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Abstract

This paper examines the contracting relationship between a manufacturer and a retailer when the retailer has ex ante private information, and is subject to limited liability. The contract takes place over two periods. In the first period, the retailer can make a report of private information, or take an action, either of which influences the manufacturer's beliefs about the distribution of demand states for a final good in the second period. In the second period, the retailer sells the manufacturers intermediate good into a final output market according to a variable fee schedule. The interaction of the limited liability constraints with incentive compatibility in the second stage gives rise to an expected surplus to the retailer, which the manufacturer can extract with a franchise fee. The franchise fee can also be used as a screening device or a means of eliciting the efficient first stage action from the retailer.

Key words: Franchise Fee, Limited Liability, Vertical Restraints

JEL Classification: D82, D86, L42

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1 Introduction

The interaction of vertically related firms is typically complicated by the presence of externalities and informational asymmetries. For example, the contract written between a manufacturer and retailer may need to account for the possibility that the retailer may not internalise the impact of its pricing decision on the manufacturer, or deal with opportunism that can arise when the retailer has an informational advantage over the manufacturer. It is well known that the inefficiencies that arise from externalities can be resolved by the use of contractual provisions, known as vertical restraints. Moreover, the performance of those vertical restraints under information asymmetries has been studied in a number of recent examinations using Principal-Agent models, where contracts involve a system of incentives and penalties.[1] However, there are two feature of franchise arrangements that have so far received little attention. First, retailers may often be bound by limited liability constraints. Hence, the manufacturer may be restricted in its enforcement of harsh penalties. Second, the retailer may be able to take an action that favourably influences the level of demand it ultimately faces, or may have private information about demand that is imprecise at the time of signing the contract.

This paper explores a franchise relationship under uncertainty when the retailer (i) has limited liability to comply with the terms of the contract once it realises its precise level of demand, and (ii) only learns private information relating to the precise demand state subsequently to signing the contract.[2] At the time of signing the contract, the retailer and manufacturer share a common belief about the probability distribution of demand states, and both know that only the retailer will eventually learn the precise demand state. This informational asymmetry confers a strategic advantage to the retailer, which the manufacturer

[1]See, for example, Rey and Tirole (1986), Blair and Lewis (1994) and Martimort and Piccolo (2007).

[2]There are a variety of different vertical relationships that can be classed as franchises. Broadly speaking, franchises can be separated into three categories: manufacturing, product and business franchises. Manufacturing franchises typically involve a technology licensing arrangement, where manufacturing firm holds intellectual property rights to a technology that it may license another manufacturer to use. A product franchise may be a distributorship, or a licensed dealer or reseller of a manufacturer's product. In a business franchise, the upstream firm typically issues the right for a downstream firm to use its business plan to retail to consumers.

can resolve by incorporating self-selection, or incentive constraints into its contracts. These constraints ensure that the retailer has an incentive to reveal its true private information to the manufacturer. The resulting contracts are administered through a fee schedule: the retailer selects a quantity from the schedule to sell and pays a corresponding fee. However, in this paper, the interaction of the incentive constraints on the manufacturer's contracts, and the impact of the retailer's limited liability on the capacity for those contracts to be implemented give rise to an expected surplus to the retailer that stems from its informational advantage. This paper demonstrates how the manufacturer can extract this expected surplus with an additional vertical restraint in its contracts: an ex ante franchise fee, that together with a fee schedule, forms a contract that is referred to in this paper as a *franchise contract*.

There are a number of reasons to expect that retailers are bound by limited liability. First, limited liability may be the result of legal implications that are embedded in local competition policy legislation.[3] It may also follow from consideration of the retailer's operating decision, or the decision to shutdown. If a franchisee finds that its average operating revenue from executing the contract is less than the amount to be paid under the terms of the contract, it may opt out. Limited liability only binds when the manufacturer would like to incorporate penalties into the contract – that is, when there is uncertainty.

Uncertainty arises here over the precise level of demand in the final market. Specifically, the precise state of final demand for the industry's output is uncertain to both the manufacturer and the retailer at the time of signing the contract, but the retailer privately observes the precise demand state in the process of serving its market. Furthermore, this paper incorporates a realistic aspect of franchise arrangements: the retailer may either have some coarse private information before signing the contract (for example, a survey of the composition of consumers in the market), or can take an action that influences the likelihood of the demand state faced (for example, a targeted marketing campaign).

[3] In Australia there are a number of provisions that amount to limited liability in the Trade Practices Act (1974). For instance, Australian franchise agreements must meet a Franchise Code of Conduct, which precludes the use of harsh penalties. Another reason for limited liability that is perhaps more applicable to business franchises is the size of the franchisees assets: typically such firms are small operators and do not have large assets.

The results will demonstrate how the franchise fee component of the conjectured franchise contract can be used to extract the retailer's perceived expected gross surplus from entering the contract. Moreover, the model explores the contracting possibilities when the retailer has the ability to engage in opportunistic behaviour before the precise demand state that it will face has been realised. It is shown how the manufacturer can use the franchise fee as a device to elicit the efficient action in the case of moral hazard, or as a screening instrument in the case of ex ante hidden knowledge. In this latter case, the results also offer an interesting insight into the value to the manufacturer of obtaining the retailer's information. The manufacturer can weigh up the expected benefit of paying the retailer for its information against the expected efficiency cost of implementing more finely-tailored contracts.

The next section reviews some of the related literature in vertical contracting. Then, to establish the intuition for the existence of the expected surplus that arises from the interaction of the retailer's limited liability constraints and incentive compatibility, a simple discrete demand state illustration is examined. Two hypotheses are drawn from this simple setting that are scrutinised in the general model of the following section. The general model then allows the issue of ex ante private information to be rigorously explored. The last section concludes.

2 Relation to the Literature

It is well known following Spengler (1950) that when vertically related firms, each with some degree of market power, make independent linear pricing decisions, a vertical externality emerges: each firm fails to take into account the effect of its own price on the other's profit. The result is an inefficiency in the form of a loss of producer surplus relative to an integrated structure. The price in the market for the industry's output embodies two mark-ups over marginal cost: double marginalisation. Any arrangement that can eliminate one of the mark-ups necessarily improves allocative efficiency. Contractual arrangements known as vertical restraints are one way for vertically related firms to restore allocative efficiency.

Contractual provisions allow for the manufacturer to correct the vertical externality. Tirole (2000) details how vertical restraints broadly include the use of resale price maintenance, quantity fixing, and use of franchise fee arrangements. These restraints respectively allow the manufacturer to control the retailer's price or quantity directly, or decentralise the pricing decision in a way that implements the integrated profit solution which the manufacturer and retailer can bargain over with the use of a fixed fee transfer. With the extreme bargaining power assumption of the Principal-Agent framework, the manufacturer will appropriate all the rent under complete information with a fixed fee.[4] However, when the relationship is exposed to uncertainty, there is scope for the retailer to behave opportunistically. Subsequent analysis must appeal to the theory of incentives.

Matthewson and Winter (1985) were among the first to study incentives in a business franchise arrangement. Two particular features of their model stand out as relevant for the study of this paper. First, they conclude that the presence of a vertical externality is essential for the use of franchise contracts to resolve agency issues. In contrast they show that horizontal externalities, which give rise to effects such as under-provision of services between competing downstream firms, do not motivate the use of franchise contracts. Only a vertical externality is at the heart of the results of this paper, since the manufacturer exploits the inefficiency from double marginalisation to reduce the expected information rent it inevitably must pay out.

Second, Matthewson and Winter describe a positive relationship between the variance of demand and the variance of the nonlinear tariffs that emerge optimally in their model. This result partly reflects their assumption that the retailer is risk averse. However, it also arises because they compare demand distributions through the lens of second order stochastic dominance: where a more favourable distribution is one with smaller variance. In this paper, demand state distributions are compared by first order stochastic dominance: distributions are ranked by their mean. Moreover, here both the manufacturer and retailer are risk neutral. Notwithstanding, this analysis arrives at a similar intuition for the fixed franchise fee. The

[4] There is a vast literature related to the theory and policy of vertical restraints. Much of the literature is devoted to a welfare analysis of the various vertical restraints that are used in practice in many industries, and the implication for competition policy. The results are not clear cut, as evidenced by the many changes that competition agencies have made to their legislation over the years. This paper does not attempt to evaluate the welfare properties of vertical restraints, but rather conjectures a variety of contract that could be employed by a manufacturer in a realistic contracting environment with a retailer.

more favourable the distribution of demand states, the lower the franchise fee. However, in contrast to Matthewson and Winter, this paper finds that the variable fees are higher for the favourable distribution, since the manufacturer is more intent on extracting information rents in that case.

Other papers that are close to this analysis are Crocker (1983), Rey and Tirole (1986), Blair and Lewis (1994), and Martimort and Piccolo (2007). Crocker (1983) describes a vertical contracting environment where the retailer already has private information about its production cost at the time of signing the contract. The thrust of his analysis is to weigh the transaction cost of the agency relationship, measured in terms of the rent extraction - efficiency trade-off, against the benefit of eliminating it through vertical integration. His results are similar to the second stage contracts of this paper, however here the retailer signs the contract before realising the precise demand state, so the timing is different.

The timing of the revelation of the uncertainty in Rey and Tirole (1986) is similar to that employed here: manufacturers and retailers are symmetrically informed about the precise demand state at the time of contracting, and retailers become privately informed about the precise demand state after the contract is signed. Hence, the retailer's attitude to risk in their paper plays a role in the efficacy of contractual provisions, and varies under different types of uncertainty. Their analysis chiefly describes the efficacy of contractual provisions in achieving the integrated profit within a vertical structure under uncertainty and retailer risk aversion. However, Rey and Tirole's model does not capture the intertemporal effect of the retailer's action on the distribution of demand states as the model presented here does. Nor does it explicitly analyse the manufacturers trade-off between extracting rents from high demand retailers on the one hand, and deliberately employing double marginalisation on the other to induce an allocation inefficiency to penalise misreporting.

In fact, the timing of the model presented here more closely resembles that of the information acquisition process in the optimal contest design literature. For example, in Fu and Lu (2010) an entry fee or subsidy induces the effort-maximising contest while extracting all the surplus from the contestants in the subgame perfect equilibrium of the game. This will prove to be similar to the role of the franchise fee in the contracts of the model presented later. In contrast however, in this model the second stage involves a revelation game where the retailer privately observes its demand, hence a rent-extraction efficiency trade-off is required for incentive compatibility.

The rent extraction-efficiency trade-off is dealt with in Blair and Lewis (1994), and Martimort and Piccolo (2007). Their papers analyse vertical delegation in a model of simultaneous adverse selection and moral hazard. In their models, the retailer is already privately informed at the time of contracting, and moral hazard is defined over the degree of promotional effort the retailer exerts to increase the demand for their product.[5] Blair and Lewis focus on establishing resale price maintenance as a provision that emerges endogenously to the informational environment of their model. Their results are similar to the second stage results of the model here, in the respect that the manufacturer optimally employs the vertical externality to their overall advantage.[6] However, both models differ from the one presented here in the respect that moral hazard in their models takes place after the retailer has learnt their private information, whereas in this paper it takes place before. Moreover, it is this difference in the timing combined with limited liability constraints on payoffs for realised demand states that creates an expected surplus for the retailer that is central to the results of this paper.

The impact of limited liability constraints in agency contracts was described in Sappington (1983). His analysis showed that in a model where the retailer signs the contract before realising its private information, limited liability constraints restrict the manufacturer's ability to impose penalties that would otherwise induce the socially efficient outcome. Hence, limited liability constraints result in a pattern of information rents that is similar to the case of interim contracting. The consequences for limited liability are very similar in this paper: ex post constraints on the payoff to the retailer in the lowest demand state skew the information rents to higher demand state types. It is exactly this skewed pattern of information rents that creates the context for a fixed franchise fee to be incorporated into the contract.

[5]Since there is no uncertainty on the part of the retailer, attitude to risk does not play any role in their results, unlike Rey and Tirole (1986).

[6]Martimort and Piccolo (2007) is technically similar to Blair and Lewis (1994), however their focus is on the difference between resale price maintenance and quantity fixing and the implications for welfare from both private and social perspectives.

3 Discrete Demand State Illustration

This section sketches the nature of the conjectured franchise contracts for discrete demand states. First, the double marginalisation problem is set out for a single demand state in a non-integrated manufacturer-retailer hierarchy and compared to a vertically integrated structure. This is followed by a brief discussion of how vertical restraints can improve efficiency. Then a two demand state problem under complete information is constructed to obtain the first-best contracts. Moving to a situation of incomplete information shows how the retailer can strategically use its private information to its advantage, and how the manufacturer employs the inefficiency from double marginalisation to ensure incentive compatibility, thus improving its ex ante position. Finally, the retailer's limited liability constraint is shown to interact with incentive compatibility to give rise to an expected surplus accruing to the retailer, which the manufacturer can appropriate with a franchise fee.

3.1 Double Marginalisation

To illustrate the inefficiency that occurs in this framework, first consider a non-integrated industry that faces a single known demand state where both the manufacturer and retailer are restricted to use linear pricing. Panel (a) on the left hand side of Figure 1 depicts the consumer demand curve, D, which the retailer faces. Suppose the retailer's marginal cost of retailing is zero. Then the manufacturer faces a derived demand, D_M which comes from the retailer's profit maximisation problem. Hence, the D_M curve in Figure 1(a) is both the manufacturer's derived demand curve and the retailer's marginal revenue curve, MR_R. Since the manufacturer is a monopolist in the upstream market, it maximises its profit by selling to the retailer a quantity x^{ni} where its marginal revenue MR_M from the sale of that quantity is equal to its constant marginal production cost, $c > 0$. The manufacturer charges the wholesale price p_w per unit to the retailer, and earns wholesale profits equal to the area π^M, depicted in panel (b) for clarity.

Since the manufacturer's problem came from a derived demand curve, it internalised the retailer's problem in its quantity decision. Since the retailer is a monopolist in the downstream market, it resells the quantity x^{ni} which it would have optimally chosen if its

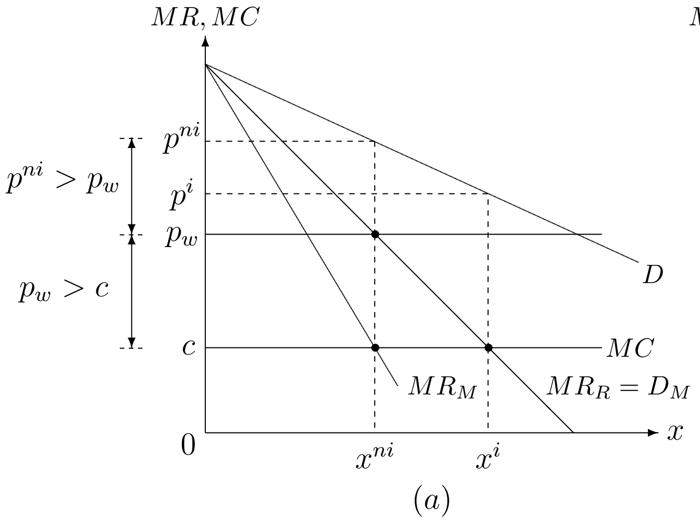

Figure 1: Double Marginalisation

marginal cost was p_w, and earns a retail profit equal to the area between its marginal revenue and marginal cost curves, denoted as π^R in Figure 1(b). The price each unit is sold for can be read off the final demand curve at x^{ni} units of the quantity, which will be denoted p^{ni}. Hence, the final price faced by consumers includes two markups over marginal cost: the manufacturer's markup $p_w > c$ and the retailer's markup: $p^{ni} > p_w$. This is Spengler's (1950) double marginalisation phenomenon.

In an integrated structure, a monopolist would choose the quantity x^i where the marginal (retail) revenue of the last unit sold is equal to the marginal (production) cost. In doing so, it would earn profit equal to the area $\pi^R + \pi^M + el$ in Figure 1(b) by charging the per unit integrated price p^i, as in panel (a). By comparing to the non-integrated case, it is straightforward to see that by making sequential markups over their respective marginal costs, the manufacturer-retailer hierarchy sells for a higher price: $p^{ni} > p^i$, and sells too few units into the final market: $x^{ni} < x^i$ relative to an integrated structure. The area el is a measure of the relative allocative efficiency loss on the producer side to a manufacturer-retailer hierarchy from the vertical externality.[7]

[7]Note that there is an associated loss of consumer surplus from the double markup, and that consumer's prefer the integrated case since more output is then sold.

3.2 Vertical Restraints

If the manufacturer is no longer restricted to use linear prices, it can employ various contractual provisions known as *vertical restraints* to overcome the efficiency loss on the producer side from double marginalisation. Vertical restraints include instruments like resale price maintenance, quantity fixing, and non-linear pricing techniques which are generally referred to as franchise fees. In an environment of complete information, all these types of vertical restraints achieve the same outcome.[8] For example, if the manufacturer stipulated that the retailer must fix the consumer's price to the integrated monopoly price p^i - resale price maintenance - then the wholesale price can be used simply to distribute the integrated monopoly profit between the manufacturer and retailer. A quantity fixing contract, which stipulates that x^i units must be sold, has the same effect. Both these methods require the manufacturer to exert direct control over the behavior of the retailer.

The manufacturer can also decentralise the retail decision making and still implement the integrated outcome by selling using a two-part tariff. It could sell as many units to the retailer as it wanted to purchase at marginal cost - in which case the retailer optimally purchases x^i units - and charge a fixed fee to extract all or part of the integrated profit that the retailer would otherwise earn. In such a set up, the fixed fee which the retailer and manufacturer bargain over is called the franchise fee.

In what follows the manufacturer is assumed to offer the retailer a type of franchise fee contract, however it will be slightly more restrictive: the manufacturer will infer the integrated monopoly quantity contingent on the demand state, and ask the retailer to sell that quantity into the final market in return for a fixed fee equal to the entire revenue obtained.[9] Note that this implicitly confers full bargaining power to the manufacturer: the retailer will make zero economic profit, and the manufacturer will receive the entire integrated monopoly profit. Hence, the manufacturer will offer a contract for each demand

[8] With incomplete information, different vertical restraints give different outcomes according to the nature of the informational asymmetry and the existence of intra-brand or inter-brand competition. See Rey and Tirole (1986).

[9] An equivalent resale price maintenance or quantity fixing contract could be found, however it would involve setting an appropriate wholesale price in each case. It will be notationally simpler to use fixed fees in the sequel.

state consisting of a fee to be paid and a quantity to be sold: $\{t, x\}$. The set of these contracts constitutes the *fee schedule*.

3.3 Franchise Contracts

Now suppose that the level of demand can take one of two possible values: high or low. Figure 2 depicts the two demand state case, where the demand curves have been omitted for clarity. If there is complete information about which demand state the retailer faces, then the manufacturer will offer one of two contracts. If the demand is low, then the contract is: $\left\{CAc, x_L^{fb}\right\}$. If the demand is high, then the manufacturer offers contract: $\left\{DBc, x_H^{fb}\right\}$. Notice that, because the marginal revenue curve is further out from the origin for the high demand type than the low demand type, $x_H^{fb} > x_L^{fb}$, and $DBc > CAc$.

If it is only the retailer that knows the state of demand, then there is incomplete information. In this case, the manufacturer knows that there are two possible states of demand, but not which is the prevailing state. The manufacturer has the common knowledge belief that demand is high with probability g, and low with probability $1 - g$, where $0 \leq g \leq 1$. Moving to a situation of incomplete information introduces an information revelation problem: if the manufacturer continues to offer the first best contracts, a retailer facing the high demand state actually prefers to take the low demand contract. Thus, the manufacturer faces an adverse selection problem. The reason is illustrated in Figure 2. By reporting that the state of demand is low, a high demand retailer earns an *information rent* equal to the difference in the total revenue they can achieve, $DEx_L^{fb}0$, and the fixed fee it must pay the manufacturer, CAc. Hence, the high demand retailer earns information rent equal to $DEAC$. Note that the source of the information rent is from the high demand retailer selling the low level of output for a higher price than the integrated monopoly price: the vertical externality returns, albeit in a slightly different guise.

In this situation, the manufacturers payoff is always CAc, since the high demand contract would never be taken. However, if the manufacturer offered to give up at least the rent $DEAC$ in the case of high demand, then the retailer in that instance would weakly prefer to accept the high demand contract. Hence, the manufacturer would prefer to offer the

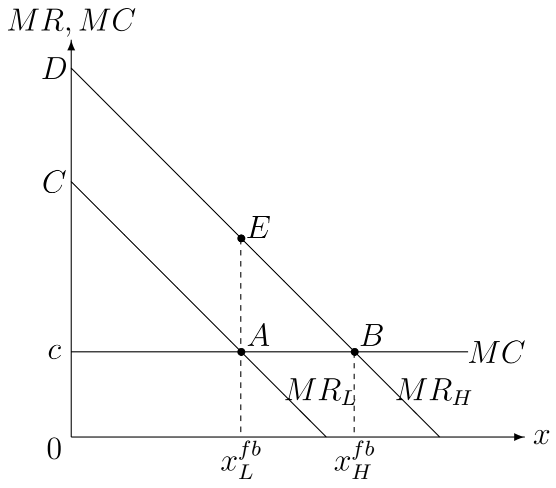

Figure 2: Incentive to Mis-report

separating contracts $\left\{CAc, x_L^{fb}\right\}$, $\left\{DBc - DEAC, x_H^{fb}\right\}$ only if:

$$\mathbb{E}\pi = (1-g)CAc + g(DBc - DEAC) \geq gDBc$$

That is, it pays the manufacturer to use separating contracts if the expected payoff from inducing the high demand retailer to report the true state of demand exceeds the payoff from only ever selling the low demand quantity.

Inspection of Figure 2 and the manufacturer's expected payoff from eliciting a truthful report from the high demand retailer suggests that the manufacturer can improve their ex ante position. It can do this by imposing an efficiency loss from double marginalisation on the low demand retailer. By reducing the quantity its asks the retailer to sell in the low demand state to the second-best quantity x_L^{sb}, the size of the information rent that the manufacturer has to give up to the high demand retailer reduces to $DFGC$ in Figure 3. The cost to the manufacturer of exploiting the vertical externality is a reduction in the revenue it can ask the low demand retailer to pay by area $GAx_L^{fb}x_L^{sb}$ in Figure 3. Hence, the manufacturer trades off extracting the information rent from the high demand retailer against an allocative efficiency loss from the low demand retailer. The manufacturer prefers second-best contracts whenever:

$$\mathbb{E}\pi^{sb} = \underbrace{(1-g)CAc + gDBc}_{(a)} - (\underbrace{(1-g)GAH}_{(b)} + \underbrace{gDFGC}_{(c)}) \geq gDBc \tag{3.1}$$

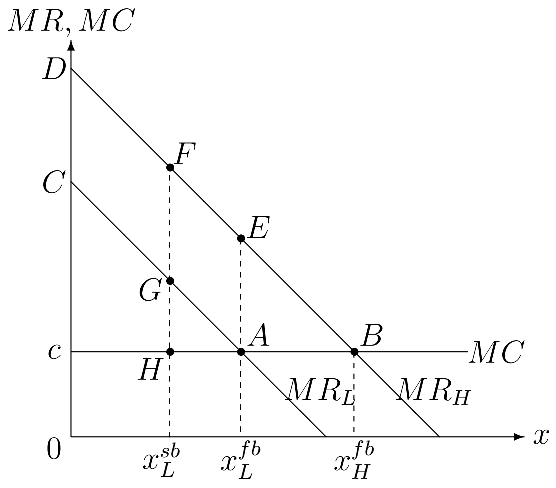

Figure 3: Incomplete Information Contracts

In the expression for expected profit in equation (3.1), the first term (a) corresponds to the first-best expected efficiency. Out of this, the manufacturer expects to lose efficiency from optimally exploiting the vertical externality for low demand types - term (b), and expects to payout an information rent to the high demand type - term (c). Importantly, the manufacturer controls the low demand retailer's quantity to improve their position. Note also that the magnitudes of the distortion in the low demand quantity and information rent depends on the probability distribution over the demand states.

Now suppose that before information is revealed to either party, the retailer can take a private action to influence the probability of the high demand state occurring. Specifically, suppose that the retailer has an action space containing two actions: $a, a' \in \mathcal{A}$. Furthermore, suppose that action a induces a greater probability of the high demand state occurring than action a': $g(a) > g(a')$. Then action a is said to induce a discrete probability distribution that first-order stochastically dominates the probability distribution induced by action a'. This is generalised in the next section to a probability distribution over a continuum of demand states.

Finally, consider the case of incomplete information where neither the manufacturer nor the retailer are informed about the precise state of demand at the time of signing the contract, but they both share the same objective belief about the likelihood of the demand states. Moreover, suppose that just after the contract is signed the retailer privately realises the state of demand. To evaluate its payoff, the retailer also must form an expectation over their payoffs in each state of demand. Then once again utilising Figure 3, the retailers expected payoff is: $u = (1 - g(a))0 + g(a)DFGC$. Note that the retailer's payoff in the low demand state is bound at zero. This is due to the assumption of limited liability. Limited liability hence interacts with incentive compatibility to create a positive gross expected surplus to the retailer. However, if the retailer's outside opportunity is normalised to zero, then the manufacturer can extract the full expected surplus with a franchise fee payable at the time of signing the contract: $T = g(a)DFGC$.

Two hypotheses can be drawn from this simple illustration of ex ante contracting and limited liability. First, there is a reciprocal relationship between the size of the inefficiency induced by the vertical externality and the expected surplus $T = g(a)DFGC$. This is a straightforward consequence of the timing of information revelation and the basic rent-extraction efficiency result that frames the equilibrium contracts. Second, the more weight the manufacturer assigns to the high demand state occurring, the greater is the inefficiency from the vertical externality that the manufacturer imposes on the low demand type. This hypothesis is the key observation of this paper. To see why it is important, consider Figure 3 again. The marginal expected benefit to the manufacturer from distorting the low demand quantity is the revenue $FEAG$ from rent extraction, and the marginal expected cost is the loss of revenue GAH from the vertical externality on the low demand retailer. Then increasing the probability of the high demand state occurring increases the manufacturer's relative benefit from distorting the low demand quantity.

From these two hypotheses, it follows that the size of the franchise fee T depends on the probability of the high demand state occurring, which in turn depends on the action taken by the retailer prior to realising the state of the world. Since the action becomes relevant before the retailer privately realises the precise demand state, there is potential for either a moral hazard problem or an adverse selection problem over the action. For instance, if the action space of the retailer consists of a set of costly efforts that the retailer can exert which are unobservable and non-verifiable, say in promoting the manufacturer's good or service, then the manufacturer's problem is one of moral hazard followed by adverse selection. If the action space of the retailer consists of a set of reports of signals that the retailer receives which are perfectly correlated with the demand distribution it will be facing, then the manufacturer's problem is one of sequential adverse selection. In either case, this paper conjectures that the manufacturer can offer a set of *franchise contracts* of the form $\{T, (t, x)\}$ that can either induce the efficient effort level, or induce truthful signal report from the retailer. Both these situations are studied later in a more general model.

The timing of the action that the retailer takes and the revelation of the precise demand state gives rise to a commitment game that will be described here for clarity. The game is slightly different depending on whether there is moral hazard or adverse selection from the outset. In the moral hazard case the manufacturer will offer a single franchise contract $\{T, (t, x)\}$. By offering this contract, both the manufacturer and retailer commit to the variable fee schedule (t, x) which is incentive compatible for reports of the precise demand state once it is realised by the retailer, contingent on the efficient action being taken. That is, the retailer must commit to the contract before it knows the precise state of demand. Since the fee schedule also must abide by ex post limited liability, the retailer's expectation of the rent it will receive from this part of the contract is positive. To solve the commitment problem, the manufacturer must design the franchise fee T in a way that simultaneously elicits the efficient action with respect to which (t, x) is designed, and that extracts the retailer's positive expected surplus from the variable fee schedule.

For the adverse selection case, the commitment game is essentially the same, except the manufacturer and retailer commit to a menu of variable fee schedules, and the manufacturer must design a corresponding menu of franchise fees that elicit truthful reports about the signal that the retailer receives. In this case, the manufacturer must also consider whether it is worthwhile to extract the retailer's signal or to remain ignorant. The manufacturer's beliefs that support it's commitment to its contracts are explored later.

The next section establishes the two hypotheses in a general model with a continuum of demand states. This then allows a characterisation of the optimal franchise contracts. The section after that discusses the use of the franchise fee component of the franchise contracts to solve ex ante informational asymmetries that might emerge.

4 Analytical Framework

The Principal-Agent problem is between a manufacturer and a retailer. The risk neutral manufacturer produces an intermediate product, x, at constant unit cost $c \geq 0$. The risk neutral retailer sells each unit supplied to it on a final output market, generating total revenues $R(x, \theta)$ where θ is the retailer's private information about the precise state of demand, which belongs to the compact support $\Theta := [\underline{\theta}, \overline{\theta}]$. The retailer's marginal retailing cost is assumed to be zero, without loss of generality. Also, the retailer is assumed to be a single price monopolist in the final output market.

The revenue function, $R(x, \theta)$ is concave in sales, x, and increasing in the level of demand, θ. Moreover, since the retailer is a monopolist in the final market, the relevant part of the total revenue curve is where it is increasing in output. Putting these restrictions more concisely: over the relevant domain of x, $R_x > 0$, $R_{xx} < 0$ and $R_\theta > 0$, where subscripts denote partial derivatives. Importantly, the retailer's marginal revenue is assumed to be increasing in the realised demand state: $R_{x\theta} > 0$: this is the Spence-Mirrlees single crossing property. The following technical assumptions are also made: $R_{x\theta\theta} \geq 0$ and $R_{\theta xx} \geq 0$ to ensure the contracts are well behaved.

The model has two time periods, indexed by $s = 1, 2$. In the first period, the retailer takes an action, $a \in \mathcal{A}$, where $\mathcal{A}$ is the set of possible actions that the retailer can take. The action a affects the probability distribution of demand states that the retailer faces in the second period. In the second period, the retailer privately realises the level of demand, $\theta \in \Theta$. Thus, the timing of the revelation of precise demand state information satisfies an ex-ante constraint: the retailer must sign to a contract with the manufacturer before the uncertainty about demand for the product is resolved. Moreover, the contract is signed before the retailer takes any action $a \in \mathcal{A}$. Given this timing, the manufacturer must construct its contracts in a way that elicits the action it finds most appropriate, and induces the retailer to truthfully report its information about the level of demand in the second period.

The Revelation Principle permits attention to be restricted to the class of direct revelation mechanisms.[10] The conjecture of this paper is that the manufacturer may offer to the

[10] Laffont and Martimort (2002) show that the Revelation Principle can be extended to cover contracts

retailer a *franchise contract*, which is defined to be the triple: $\{T, (t(\theta), x(\theta))_{\theta\in\Theta}\}$. The first component, T, is a franchise fee that the retailer is liable for when the contract is signed in the first period. The second component is a menu of contracts, $(t(\theta), x(\theta))$, consisting of the variable fee $t(\theta)$ payable by the retailer to the manufacturer for a quantity $x(\theta)$ of the intermediate product that the retailer of type θ purchases from the manufacturer in the second period. Then the retailer's second period surplus from the fee schedule $(x(\theta), t(\theta))$ is given by:

$$U(\theta) := R(x(\theta), \theta) - t(\theta) \tag{4.1}$$

The fee and quantity profile represent a fee schedule that the manufacturer and retailer commit to using when the contract is signed. The exact timing of the contracts are detailed in Figure 4.

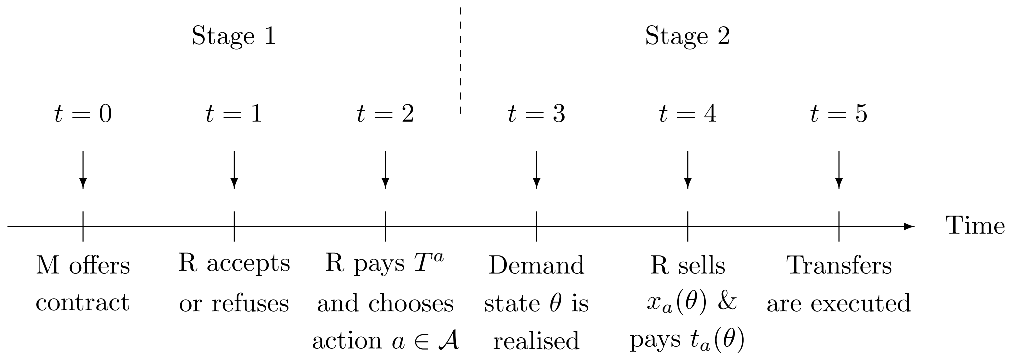

Figure 4: General Timing

Stage 1 encompasses the first three steps, where along the equilibrium path, the contract is offered and accepted and the retailer chooses some action $a \in \mathcal{A}$. Stage 2 encompasses the last steps in the contract, where the demand state θ is privately realised by the retailer and the retailing decision is made according to the fee schedule.

The manufacturer chooses its contracts to maximise its expected payoff given by:

$$V = T + \int_{\Theta} (t(\theta) - cx(\theta))g_a(\theta)d\theta$$

The manufacturer designs the contract using backward induction. Hence, the fee schedule $(t(\theta), x(\theta))$ is chosen first to maximise the second term in its expected payoff above. Then,

with sequential information revelation (p. 274).

given this second period behavior, the manufacturer optimally chooses the franchise fee that elicits its most preferred, or efficient, action from the retailer through use of the franchise fee component. The retailer's first stage action influences the probability distribution over demand states in the sense of first order stochastic dominance.

More precisely, when the retailer exerts action $a \in \mathcal{A}$, the demand state $\theta \in \Theta$ is distributed according to the continuous, conditional cumulative distribution function $G_a(\theta) := G(\theta|a)$ on the compact interval $\Theta := [\underline{\theta}, \overline{\theta}]$ with conditional density function $g_a(\theta) := g(\theta|a) > 0$. For technical convenience, $g(\underline{\theta}|a) = g(\underline{\theta}|a')$ and $g(\overline{\theta}|a) = g(\overline{\theta}|a')$ for $a, a' \in \mathcal{A}$. These distributions are assumed to satisfy the monotone hazard rate condition:

$$(MHR) \qquad \frac{d}{d\theta}\frac{g(\theta)}{1 - G(\theta)} \geq 0, \qquad \forall \theta \in \Theta$$

Some actions are assumed to induce favorable demand state distributions, so a means of ranking probability distributions is needed. For this purpose, the cumulative distribution property is required to satisfy hazard rate dominance. If the distribution induced by action a is more favourable distribution than the distribution induced by action a', then:

$$(HRD) \qquad \frac{g_a(\theta)}{1 - G_a(\theta)} \leq \frac{g_{a'}(\theta)}{1 - G_{a'}(\theta)}, \qquad \forall \theta \in \Theta$$

HRD means that the hazard rate of the favourable distribution is point-wise lower than the hazard rate of the unfavourable distribution. Intuitively, this means the conditional probability of eliminating a demand type in the interval $[\theta, \theta + d\theta]$ is smaller for the more favorable distribution.

A convenient implication of assuming HRD is that it also implies that demand realisations are correlated with the action of the retailer in the sense of first-order stochastic dominance. That is, an action a induces a more favourable distribution than action a' if:

$$(FOSD) \qquad G_a(\theta) \leq G_{a'}(\theta), \qquad \forall \theta \in \Theta$$

Lemma 1 proves this stochastic ordering:

Lemma 1 *(Stochastic Orders) HRD $\Rightarrow$ FOSD.*

The next section establishes the manufacturer's second period fee schedule as the solution to a second period maximisation problem that takes as given an action $a \in \mathcal{A}$ that the retailer takes in the first period. Hence, the optimal second period fee schedule is the a-contingent solution $(t^a(\theta), x^a(\theta))$ for some $a \in \mathcal{A}$. After that, the franchise fee is derived for various action spaces $\mathcal{A}$.

4.1 Stage 2: Fee Schedule

The limited liability constraint requires retailers to *realise* a level of surplus at least as good as their outside opportunity, which is normalised to zero. Note that limited liability is different from an individual rationality constraint. Individual rationality applies to the entire contract, so will constrain the franchise fee, as will be shown in the next section. Individual rationality will therefore apply before the demand state θ is realised by the retailer, whereas the limited liability constraint must be satisfied after the demand state θ is realised. The second period limited liability constraint is:

$$(LL2) \qquad U(\theta) = R(x(\theta), \theta) - t(\theta) \geq 0, \qquad \forall \theta \in \Theta \tag{4.2}$$

To ensure that a retailer with a high realisation of demand does not take a contract designed for a low demand state retailer, self-selection or second period incentive compatibility constraints apply. By the Revelation Principle, attention is restricted to the class of direct mechanisms where the retailer is asked to make a truthful report on their demand type, θ. Using the notation that a retailer of demand type θ reports their demand state as type $\hat{\theta}$, then their second period utility is $U(\theta, \hat{\theta})$. Incentive compatibility requires:

$$U(\theta, \theta) \geq U(\theta, \theta^{'}) \qquad \forall \text{ pairs } (\theta, \theta^{'}) \in \Theta \times \Theta$$

That is, the contracts are constructed so that truthfully reporting is optimal for the retailer. Formally, locally incentive compatibility requires that the truthful report satisfies:

$$(IC2) \qquad \theta \in \arg\max_{\hat{\theta} \in \Theta} \left\{ R(x(\hat{\theta}), \theta) - t(\hat{\theta}) \right\}, \qquad \forall \theta \in \Theta \tag{4.3}$$

Intuitively, this means that the manufacturer must give up an information rent equal to the marginal revenue that a retailer of type θ could obtain by reporting its realised demand state to be $\theta - d\theta$. Lemma 2 formalises this intuition:

Lemma 2 *(Necessary & Sufficient Conditions for IC2) Necessary and sufficient conditions for incentive compatibility are: (i)* $U_\theta(\theta) = R_\theta(x(\theta), \theta)$ *and (ii)* $x_\theta(\theta) > 0$.

Part (ii) of Lemma 2 simply implies that for truth-telling to be locally (and globally), optimal, the contract must specify that higher demand state retailers sell greater quantities of the manufacturer's good or service. The proof of the Lemma shows that part (ii) is a global condition.

A corollary of Lemma 2 that is immediately apparent is the existence of limited liability-induced information rents that the manufacturer has to give up to the retailer in order to prevent high demand states from being mis-reported. The rents arise due to the interaction of the ex-post limited liability constraints and the implementability condition for separating second stage contracts. This is because limited liability implies that the manufacturer cannot penalise the retailer if a low demand state occurs. In fact, if the retailer faces the lowest demand state, then its ex post payoff must be non-negative: $U(\underline{\theta}) = 0$. This skews the information rents that must be given up to higher demand types to ensure incentive compatibility.

Corollary 1 *(Demand State Information Rents) The second period information rent accruing to a retailer that realises demand state* θ *is:*

$$U(\theta) = \int_{\underline{\theta}}^{\theta} R_\theta(x(\tilde{\theta}), \tilde{\theta}) d\tilde{\theta} \tag{4.4}$$

The analogous information rents in the discrete demand state example were $U_H = DFGC$ and $U_L = 0$ in Figure 3.[11]

Note that the expression for the retailer's information rents depends cumulatively on the quantity profile for any given demand state. The manufacturer must inevitably give up the information rents if it wishes to achieve incentive compatibility, but by choosing an appropriate profile of quantities, the manufacturer can distort the distribution of information rents to its advantage. Recall that the manufacturer wishes to maximise its expected profit, conditional on an action that the retailer takes. Recall that the manufacturers expected profits are $V = T + \int_\Theta (t(\theta) - cx(\theta)) g_a(\theta) d\theta$. Hence, the manufacturer's problem is to

[11]Note that $DFGC$ may also be interpreted as the marginal revenue in the discrete state as well.

maximise its a-contingent expected profits subject to part (i) of Lemma 2 and $(LL2)$:

$$(P) \quad \max_{\{x,U\}} V = \int_{\Theta} (R(x(\theta), \theta) - U(\theta) - cx(\theta)) g_a(\theta) d\theta \tag{4.5}$$

$$\text{subject to:} \quad (LL2) \;\; U(\theta) \geq 0 \quad and \quad (IC2) \;\; U_\theta(\theta) = R_\theta(x(\theta), \theta)$$

In the optimal control formulation, $U(\theta)$ is the state variable, $x(\theta)$ is the control variable and $\mu(\theta)$ is the costate variable. The transversality conditions are: (TV) $U(\underline{\theta}) = 0$ and $\mu(\overline{\theta}) = 0$. Using the individual rationality constraint to eliminate $t(.)$ from the manufacturer's profit function, the Hamiltonian is:

$$\mathcal{H} = (R(x(\theta), \theta) - U(\theta) - cx(\theta)) g_a(\theta) + \mu(\theta) R_\theta(x(\theta), \theta) \tag{4.6}$$

Application of the Maximum Principle, as described in Leonard & Long (1992), yields the following first order conditions:

$$\frac{\partial \mathcal{H}}{\partial x} = 0 \Leftrightarrow (R_x - x^a) g_a(\theta) + \mu(\theta) R_\theta(x^a(\theta), \theta) = 0 \tag{4.7}$$

$$\mu_\theta(\theta) = \frac{\partial \mathcal{H}}{\partial U^a} \Leftrightarrow \mu_\theta(\theta) = g_a(\theta) \tag{4.8}$$

$$U^a_\theta(\theta) = \frac{\partial \mathcal{H}}{\partial \mu} = R_\theta(x^a(\theta), \theta) \tag{4.9}$$

The optimally controlled solution to the manufacturer's second stage problem is an a-contingent menu of contracts, or fee schedule $(t^a(\theta), x^a(\theta))_{\theta \in \Theta}$. The retailer's rents when evaluated at the optimally controlled solution yields the reaslised a-contingent surplus: $U^a(\theta) := U(\theta, t^a(\theta), x^a(\theta))$. The precise nature of the fee schedule is given in the following Proposition:

Proposition 1 *(Fee Schedule) The a-contingent contracts in the second period with incomplete information and limited liability, contingent on the retailer choosing action $a \in \mathcal{A}$ in the first period, are given by the fee schedule $(t^a(\theta), x^a(\theta))$ where:*

$$t^a(\theta) = R(x^a(\theta), \theta) - \int_{\underline{\theta}}^{\theta} R_\theta(x^a(\tilde{\theta}), \tilde{\theta}) d\tilde{\theta}, \qquad \& \qquad R_x - c = \tfrac{1 - G_a(\theta)}{g_a(\theta)} R_{\theta x}$$

where $x^a(\theta)$ is the implicit solution to the first-order condition on the right.

Inspection of the right hand condition shows the implicit solution to the optimal quantity profile. For the highest demand state, the manufacturer requires the retailer to sell into the market the (first-best) integrated monopoly quantity, where $MR(x^a(\overline{\theta})) = c$. This is the largest profit that can be made in the output market. The fee that the retailer must pay to the manufacturer, $t^a(\overline{\theta})$, is equal to the entire monopoly revenue that it makes from the output market, $R(x^a(\overline{\theta}), \overline{\theta})$, less the information rent that the manufacturer must give up to it to elicit a truthful report of the demand state, $U^a(\theta) = \int_{\underline{\theta}}^{\theta} R_\theta(x^a(\tilde{\theta}), \tilde{\theta})d\tilde{\theta}$. The analogous contract in the discrete demand case was $\left\{DBc - DFGC, x_H^{fb}\right\}$.

Compare this to the lowest demand state. Since the manufacturer cares about leaving information rents to higher demand state retailers, it sacrifices allocative efficiency by distorting the quantity it asks the lowest demand retailer (thereby inducing a form of the vertical externality). To see this note that for the lowest demand state $\underline{\theta}$, the right hand side of the implicit quantity solution is the largest. As such, there is wedge between marginal revenue and marginal cost: $MR(x^a(\underline{\theta})) > c$. However, by sacrificing allocative efficiency on low demand states, the manufacturer is able to save on information rents that it must give up in the event of higher demand states occurring. In addition, since the manufacturer obtains no benefit to leaving information rents to the lowest demand state retailer, it extracts all the profit from the output market in that state of the world: $t^a(\underline{\theta}) = R(x^a(\underline{\theta}), \underline{\theta})$. The analogous contract in the discrete demand case was $\left\{CGHc, x_L^{sb}\right\}$.

Part (ii) of Lemma 2 required that the control path for the quantity schedule is monotonically increasing for the variable component of the franchise contract to be implementable. This is shown to be true in Lemma 3:

Lemma 3 *(Implementability) The optimal a-contingent quantity schedule $x^a(\theta)$ is monotone increasing in θ.*

The equilibrium franchise contract can now be defined as the triple $\{T^a, (t^a(\theta), x^a(\theta))_{\theta\in\Theta}\}$ for some $a \in \mathcal{A}$. This section has just characterised the a-contingent fee schedule, $(t^a(\theta), x^a(\theta))_{\theta\in\Theta}$ as the solution to the manufacturer's optimal control problem. The next section takes a step back in the timing to where the retailer has not yet realised the demand state. This will enable a characterisation of the a-contingent franchise fee component of the contract: T^a.

4.2 Stage 1: Franchise Fee Component

In the first period the manufacturer wishes to charge a franchise fee to extract all the *expected* surplus from the second period retailing. The expected second period surplus is non-zero since the manufacturer must abide by ex-post limited liability constraints on retailing. Hence, while it will give up information rents in the second period, it can extract a first period value equal to the retailer's expected value of engaging in the retailing activity. To simplify the expressions, the following definition is employed:

$$\mathbb{E}_{\theta,a}U^{\hat{a}}(\theta) := \int_{\Theta} U^{\hat{a}}(\theta^*)g_a(\theta^*)d\theta^* = \int_{\Theta}\int_{\underline{\theta}}^{\theta^*} R_\theta(x^{\hat{a}}(\tilde{\theta}),\tilde{\theta})g_a(\theta^*)d\tilde{\theta}d\theta^*, \qquad a,\hat{a}\in\mathcal{A}$$

This represents the retailer's expected rent from signing to an $\hat{a}$-contingent fee schedule if it chooses action $a \in \mathcal{A}$. Hence, while the second period information rents are generated from the $\hat{a}$-contingent fee schedule that the manufacturer optimally controls for that action, the expectation over realisations of θ is evaluated according to the distribution induced by the action $a \in \mathcal{A}$ that the retailer takes. This reflects the scope for the retailer to act contrary to the designs of the manufacturer, the subject of the next section. Note that in the discrete demand case, the analogous expression was: $\mathbb{E}_{g,a}U = (1-g(a))0 + g(a)DFGC$, where the size of $DFGC$ depended on the action that the manufacturer believes the retailer to have taken. In that case, the manufacturer charged a fixed fee to extract this expected surplus and leave the retailer indifferent to entering the contract. The same task is undertaken here.

It will be convenient in what follows to define the *efficient action* a^* from the manufacturer's perspective:[12]

$$a^* := \left\{a\in\mathcal{A}|V^a \geq V^{a'} \;\; \forall\, a'\in\mathcal{A}\right\}$$

Where V^a is the manufacturer's surplus evaluated at the equilibrium solution. That is, a^* is the action that is most favoured by the manufacturer.

For the continuum of demand states case, first period payoffs to the retailer are:

$$u^a := -T^a + \mathbb{E}_{\theta,a}U^{\hat{a}}(\theta), \qquad a,\hat{a}\in\mathcal{A}$$

where T^a is the franchise fee charged by the manufacturer to the retailer. The manufacturer must respect a first period individual rationality constraint. To induce the retailer to sign

[12] It is assumed that the set a^* is single-valued. That is, there is a unique efficient action.

the contract, the manufacturer must at least provide the retailer with their ex ante reservation value, which is normalised to zero when the retailer takes the efficient action. So individual rationality means that the retailer's *expected value* of entering the contract with the manufacturer when the contract induces the efficient action a^* must be greater than zero:

$$(IR1) \quad u^{a^*} = -T^{a^*} + \mathbb{E}_{\theta,a^*} U^{a^*}(\theta) \geq 0 \tag{4.10}$$

Before turning to examine the consequences of specific action spaces in the next section, first the case where no action can be taken by the retailer is examined as a benchmark.

4.2.1 Empty Action Space: $a = \phi = \mathcal{A}_1$

In this case, the retailer cannot take an action in the first period that influences the distribution of the demand states. Hence, both the manufacturer and retailer believe the probability distribution of demand states to be given by the unconditional cumulative distribution function $G(\theta)$. The manufacturer then simply offers a contract that consists of a single franchise fee, and a fee schedule as in Proposition 1. The franchise contract in this case corresponds to a generalised two-part tariff case, where the franchise fee serves to extract expected surplus. The contract in this case must abide by $(IR1)$. Then the franchise fee is set to extract the expected surplus that accrues to the retailer from its operation in the final output market due to the ex post limited liability constraint.

Proposition 2 *(Empty Action Space) The optimal franchise contract when the retailer cannot take an action prior to the realisation of the demand state is given by* $\left\{T^\phi, (t^\phi(\theta), x^\phi(\theta))\right\}$, *where:*

$$T^\phi = \mathbb{E}_{\theta,a} U^\phi(\theta), \qquad t^\phi(\theta) = R(x^\phi(\theta), \theta) - \int_{\underline{\theta}}^{\theta} R_\theta(x^\phi(\tilde{\theta}), \tilde{\theta}) d\tilde{\theta}, \qquad R_x - c = \tfrac{1-G(\theta)}{g(\theta)} R_{\theta x}$$

where $x^\phi(\theta)$ *is the implicit solution to the first order condition on the right.*

The interpretation of the franchise contract is straightforward. The fee schedule $(t^a(\theta), x^a(\theta))$ still embodies the incentive compatibility and limited liability constraints, as in Proposition 1. However, note that when committing to choosing a second period contract from that menu, the retailer imputes its ex ante payoff to be the expectation over all the rents it could possible receive, according to the unconditional probability distribution in this case. Since the $(LL2)$ constraint results in a positive expected payoff from the fee schedule, the manufacturer can charge a franchise fee T^{ϕ} equal to the size of the retailers expected payoff, and just induce the retailer to sign the ex ante contract. In the discrete demand formulation, the franchise fee was found to be $T = g.DFGC = \mathbb{E}_g U$, which is exactly analogous to the franchise fee in Proposition 2 for the continuum of demand states.

Recall the second hypothesis from the discrete demand state illustration: "the more weight the manufacturer assigns to the high demand state occurring, the greater is the inefficiency from the vertical externality that the manufacturer imposes on the low demand type". Proposition 2 can be used to formalise this hypothesis by examining the impact of different actions on the franchise contracts. First, recall that favourable actions induce HRD. Hence, if the retailer takes an action a that is favourable instead of an action a', then the inverse hazard rates respect the inequality: $\frac{1-G_{a'}(\theta)}{g_{a'}(\theta)} \leq \frac{1-G_a(\theta)}{g_a(\theta)}$. Using this and the concavity of the revenue function in x, $R_{xx} < 0$, in the right hand condition of Proposition 2 shows that the distortion in the quantity profile for the favourable distribution is more severe: $x^a(\theta) \leq x^{a'}(\theta)$. From Corollary 1, the information rent depends accumulatively on the marginal revenues in output. Hence, a direct consequence of the difference in the output distortions is that the variable fee $t(\theta)$ is pointwise higher for the favourable distribution because the manufacturer extracts greater rents for each type due to the relatively lower expected cost of allocative inefficiency.

Broadly speaking, the manufacturer cares more about extracting rents when it believes it is facing a favourable demand state distribution. However, when it believes it is facing an unfavourable demand state distribution, it cares more about allocative efficiency: it prefers not to penalise the retailer as much by exploiting the vertical externality for incentive compatibility.

Now all components of the franchise contract have been accounted for. Using this characterisation, the next step is to examine two different non-empty action spaces that could feasibly arise in franchise contracting arrangements. This is undertaken in the next section.

5 Action Spaces

This section analyses the retailer's choice of action $a \in \mathcal{A}$, when the action space consists of (i) a set of efforts: $\mathcal{A}_2 = \{e_L, e_H\}$ that influence the conditional probability distributions over demand states, and (ii) a set of reports of signals: $\mathcal{A}_3 = \{\sigma_L, \sigma_H\}$ that are perfectly correlated with the likelihood of the distribution of demand states. The first case corresponds to a situation of moral hazard followed by adverse selection. It describes a situation where the retailer could engage in, for example, a marketing campaign in its territory, or where the retailer's managers could stake their reputation on the manufacturers product. In both cases, the level of effort exerted by the retailer is costly, and is not something that can be written into a contract: the reward through the contract must induce the efficient action. The second case corresponds to a situation of sequential adverse selection. It describes a situation where the retailer already possesses some private information about the likelihood of the demand state distribution before the contract is signed. For example, the retailer may have better knowledge of the composition of residential and business consumers for the manufacturers good or service in its territory. In this case the manufacturer may find it useful to offer two types of franchise contract to screen retailers that have favourable information.

5.1 Inducing Effort with Franchise Fee: $a \in \{e_H, e_L\} = \mathcal{A}_2$

Here it is demonstrated how the manufacturer can devise a franchise contract that is guaranteed to elicit the efficient level of effort from the retailer when the first stage is characterised by moral hazard. Figure 5 illustrates the timing of the contract. The action space for the

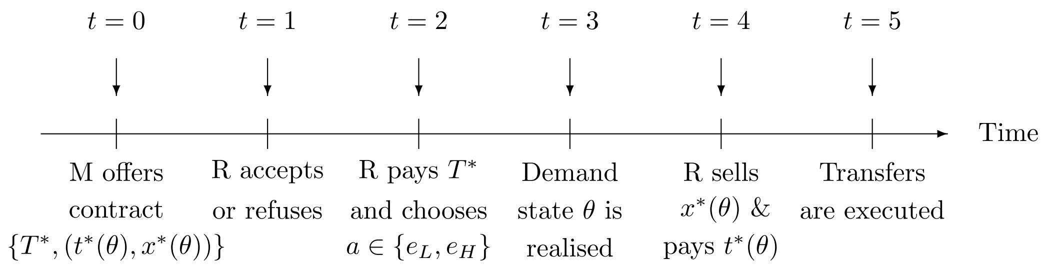

Figure 5: Timing - Stage 1 Moral Hazard

retailer in this case consists of two efforts levels, $a \in \{e_H, e_L\}$ with $e_H > e_L$. The effort level

e_H is considered more favourable than e_L, since when the retailer exerts more effort, say in marketing activities, it induces a more favourable demand state distribution. However, effort is costly to the retailer, with respective costs equal to:

$$\psi^a = \begin{cases} \psi^H = \psi & \text{if } e = e_H, \\ \psi^L = 0 & \text{if } e = e_L. \end{cases}$$

with $\psi > 0$. The manufacturer wishes to induce one of the actions, e_H or e_L. Hence, it offers one optimal contract: $\{T^*, (t^*(\theta), x^*(\theta))_{\theta\in\Theta}\}$, which can take on two values, corresponding to $a^* = e_H$ and $a^* = e_L$. Then in equilibrium, the retailer's first-period utility from exerting effort level a is:

$$u^a = -T + \mathbb{E}_{\theta,a}U^a(\theta) - \psi^a, \qquad a \in \{e_H, e_L\}$$

Notice that by assumption, there is not a stochastic relationship between effort and the outcome of the effort in this setup. For instance, if the retailer exerts high effort, then the outcome is the favorable distribution over demand states. For incentive compatibility, the retailer's first period optimal effort choice is the action $a^* \in \mathcal{A}$ that the manufacturer targets with franchise contract $\{T^*, (t^*(\theta), x^*(\theta))_{\theta\in\Theta}\}$:

$$a^* \in \underset{a\in\{e_H,e_L\}}{\operatorname{argmax}} \left\{-T^* + \mathbb{E}_{\theta,a}U^{a^*}(\theta) - \psi^a\right\}, \quad a^*, a \in \{e_H, e_L\} \tag{5.1}$$

It is assumed that the exertion of effort is unobservable by the manufacturer and non-verifiable by a court of law. This means that the manufacturer cannot simply write a contract on the amount of effort. Even so, it need only write one franchise contract, and the retailer will be induced to exert the efficient amount of effort under the circumstances. In doing this, the manufacturer must therefore respect the individual rationality constraint on the action it wishes to induce and the incentive compatibility constraint from above.

From Proposition 1 and the subsequent discussion, for each demand state, the information rent accruing to the retailer is smaller when it exerts effort since the manufacturer distorts the quantity schedule away from first best more rapidly for lower demand states. However, ex-ante more weight is placed on realising a higher demand state when effort is exerted. As a consequence, there is ambiguity in the ex ante expected surplus for each effort level: $\mathbb{E}_{\theta,H}U^H(\theta) \lesseqgtr \mathbb{E}_{\theta,L}U^L(\theta)$. Therefore, there are two cases that emerge in this situation, as captured in Lemma 4 below:

Lemma 4 *The optimal franchise fee elicits the high level of effort only when $\mathbb{E}_{\theta,H}U^H(\theta) - \mathbb{E}_{\theta,L}U^L(\theta) \geq \psi$, otherwise the manufacturer optimally induces the retailer to exert low effort.*

Without imposing more structure on the conditional probability distributions, no finer prediction about the level of effort the retailer should be induced to exert can be made. However, Lemma 4 does help in describing the two contracts that are feasible for the manufacturer to offer. These are stated in the following Proposition.

Proposition 3 *(Moral Hazard Franchise Contract) The optimal franchise contract when the retailer's action space is $\mathcal{A} = \{e_H, e_L\}$ is given by the single contract:*

$$\left\{T^j, (t^j(\theta), x^j(\theta))\right\}, \ \ where: \ \ j = \begin{cases} H \quad if \quad \mathbb{E}_{\theta,H}U^H(\theta) - \mathbb{E}_{\theta,L}U^L(\theta) \geq \psi \\ L \quad if \quad \mathbb{E}_{\theta,H}U^H(\theta) - \mathbb{E}_{\theta,L}U^L(\theta) < \psi \end{cases}$$

and where the components of the franchise contract are determined by:

$$T^j = \mathbb{E}_{\theta,e_j}U^j(\theta) - \psi^j, \qquad t^j(\theta) = R(x^j(\theta),\theta) - \int_{\underline{\theta}}^{\theta} R_\theta(x^j(\theta),\theta)d\theta, \qquad R_x - c = \tfrac{1-G_j(\theta)}{g_j(\theta)}R_{\theta x}$$

where $x^j(\theta)$ is the implicit solution to the first order condition on the right.

A nice feature of the optimal moral hazard franchise contract is that there is only one franchise fee: the manufacturer can elicit whichever level of effort it finds in its interests with a single upfront fee and a commitment to sell its good or service from the corresponding fee schedule. However, as will be shown in the next section, when the first stage informational environment is best described by adverse selection, the manufacturer may need to offer a menu of franchise contracts to ensure efficient screening of information.

5.2 Screening by Franchise Fees: $a \in \{\sigma_L, \sigma_H\} = \mathcal{A}_3$

In this section, the action space of the retailer is a report $a \in \{\sigma_H, \sigma_L\}$, where $\sigma_H > \sigma_L$ are privately observed signals that the retailer receives prior to accepting the manufacturer's contract. Figure 6 illustrates the timing of the contract. The manufacturer's beliefs about which signal the retailer received are: $Pr(\sigma = \sigma_H) = \lambda$ and $Pr(\sigma = \sigma_L) = 1 - \lambda$ where $0 \leq \lambda \leq 1$. Since the signal is private information, the manufacturer designs the franchise contracts in a way that induces truthful revelation of the signal. This means that the

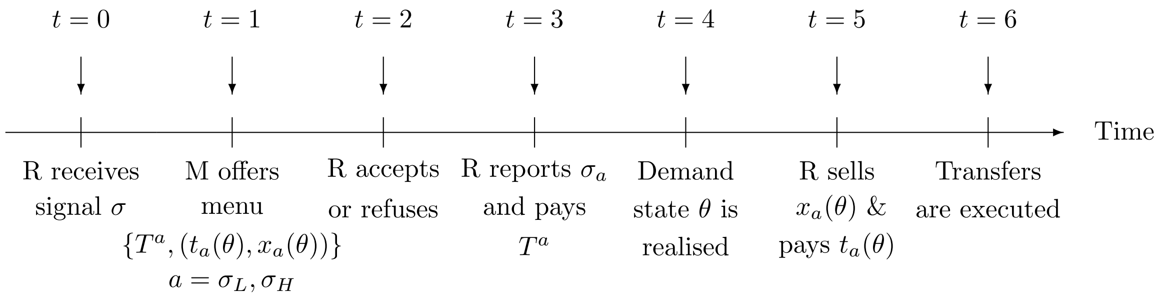

Figure 6: Timing - Stage 1 Adverse Selection

manufacturer offers contracts that are contingent on the outcome of the retailer's action. Suppose that $u(\sigma, \hat{\sigma})$ represents a retailer of type σ making a report that it is type $\hat{\sigma}$. Then first-period individual rationality conditions ($IR1$) are:

$$u(\sigma_i, \sigma_i) \geq 0, \quad i, j = \{L, H\} \tag{5.2}$$

First-period incentive compatibility conditions ($IC1$) are:

$$u(\sigma_i, \sigma_i) \geq u(\sigma_i, \sigma_j) \quad i, j = \{L, H\}\,, \quad i \neq j \tag{5.3}$$

If the manufacturer did not charge franchise fees, first-period high-signal types of retailers have an incentive to mimic low-signal types. This result is proved in Lemma 5:

Lemma 5 *A high-signal retailer has more to gain from misreporting its signal than does a low-signal retailer.*

The intuition for Lemma 5 as to why high-signal retailers have more to gain from misreporting their signal follows from considering the rent extraction efficiency trade-off under the manufacturer's belief about the distribution of demand states the retailer is facing. When the manufacturer believes the retailer is facing the unfavorable distribution of demand states, it places more weight on allocative efficiency, since it believes the probability of the retailer realising a high demand state is relatively low. Hence, it does not distort the quantity profile as much as when the distribution of demand states is favorable. This means that a retailer in any given demand state deciding whether to mimic a lower demand state retailer could sell more into the market if the quantity profile is not as distorted, and pay a smaller fee on that contract. Hence, high-signal type retailers will earn an expected rent premium on their first-period information.

In view of Lemma 5, the first-period low-signal retailer's $(IR1)$ constraint is binding, and the first-period high-signal retailer's $(IC1)$ constraint is binding. The following Lemma then characterises the incentive feasible franchise fees:

Lemma 6 *The incentive-feasible franchise fees for the low and high-signal retailers are:*

$$\begin{cases} T^L = \mathbb{E}_{\theta,L} U(\sigma_L) \\ T^H = \mathbb{E}_{\theta,H} U(\sigma_H) - \mathbb{E}_{\theta} \Delta U(\sigma_L) \end{cases} \qquad \textit{with} \quad T^H \leq T^L \tag{5.4}$$

where $\mathbb{E}_\theta \Delta U(\sigma_L) := \mathbb{E}_{\theta,H} U(\sigma_H, \sigma_L) - \mathbb{E}_{\theta,L} U(\sigma_L, \sigma_L)$ *is the first-period information rent.*

The expression $\mathbb{E}_\theta \Delta U(\sigma_L) := \mathbb{E}_{\theta,H} U(\sigma_H, \sigma_L) - \mathbb{E}_{\theta,L} U(\sigma_L, \sigma_L)$ is the expression for the expected rent premium. The first term is the expected payoff to a high-signal retailer from selling the low-signal contract quantities weighted with the favorable probability distribution. The second term is simply the franchise fee, T^L, payable by the low-signal retailer. Hence, the expected rent premium is just the excess over the low-signal franchise fee that the high-signal retailer could earn for itself by mimicking the low-signal retailer.

Note that there is a priori no reason why T^H cannot be negative: in that case, the manufacturer subsidises the high-signal retailer to sign a contract, and recoups the loss with a higher variable fee. The optimal adverse selection franchise contracts are stated in Proposition 4:

Proposition 4 *(Adverse Selection Franchise Contracts) The optimal menu of franchise contracts when the retailer's action space is* $\mathcal{A} = \{\sigma_H, \sigma_L\}$ *is given by:*

$$\left\{T^H, (t^H(\theta), x^H(\theta))\right\}, \left\{T^L, (t^L(\theta), x^L(\theta))\right\}$$

where the components of the franchise contract are determined by Lemma 6 and:

$$t^j(\theta) = R(x^j(\theta), \theta) - \int_{\underline{\theta}}^{\theta} R_\theta(x^j(\theta), \theta) d\theta, \quad \& \quad R_x - c = \tfrac{1-G_j(\theta)}{g_j(\theta)} R_{\theta x}$$

for $j = L, H$ *and where* $x^j(\theta)$ *is the implicit solution to the first order condition on the right.*

From the manufacturer's perspective, screening signal types may not always be attractive. The price of limiting information rents under a favorable demand state distribution is a relatively severe distortion in allocative efficiency. So committing to a fee schedule that embodies the severe rent extraction efficiency trade-off may only be worthwhile if the probability that a retailer receives a high-signal is great enough. Otherwise the manufacturer may prefer to remain ignorant of the retailer's first period information. This idea is explored in the next section.

5.2.1 Screening or Ignorance?

Lemma 6 shows that separating franchise fees exist, and will allow for franchise contracts that are characterised by first stage adverse selection to be written, as in Proposition 4. However, if the manufacturer's beliefs about the signal received by the retailer place a large weight on the low signal, then separating the types may not be optimal. To see this, remember that when beliefs about the distribution of the demand state are conditional on the low signal being received, the manufacturer considers achieving allocative efficiency to be relatively more important than rent extraction.

However, if the manufacturer did not revise its beliefs, it would prefer to distort the quantity profile more steeply than it otherwise would if it had received the low signal: it would be more concerned with giving up information rents. Hence, using separating franchise contracts when the probability of receiving a high-demand signal is small may leave the manufacturer with a smaller expected surplus than if they did not try to separate out the signal types.

It is possible to establish the critical probability λ^* where the manufacturer is indifferent between offering separating franchise contracts and being ignorant of the retailer's first stage signal. To make the exposition simpler, assume without further loss of generality that the manufacturer's constant marginal production cost, c, is zero. Then the risk neutral manufacturer's ex ante profit is:

$$\mathbb{E}_{\theta,k} V = T^k + \int_{\Theta} t(\theta_k) g_k(\theta) d\theta, \qquad k = \phi, L, H$$

Making the substitutions for $k = \phi, L, H$ yields:

$$\begin{aligned}
\mathbb{E}_{\theta,\phi}V^{\phi} &= \int_{\Theta} R(x^{\phi}(\theta))g_{\phi}(\theta)d\theta = \mathbb{E}_{\theta,\phi}R^{\phi} \\
\mathbb{E}_{\theta,L}V^{L} &= \int_{\Theta} R(x^{L}(\theta))g_{L}(\theta)d\theta = \mathbb{E}_{\theta,L}R^{L} \\
\mathbb{E}_{\theta,H}V^{H} &= -\Delta\mathbb{E}_{\theta}U^{L}(\theta) + \int_{\Theta} R(x^{H}(\theta))g_{H}(\theta)d\theta = -\Delta\mathbb{E}_{\theta}U^{L}(\theta) + \mathbb{E}_{\theta,H}R^{H}
\end{aligned}$$

Then total manufacturer payoffs are:

$$\mathbb{E}_{\theta,H}V^{H} + T^{H} = \mathbb{E}_{\theta,H}R^{H} - \mathbb{E}_{\theta}\Delta U^{L}(\theta), \quad \mathbb{E}_{\theta,L}V^{L} + T^{L} = \mathbb{E}_{\theta,L}R^{L}, \quad \mathbb{E}_{\theta,\phi}V^{\phi} + T^{\phi} = \mathbb{E}_{\theta,\phi}R^{\phi}$$

A separation of signal types is only worthwhile for the manufacturer if the expected payoff from the high-signal retailer is greater than from the low-signal retailer, which requires: $\mathbb{E}_{\theta,H}R^{H} - \mathbb{E}_{\theta,L}R^{L} \geq \mathbb{E}_{\theta}\Delta U^{L}(\theta)$. Moreover, the manufacturer prefers offering a screening franchise contract to remaining ignorant whenever: $\lambda\mathbb{E}_{\theta,H}R^{H} + (1-\lambda)\mathbb{E}_{\theta,L}R^{L} - \lambda\mathbb{E}_{\theta}\Delta U^{L}(\theta) - \mathbb{E}_{\theta,\phi}V^{\phi} \geq 0$. These two inequalities taken together imply a lower bound on λ that is required for existence of screening franchise contracts:

$$\lambda \geq \frac{\mathbb{E}_{\theta,\phi}R^{\phi} - \mathbb{E}_{\theta,L}R^{L}}{\mathbb{E}_{\theta,H}R^{H} - \mathbb{E}_{\theta,L}R^{L} - \mathbb{E}_{\theta}\Delta U_{L}(\theta)} \equiv \lambda^{*}$$

From the definition of λ^{*}, existence of screening contracts requires the following inequalities to be satisfied: $\mathbb{E}_{\theta,\phi}R^{H} \geq \mathbb{E}_{\theta,\phi}R^{\phi} \geq \mathbb{E}_{\theta,L}R^{L}$, and $\mathbb{E}_{\theta,\phi}R^{H} - \mathbb{E}_{\theta,L}R^{\phi} > \mathbb{E}_{\theta}\Delta U_{L}(\theta)$. In this case, $\lambda^{*} \in (0,1)$. The single "ignorant" franchise contract is exactly as in Proposition 2, where the retailer cannot take an action. The following Proposition gathers the results on screening franchise contracts.

Proposition 5 *If $\lambda \in [\lambda^{*}, 1]$, the manufacturer offers the menu of screening franchise contracts in Proposition 4. If $\lambda \in [0, \lambda^{*})$ then the manufacturer offers the pooling franchise contract in Proposition 2.*

There is no guarantee of the existence of $\lambda^{*} \in (0,1)$. Depending on the structure imposed on the parameters of the model, it may be that the manufacturer always prefers to screen, or always prefers to remain ignorant.

6 Conclusion

This paper has demonstrated how a manufacturer can use franchise contracts in a vertical relationship with a retailer when the retailer has ex ante private information, and the manufacturer's contracts are bound by the retailer's limited liability. It was shown how the interaction of second stage incentive compatibility constraints with limited liability constraints yield an expected surplus to the retailer, and how the manufacturer may employ a franchise fee to appropriate it. This franchise fee, combined with the fee schedule was defined as a franchise contract.

The franchise fee can be used as an instrument for the manufacturer to either elicit an efficient action to influence the distribution of demand states, or screen ex ante private information from the retailer, according to the type of ex ante information that the retailer possessed. In the context of first stage moral hazard, it was shown that the manufacturer can always offer a single franchise contract that elicits the most efficient level of effort in the first stage and induces a truthful demand state report in the second stage. In the context of first stage adverse selection, it was shown that a pair of screening franchise contracts performs better than when the manufacturer is ignorant of the retailer's first stage information only where the manufacturer's beliefs about the probability that the retailer has received the high-demand signal is larger than some critical threshold, λ^*.

The model in this paper is static: it does not account for situations in which the manufacturer and retailer contract repeatedly. In such situations, the retailer's concern for its reputation may solve the moral hazard issue. Furthermore, if the demand state that the retailer faces is correlated over time, then the simple franchise contract outlined in this paper may no longer allow the manufacturer to screen privately informed retailers. Issues of renegotiation and commitment arise, so that the mechanics of dynamic contracting must be employed.[13] These matters are left for future research.

The conclusion that the manufacturer may prefer to remain ignorant rather than screen retailers with different signals about the likelihood of the demand state distribution raises an issue for further research. As there is expected surplus at stake, it may be possible to

[13]See for example Hart and Tirole (1988) and Laffont and Tirole (1988).

construct a game where the strategies of the manufacturer are to attempt to "muddy the waters" or signal or screen information. The mechanics of the timing of the model and the use of limited liability constraints could easily be applied to other incentives theory contexts, for example, in non-linear pricing or in credit rationing contracts.

7 Appendix

7.1 Proof of Lemma 1

First note that: $\frac{d}{d\theta}ln(1-G(\theta)) = -\frac{g(\theta)}{1-G(\theta)}$.

Integrating this and rearranging for $G(\theta)$ yields: $G(\theta) = 1 - e^{-\int_{\theta}^{\overline{\theta}} \frac{g(\theta)}{1-G(\theta)} d\theta}$.

By hypothesis, the HRD condition holds point-wise, so integrating both sides of the inequality results in:

$$\int_{\theta}^{\overline{\theta}} \frac{g_{a'}}{1-G_{a'}} d\theta \leq \int_{\theta}^{\overline{\theta}} \frac{g_a}{1-G_a} d\theta \Rightarrow 1 - e^{-\int_{\theta}^{\overline{\theta}} \frac{g_{a'}}{1-G_{a'}} d\theta} \leq 1 - e^{-\int_{\theta}^{\overline{\theta}} \frac{g_a}{1-G_a} d\theta} \Rightarrow G_{a'}(\theta) \leq G_a(\theta)$$

which holds $\forall \theta \in \Theta$. ■

7.2 Proof of Lemma 2

For part (i), differentiating $R(x(\hat{\theta}), \theta) - t(\hat{\theta})$ with respect to the retailer's report $\hat{\theta}$ yields:

$$R_x(x(\hat{\theta}), \theta)x_\theta - t_\theta(\hat{\theta}) = 0 \tag{7.1}$$

For truth-telling to be optimal, (7.1) becomes the identity $R_x(x(\theta), \theta)x_\theta - t_\theta(\theta) = 0, \forall \theta \in \Theta$. Now differentiating the argument of (4.3) with respect to θ and evaluating at $\hat{\theta} = \theta$ yields:

$$U_\theta = R_x x_\theta + R_\theta - t_\theta$$

Using the first order condition for truth-telling in this expression yields the result.

For part (ii) of Proposition 4.3, the second derivative of (7.1) again with respect to the retailers report $\hat{\theta}$ must be less than or equal to zero:

$$R_{xx}(x(\hat{\theta}), \theta)x_\theta^2 + R_x(x(\hat{\theta}), \theta)x_{\theta\theta} - t_\theta(\hat{\theta}) \leq 0 \tag{7.2}$$

Now since the truth-telling identity is always zero, differentiating again with respect to θ yields: $R_{xx}(x(\theta),\theta)x_\theta^2 + R_x(x(\theta),\theta)x_{\theta\theta} + R_{x\theta}(x(\theta),\theta) - t_\theta(\hat{\theta}) = 0$. Plugging this into (7.2) yields:

$$-R_{x\theta}(x(\theta),\theta)x_\theta \leq 0$$

Recalling the assumption that $R_{x\theta} > 0$ yields the local incentive result.

For global incentive compatibility, note that for any pair $\theta, \theta' \in \Theta$, with $\theta > \theta'$, that:

$$R(x(\theta),\theta) - t(\theta) \geq R(x(\theta'),\theta) - t(\theta') \quad \& \quad R(x(\theta'),\theta') - t(\theta') \geq R(x(\theta),\theta') - t(\theta)$$

Adding these together and rearranging yields:

$$R(x(\theta),\theta) - R(x(\theta'),\theta) \geq R(x(\theta),\theta') - R(x(\theta'),\theta')$$

Rewriting yields:

$$\int_{x(\theta')}^{x(\theta)} R_x(z,\theta)dz \geq \int_{x(\theta')}^{x(\theta)} R_x(z,\theta')dz$$

Then finally:

$$\int_{\theta'}^{\theta}\int_{x(\theta')}^{x(\theta)} R_{x\theta}(z,y)dzdy \geq 0$$

Since $\theta > \theta'$ and $R_{x\theta} > 0$ by assumption, then this inequality holds only if $x(\theta) > x(\theta')$. ■

7.3 Proof of Corollary 1

First note that Part (i) of Lemma 2 is a continuum of constraints, given by the solution to the differential equation: $U^a(\theta) = R_\theta(x(\theta),\theta)$. A particular solution to this differential equation requires an initial condition. Since $R_\theta(.) > 0$ by assumption, then the utility profile is always upward sloping, hence the (LL) constraint will only bind at the lowest type in the interval, so $U(\underline{\theta}) = 0$. Integrating Lemma 2(i) and using the initial condition $U(\underline{\theta}) = 0$ yields:

$$U(\theta) - U(\underline{\theta}) = U(\theta) = \int_{\underline{\theta}}^{\theta} R_\theta(x(\tilde{\theta}),\tilde{\theta})d\tilde{\theta}$$

which is the required result. ■

7.4 Proof of Proposition 1

First note that due to the concavity of the manufacturer's problem and Lemma 2, these necessary conditions are also sufficient. Equation (4.8) and the TV, $\mu(\overline{\theta}) = 0$, defines a differential IVP whose solution is:

$$\mu(\overline{\theta}) - \mu(\theta) = \int_{\theta}^{\overline{\theta}} g_a(\tilde{\theta})d\tilde{\theta} = (1 - G_a(\theta)) \Rightarrow \mu(\theta) = -(1 - G_a(\theta)) \tag{7.3}$$

Putting this expression into equation (4.7) yields:

$$(R_x - c)g_a(\theta) = \tfrac{1-G_a(\theta)}{g_a(\theta)} R_{\theta x}(x^a(\theta), \theta)$$

which is the left hand side equation in the Proposition. For the right hand side equation, note that for each $\theta \in \Theta$ the expression in (4.2) must hold. Rearranging and replacing the surplus $U^a(\theta) := U(\theta, t^a(\theta), x^a(\theta))$ with the corresponding information rent from Corollary 1 yields the result. ■

7.5 Proof of Lemma 3

Totally differentiating equation (4.7) yields: $R_{\theta x} + R_{xx}x_\theta^a = \frac{d}{d\theta}(\frac{1-G_a(\theta)}{g_a(\theta)}) + \frac{1-G_a(\theta)}{g_a(\theta)}(R_{\theta\theta x} + R_{\theta xx}x_\theta$. Solving this expression for x_θ^a gives:

$$x_\theta^a = \frac{\left(\frac{d}{d\theta}\frac{1-G_a(\theta)}{g_a(\theta)} - 1\right) R_{\theta x} + \frac{1-G_a(\theta)}{g_a(\theta)} R_{\theta\theta x}}{R_{xx} - \frac{1-G_a(\theta)}{g_a(\theta)} R_{\theta xx}} \tag{7.4}$$

Since the HR is monotone increasing, the numerator of this expression is monotone decreasing. By assumption, $R_{\theta x} > 0$, $R_{xx} < 0$, $R_{\theta\theta x} \leq 0$ and $R_{\theta xx} \geq 0$. Then it follows that $x^a(\theta)$ is monotone increasing in θ. Note that under complete information, $x_\theta^{a*} = -\frac{R_{\theta x}}{R_{xx}} > 0$, and that due to incomplete information, $x_\theta^a < x_\theta^{a*}$. ■

7.6 Proof of Lemma 4

The proof simply requires checking the incentive constraint for effort exertion for the retailer. When the manufacturer sets the franchise fee to T, the payoffs under each level of effort are:

$$\begin{aligned} u^{e_H} &= -T + \mathbb{E}_{\theta,H}U^H - \psi \\ u^{e_L} &= -T + \mathbb{E}_{\theta,L}U^L \end{aligned}$$

So first, if $\mathbb{E}_{\theta,H}U^H < \mathbb{E}_{\theta,L}U^L$ then for the franchise fee $u^H < u^L$. The manufacturer chooses the fee to satisfy the IR1 constraint on the low-effort level, $T = \mathbb{E}_{\theta,L}U^L(\theta)$. In this case, the retailer exerts no effort.

If $\mathbb{E}_{\theta,H}U^H > \mathbb{E}_{\theta,L}U^L$ then there are two cases. Case (i): $0 \leq \mathbb{E}_{\theta,H}U^H(\theta) - \mathbb{E}_{\theta,L}U^L(\theta) \leq \psi$. In this case the retailer has no incentive to exert effort and the best the manufacturer can do is to set the franchise fee to extract the expected rent at the low-effort level: $T = \mathbb{E}_{\theta,L}U^L(\theta)$. Case (ii): $\mathbb{E}_{\theta,H}U^H(\theta) - \mathbb{E}_{\theta,L}U^L(\theta) \geq \psi$. Hence, $u^H \geq u^L$. In this case the manufacturer sets the franchise fee so that IR1 binds for the high-effort level: $T = \mathbb{E}_{\theta,H}U(\theta) - \psi$ and the retailer always exerts effort. ■

7.7 Proof of Lemma 5

The expected gain to the high-signal type of retailer from reporting a low-signal is the gain from having the manufacturer believe that its consumption valuations in period 2 comes from the distribution $G_L(\theta)$ rather than $G_H(\theta)$:

$$\Delta_H U := \mathbb{E}_{\theta,H}U(\sigma_H, \hat{\sigma}_L) - \mathbb{E}_{\theta,H}U(\sigma_H, \hat{\sigma}_H) = \int_\Theta U_L(\theta)g_H(\theta)d\theta - \int_\Theta U_H(\theta)g_H(\theta)d\theta$$

Similarly, the gain to the low-signal retailer from reporting a low-signal is the gain from having the manufacturer believe that their consumption value in period 2 (correctly) comes from the distribution $G_L(\theta)$ rather than $G_H(\theta)$:

$$\Delta_L U := \mathbb{E}_{\theta,L}U(\sigma_L, \hat{\sigma}_L) - \mathbb{E}_{\theta,L}U(\sigma_L, \hat{\sigma}_H) = \int_\Theta U_L(\theta)g_L(\theta)d\theta - \int_\Theta U_H(\theta)g_L(\theta)d\theta$$

Then the difference in the expected difference in surplus is:

$$\Delta_H U - \Delta_L U = \int_\Theta (U_L(\theta) - U_H(\theta))(g_H(\theta) - g_L(\theta))d\theta$$

Integration by parts gives:

$$\Delta_H U - \Delta_L U = (U_L(\theta) - U_H(\theta))(G_H(\theta) - G_L(\theta))|_\Theta - \int_\Theta (\tfrac{\partial U_L(\theta)}{\partial \theta} - \tfrac{\partial U_H(\theta)}{\partial \theta})(G_H(\theta) - G_L(\theta))d\theta$$

Using Leibniz' rule that $\frac{\partial U_k(\theta^*)}{\partial \theta^*} = R_\theta(x_k(\theta^*)) > 0$ for $k = L, H$:

$$\Delta_H U - \Delta_L U = \int_\Theta (R_\theta(x_L(\theta)) - R_\theta(x_H(\theta)))(G_L(\theta) - G_H(\theta))d\theta > 0$$

where the last inequality follows from (i) FOSD and (ii) $x^L(\theta) \geq x^H(\theta)$ pointwise for $\theta \in \Theta$.

■

7.8 Proof of Lemma 6

The low-signal retailer franchise fee follows directly since (IR1) binds for the low-signal retailer. For the high-signal retailer franchise fee, since (IC1) binds:

$$\begin{aligned} u(\sigma_H, \sigma_H) &= -T^L + \mathbb{E}_{\theta,L} U(\sigma_L, \sigma_L) - \mathbb{E}_{\theta,L} U(\sigma_L, \sigma_L) + \mathbb{E}_{\theta,H} U(\sigma_H, \sigma_L) \\ &= u(\sigma_L, \sigma_L) + \mathbb{E}_\theta \Delta U(\sigma_L) \end{aligned}$$

The high-signal retailer franchise fee can be derived using the definition of $u(\sigma_H, \sigma_H)$ and again, since (IR1) binds for the low-signal retailer.

To show that $T^H \leq T^L$, first write out T^H in full:

$$T^H = \int_\Theta U_H(\theta^*) g_H(\theta^*) d\theta^* - \left(\int_\Theta U_L(\theta^*) g_H(\theta^*) d\theta^* - \int_\Theta U_L(\theta^*) g_L(\theta^*) d\theta^* \right)$$

where the last two terms comprise $\mathbb{E}_\theta \Delta U(\sigma_L)$, which itself can be more usefully expressed using integrations by parts:

$$\begin{aligned} \mathbb{E}_\theta \Delta U(\sigma_L) &= \int_\Theta U_L(\theta^*) \left[g_H(\theta^*) - g_L(\theta^*) \right] d\theta^* \\ &= U_L(\theta^*) \left[G_H(\theta^*) - G_L(\theta^*) \right] |_\Theta - \int_\Theta \left(\tfrac{\partial U_L(\theta^*)}{\partial \theta^*} \right) \left[G_H(\theta^*) - G_L(\theta^*) \right] d\theta^* \end{aligned}$$

Using Leibniz' rule: $\frac{\partial U_L(\theta^*)}{\partial \theta^*} = R_\theta(x_L(\theta^*)) > 0$. Hence, by the property of FOSD:

$$\mathbb{E}_\theta \Delta U(\sigma_L) = \int_\Theta R_\theta(x_L(\theta^*)) \left[G_L(\theta^*) - G_H(\theta^*) \right] d\theta^* > 0 \tag{7.5}$$

Returning to the expression for T^H and focusing on the first term in the expression, using integration by parts yields:

$$\begin{aligned} \int_\Theta U_H(\theta^*) g_H(\theta^*) d\theta^* &= U_H(\theta^*) G_H(\theta^*) |_\Theta - \int_\Theta \left(\tfrac{\partial U_H(\theta^*)}{\partial \theta^*} \right) G_H(\theta^*) d\theta^* \\ &= U_H(\overline{\theta}) - \int_\Theta R_\theta(x_H(\theta^*)) G_H(\theta^*) d\theta^* \end{aligned} \tag{7.6}$$

Recalling that $U_H(\overline{\theta}) = \int_\Theta R_\theta(x_H(\theta)) d\theta$ and combining equations (7.5) and (7.6) into T^H gives:

$$T^H = \int_\Theta R_\theta(x_H(\theta)) d\theta - \int_\Theta R_\theta(x_L(\theta)) G_L(\theta) d\theta - \int_\Theta \left[R_\theta(x_H(\theta)) - R_\theta(x_L(\theta)) \right] G_H(\theta) d\theta$$

Now, recall that:

$$T^L = U(\sigma_L) = \int_\Theta U_L(\theta)g_L(\theta)d\theta = U_L(\overline{\theta}) - \int_\Theta R_\theta(x_L(\theta))G_L(\theta)d\theta$$

where integration by parts has been used again. Substituting and rearranging yields:

$$T^H = \int_\Theta R_\theta(x_H(\theta))d\theta + T^L - U(\overline{\theta}|\sigma_L) - \int_\Theta \left(R_\theta(x_H(\theta)) - R_\theta(x_L(\theta))\right) G_H(\theta)d\theta$$

Rearranging yields:

$$\begin{aligned} T^H - T^L &= \int_\Theta R_\theta(x_H(\theta))d\theta - \int_\Theta R_\theta(x_L(\theta))d\theta - \int_\Theta \left(R_\theta(x_H(\theta)) - R_\theta(x_L(\theta))\right) G_H(\theta)d\theta \\ &= \int_{\Theta 2} \left(R_\theta(x_H(\theta)) - R_\theta(x_L(\theta))\right)(1 - G_H(\theta))d\theta \end{aligned}$$

Rearranging and noting from Proposition 1 that $R_\theta(x_L(\theta)) - R_\theta(x_H(\theta)) > 0$ for all θ, gives the required result:

$$T^H = T^L - \int_\Theta \left(R_\theta(x_L(\theta)) - R_\theta(x_H(\theta))\right)(1 - G_H(\theta))d\theta$$

From Proposition 1(i), $x_L(\theta) \geq x_H(\theta)$. Hence, $T^H \leq T^L$. ■

References

[1] Blair, B.F. and T.R. Lewis, (1994), "Optimal Retail Contracts with Asymmetric Information and Moral Hazard," *The RAND Journal of Economics*, Vol. 25(2), pp. 284-296.

[2] Crocker, K.T., (1983), "Vertical Integration and the Strategic use of Private Information," *The Bell Journal of Economics*, Vol. 14(1), pp. 236-248.

[3] Fu, Q. and J. Lu (2010), "Contest Design and Optimal Endogenous Entry," *Economic Enquiry*, Vol. 48, pp. 80-88.

[4] Hart, O. and J. Tirole (1988), "Contract Renegotiation and Coasian Dynamics," *Review of Economic Studies*, Vol. 55, pp. 509-540.

[5] Laffont, J-J. and D. Martimort, (2002), *The Theory of Incentives: The Principal-Agent Model*, Princeton University Press.

[6] Laffont, J-J. and J. Tirole (1988), "The Dynamics of Incentive Contracts," *Econometrica*, Vol. 56, pp. 1153-1175.

[7] Leonard, D. and N.V. Long (1992), *Optimal Control Theory and Static Optimisation in Economics*, Cambridge University Press.

[8] Martimort, D. and S. Piccolo, (2007), "Resale Price Maintenance Under Asymmetric Information," *International Journal of Industrial Organization*, Vol. 25, pp. 315-339.

[9] Mathewson, G.F and R.A. Winter, (1985), "The Economics of Franchise Contracts," *Journal of Law and Economics*, Vol. 28, pp. 503-526.

[10] Rey, P. and J. Tirole, (1986), "The Logic of Vertical Restraints," *The American Economic Review*, Vol. 76(5), pp. 921-939.

[11] Sappington, D., (1983), "Limited Liability Contracts Between Principal and Agent," *Journal of Economic Theory*, Vol. 29, pp. 1-21.

[12] Spengler, L.G., (1950), "Vertical Integration and Anti-trust Policy," *Journal of Political Economy*, Vol. 63, pp. 347-352.

[13] Tirole, J., (2000), *The Theory of Industrial Organization*, MIT Press.

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