.04	5.20	0.00	−0.07	0.11	0.50
ARt-MSC	0.99	1.03	1.03	1.02	0.21	−0.08	0.12	0.57
AR-CMS	0.99	1.00	1.02	1.03	0.11	0.01	−0.08	−0.01
ARt-CMS	0.97	1.03	1.04	1.04	0.21	−0.10	0.09	0.54
AR-TVPMS	1.02	1.03	1.08	1.14	0.11	0.00	0.03	0.11
ARt-TVPMS	0.99	1.05	1.09	1.13	0.20	−0.08	0.10	0.52
AR-MSSV	0.99	1.01	1.00	1.00	0.17	0.09	0.23	0.59
ARt-MSSV	0.98	1.03	1.03	1.02	0.29	0.01	0.18	0.61

Note: Results for AR benchmark are actual values and the results for the remaining models are reported relative to the benchmark. Values in gray cells indicate the best relative MAFE and ALPL.

Table 4: Forecast performance relative to the standard AR for Japan.

	relative MAFE				relative ALPL			
	m=1	m=3	m=6	m=9	m=1	m=3	m=6	m=9
AR	3.80	8.80	15.17	19.44	−3.11	−3.93	−4.81	−5.26
AR-SV	1.00	0.99	0.99	0.98	0.11	0.14	0.39	0.56
ARt	1.00	1.01	1.00	1.00	0.02	0.03	0.20	0.27
ARt-SV	1.00	0.99	0.99	0.98	0.11	0.13	0.38	0.55
AR-TVP	1.04	1.08	1.14	1.16	0.00	0.01	0.03	−0.01
ARt-TVP	1.04	1.09	1.16	1.20	0.02	−0.02	0.12	0.22
AR-TVPSV	1.02	1.05	1.09	1.10	0.10	0.09	0.30	0.42
ARt-TVPSV	1.03	1.05	1.09	1.10	0.10	0.08	0.29	0.42
AR-MS	1.01	1.03	1.09	1.19	−0.01	0.08	0.30	0.37
ARt-MS	1.01	1.02	1.03	1.03	0.03	0.07	0.30	0.39
AR-DMS	1.04	1.06	1.11	1.18	−0.01	0.00	0.19	0.30
ARt-DMS	1.03	1.06	1.10	1.14	0.01	0.00	0.21	0.34
AR-MSC	1.03	1.02	1.02	1.00	0.00	0.03	0.25	0.44
ARt-MSC	1.02	1.00	1.00	0.99	0.02	0.05	0.29	0.45
AR-CMS	1.00	1.00	1.00	1.00	0.00	0.01	−0.07	0.05
ARt-CMS	1.00	1.01	1.00	1.01	0.01	0.02	0.23	0.30
AR-TVPMS	1.04	1.08	1.14	1.16	0.00	0.01	0.05	−0.02
ARt-TVPMS	1.04	1.09	1.16	1.19	0.01	−0.01	0.10	0.14
AR-MSSV	1.00	1.00	0.99	0.97	0.11	0.13	0.37	0.53
ARt-MSSV	0.99	0.99	0.98	0.96	0.10	0.14	0.40	0.58

Note: Results for AR benchmark are actual values and the results for the remaining models are reported relative to the benchmark. Values in gray cells indicate the best relative MAFE and ALPL.

5 Full Sample Estimation Results

As shown in the previous section, the ARt-MSSV has a relatively good forecasting performance compared with the other models. We then estimate the ARt-MSSV model on the full sample of different natural gas series in US, EU and Japan. Figure 3 plots the estimated posterior distributions of ν. It is evident that allowing a t-distributed error seems to be important in modelling the natural gas prices for the US and Japan. While the posterior distribution of ν for the EU market data is centered between 1 and 3 the cor-

responding for the US and Japan are wider, concentrated between 5 and 20 or more. The results indicate specifying a heavy-tailed distribution is favored by the data, especially for EU markets.
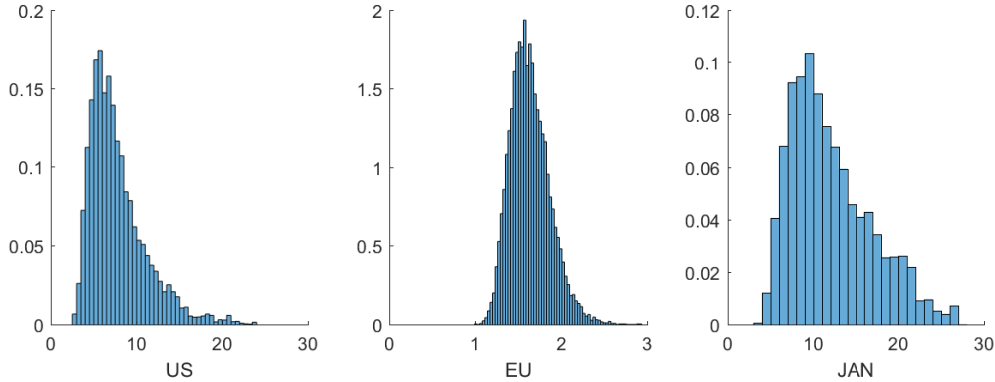

Figure 3: Estimated posterior distributions of ν.

Figure 4 plots the estimated posterior means and the 90% credible intervals for the standard error, $e^{\frac{h_t}{2}}$. For the U.S. market, the variance of natural gas price has been steadily increasing since the Decontrol Act 1989 and fluctuates greatly for the first decade of this century. Interestingly, the variance becomes much more stable after 2010. A possible explanation is that the shale gas has gradually become a major source of natural gas supply in the U.S. and this makes the U.S. market much less susceptible to any supply shocks to the market.[1] For the EU market, the variance of natural gas price stays relatively low and stable until 2014. After the Russo-Ukrainian War broke out in 2014, the variance has been increasing steadily, which is likely the combined result of both EU's reliance on Russian natural gas and the increasing tension between the NATO member states and Russia. For the Japan market, the variance also stays relatively low and stable but increases drastically around 2011 and 2016. These drastic increases in volatility are likely the results of the 2011 Fukushima accident and 2016 Fukushima earthquake. After the 2011 Fukushima accident and subsequent nuclear shutdown, Japan's LNG demand quickly increased by 30% despite a near doubling in price. The 2016 Fukushima earthquake, although not causing any major damage to the Fukushima nuclear power station, likely pushed back the restarts of the reactors. Overall, the U.S. market is evidently much more volatile compared with the EU and Japan markets.

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[1]In 2000 shale gas provided only 1% of the U.S. natural gas production and by 2010 it was over 20%.
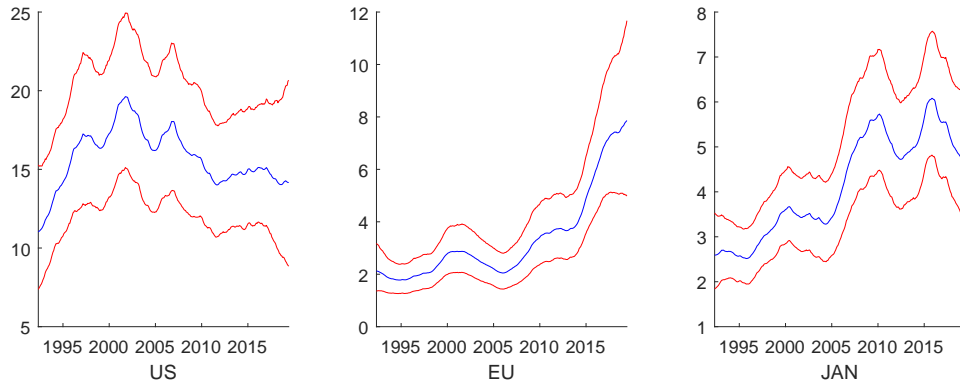
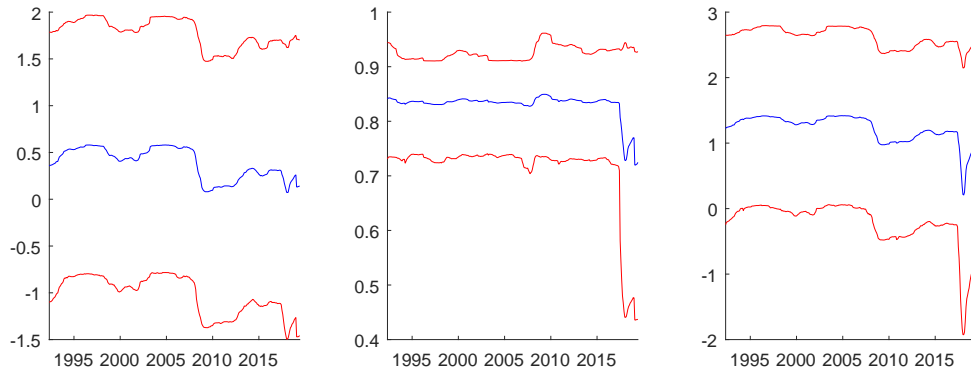Figure 4: Estimated posterior distributions of $e^{\frac{h_t}{2}}$.

Figure 5: Estimated posterior means and 90% credible intervals of the AR intercept (left), largest AR root (middle) and the sum of AR coefficients (right) for the US.
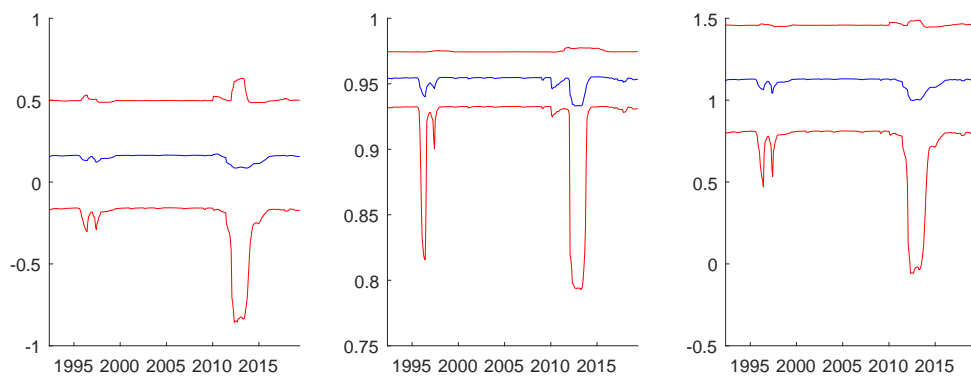

Figure 6: Estimated posterior means and 90% credible intervals of the AR intercept (left), largest AR root (middle) and the sum of AR coefficients (right) for the EU.
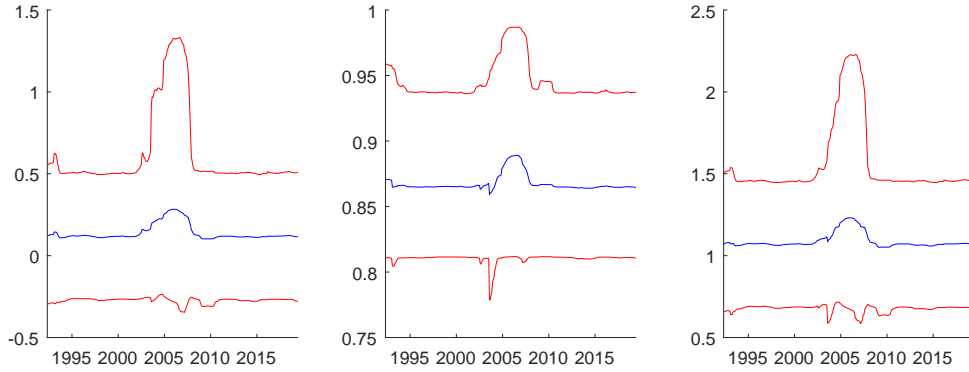
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Figure 7: Estimated posterior means and 90% credible intervals of the AR intercept (left), largest AR root (middle) and the sum of AR coefficients (right) for the JAN.

Figure 5 - Figure 7 display the estimated posterior means and 90% credible intervals for (a) the AR intercept, (b) the largest AR root (LAR), and (c) the sum of the AR coefficients. The two latter serve as our measures of persistence. We see that the intercepts for all three markets stay relatively low and constant with a wide credible interval. Both persistence measures for the U.S. Market stay rather constant, but drop significantly after 2017. Before the drop, the sum of AR coefficients is about 1.2, and the LAR is around 0.84. For the EU market, both persistence measures stay rather constant with two dips around 1996-1997 and 2012-2014. The sum of AR coefficients is about 1.1, and the LAR is a little above 0.95. For the Japan market, both persistence measures stay rather constant with one small bump between 2004 and 2008. The sum of AR coefficients is about 1.1, and the LAR is around 0.86.

6   Conclusion

In this paper, we evaluate the out-of-sample forecasting performance of a wide class of time series models for natural gas prices across the US, European and Japanese natural gas markets. To fully capture the possible time-varying dynamics in mean and variance, in addition to the traditional TVPSV and MS models, we propose a class of hybrid TVPSV-MS models that allow for a combination of gradual changes in mean and abrupt changes in variance, and vice versa. We also allow for heavy-tailed distributed disturbance in our models, for its ability in capturing the occurrence of extreme prices compared to the normal distribution (Chang, 2012).

The results shed light on the fundamentals of the three natural gas markets. For both Japan and EU, time-varying parameter models perform better than static models. In particular, the best forecasting performance is found when allowing for drastic changes in the conditional mean and gradual changes in the conditional volatility. For the US,

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however, no model performs systematically better than the simple AR model. In addition, allowing t-distributed error is important in modelling the natural gas prices, especially for EU markets. The results further suggest that the EU and Japan markets are similar in terms of the fundamental time-varying dynamics, even though the European market has been in the process of disintegrating from the oil market. The US market, where gas prices are set through gas-on-gas competition at the Henry Hub and not oil-indexed, exhibits completely different time-varying dynamics than the EU and Japan markets.

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