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Quality, Market Structure and Externalities

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Abstract

This paper extends the full information model of goods quality (Spence, 1975 and Sheshinski, 1976) to allow for oligopolistic competition and the presence of two types of externalities caused by production. The choice of output and quality per item under monopoly, competition and Cournot duopoly are derived. These are compared to the efficient levels of output and quality per item. (Firms output is interpreted as being units of quality, and these are chosen according to familiar rules. The choice of quality per item is related to the level of quality that minimises the private cost of producing the optimal number of units of quality.) The efficient tax levels are derived. In general a tax on both output and quality per item are required to ensure efficiency.

JEL Classification: L11

QUALITY, MARKET STRUCTURE AND EXTERNALITIES

This paper concerns the firm's choice of quality of their goods under perfect information. The pioneering analysis of quality choice was conducted during the 1970s This work largely considered the relationship between the monopolists choice of goods' quality and the efficient level, with an eye to the regulation of public utilities. Spence (1975) and Sheshinski, (1976) provided a systematic treatment of the regulation of monopolies when quality is endogenous. Subsequent development of their fundamental models has been sporadic.[1] The aim of this paper is to extend these models, in a systematic way, to incorporate the effects of oligopolistic competition and the presence of two types of externality that commonly accompany the provision of goods.

The first type of externality occurs when an increase in total consumption of a good lowers the perceived quality of that good. For example, an increase in the level of car ownership can lead to road congestion that, in turn, lowers the value of car ownership. These types of externalities are called the consumer to consumer (c-c externality).[2] The second type of externality occurs when total consumption of a good lowers the value of activities that are not related to the consumption of that good. For example increased car ownership may result in increased air pollution. This type of externality is referred to as the non-consumption (n-c) externality.

Following Spence and Sheshinksi it is assumed the quality of firms' goods is both common knowledge and common to all consumers. Spence and Sheshinksi implicitly assume consumer utility is an arbitrary function of the number of items consumed and quality per item. In this paper consumer utility is restricted to be a function of the number

[1] In order to make analytic progress the literature adopts simplifying assumptions such consumers only demand one unit of the good. See for example Mussa and Rosen (1978) and Shaked and Sutton (1983). However variation in demand per customer is a fundamental characteristic of many market, for example those supplied by public utilities.

[2] This paper assumes negative externalities. However network externalities are could be considered a positive c-c externality. For example, an increase in the level of computer ownership can, though extending the reach of the Internet, increase the value of computer ownership.

of units of quality consumed: ie the product of quality per item and number of items (quantity) consumed. (Levari and Peles, 1973, and Kihlstrom and Levari, 1977).[3] The restriction is often realistic. Many, if not most, goods have the property that consumer benefit is related to the overall pleasure (units of quality) gained from consumption rather than the quality per item alone or number of items alone. For example, the utility gained from drinking a particular wine is related to product the number of bottles consumed and the quality of each bottle.

Under this assumption, it is natural to think of consumers choosing their optimal number of units of quality and firms their profit maximizing price of quality, rather than the number of items and price per item respectively. Cost can be written as a function of units of quality (uoq) and quality per item (qpi). By conducting the analysis this way it is readily established that (i) the monopoly and efficient uoq are chosen according to the usual 'equating at the margin' rules and (ii) both the monopoly and efficient levels of qpi are those levels that minimize the cost of producing the relevant number of uoq (Levari and Peles, 1973). There exists a cost minimizing level of qpi because an increase in qpi has two effects on cost. One is the direct cost of producing increased qpi. The second effect is that the quantity required to produce a given number of uoq is reduced, thereby reducing cost. Cost is minimized at the qpi where the former effect just outweighs the latter effect.

The relationship between the efficient and monopoly levels of qpi therefore depends on how the cost minimizing level of qpi varies with number of items (noi). The cost minimizing level of qpi is independent of the number of uoq when the cost function exhibits 'constant returns to scale'. This assumption on the cost function is usually reasonable when qpi is rival. In this case a monopoly produces the efficient level of qpi. This conclusion is known as the 'Swan independence result'. On the other hand when cost is additively separable, and marginal cost is increasing in both qpi and quantity, the cost minimizing level of qpi increases with noi. This assumption is often warranted when

[3] The details of consumers utility, and the demand derived from that, is given in Appendix A. Technically, for the results of this paper to hold, all is required is that utility can be written as a function of the product of average qpi and noi consumed. Thus the formulation is more general than described in these introductory remarks. Sibly (2003) argues that this formulation is often a reasonable restriction on the utility function.

qpi is non-rival, and means the monopolist (in the absence of externalities) produces a level of qpi that is less than the efficient level.

In the model developed in this paper the provision of the good differs from the efficient level because of the presence of three types of distortion: (i) the market power of firms, (ii) the c-c externality and (iii) the n-c externality. It is shown that in the absence of the two externalities oligopolistic firms chooses the cost minimising level of qpi. Therefore an oligopoly produces the efficient level of qpi under the conditions of the Swan invariance principle, but under provide qpi when cost is additively separable.

The externalities considered in this paper are detrimental to welfare, and increase with the total noi. Thus, for a given number of uoq produced, externalities reduce the marginal social cost of qpi. Consequently, for each number of uoq produced, firms under provide qpi. By increasing qpi, a lower noi is required to achieve a given number of uoq

Market structure influences the degree of under provision of quality per unit caused by the c-c externality. The presence of the c-c externality means a firm's choice of noi affects the perceived qpi produced by all other firms. Two cost functions are defined: (i) private cost, which does not internalise this externality and (ii) market cost, which does internalise this externality. Firms do not take account of the impact of their noi decision on the qpi of other firms, and therefore choose their qpi to minimise private cost. There is no distinction between private and market cost for the monopolist, it therefore fully internalises the c-c externality. However if firms face some competition, they do not fully internalise the c-c externality. Increased competition reduces the extent to which the firm internalises the c-c externality, which thereby provides and incentive to increase noi. This increased noi means less qpi is required to provide the profit maximising number of uoq. Consequently increased competition has the effect of reducing the private cost minimising level of qpi. Therefore under the conditions of the Swan independence result, increased competition leads to increased under provision of qpi. If however cost is additively separable, the impact of increased competition is ambiguous. An increase in noi increases the market cost minimising level of qpi, but externalising of the c-c externality reduces qpi.

The n-c externality is not internalised under any market structure. Therefore its presence tends to increase the efficient level of qpi irrespective of market structure. In particular, under the conditions of the invariance principle a monopolist under provides qpi.

In general the market produces neither the efficient uoq nor qpi. An efficient outcome can be achieved by applying taxes. Usually two taxes must be applied: for example one on each uoq and one on the qpi. The sign of the efficient tax on uoq is ambiguous for the usual reason: oligopoly inefficiently restricts the uoq but the presence of externalities means the market over provides uoq. The efficient tax on qpi however is usually negative, because firms inefficiently under provide qpi. However if the market is monopolistic and the n-c externality is not present, the efficient tax on qpi is zero. This is because the c-c externality is internalised by the monopolist. In this case only one tax, levied on each uoq, is required to ensure efficiency.

Network externalities are an example of positive c-c externality. Lambertini and Orsini (2001) consider the provision of qpi by a single product monopolist in the presence of a network externality. Their model differs from the one analyzed in the is paper in that (a) their utility function is of a more restrictive (linear) form, (b) their cost function is more restrictive (constant returns to scale) and (c) the network externality in Lambertini and Orisini appears directly, and linearly, in the utility function of consumers, whereas in this paper the c-c externality influences the qpi in a more general way. The assumptions used by Lambertini and Orsini do not allow the formulation of their model in terms of choice of uoq and qpi as is done in this paper. Nonetheless, under their assumptions Lambertini and Orsini show that network externalities increase the tendency of monopolists toward over provision of qpi, a result which is consistent with those obtained in this paper.

Spence and Sheshinksi's model of quality choice is closely related to Swan's model of durability choice. The primary difference is that the former is a static model, while the latter models the dynamics of product durability. The Spence and Sheshinksi model is therefore best suited to describing the choice of qpi for non-durable goods or services. However the model of quality per choice can act as a guide to durability choice, particularly when the details of the dynamic adjustment of durability are not important. This is often the case when, as is done by many authors, only changes to the steady state level of durability are analyzed.

A number of studies have extended Swan's model of durability choice of a monopolist to incorporate externalities and oligopolistic competition. Auernheimer and Saving (1977) consider the provision of durability in the presence of pecuniary externalities, and conclude that monopoly produces the efficient level of durability. However Abel (1983 p.631) models the impact of a non-pecuniary externality in production, and concludes that a competitive industry produces a lower level of quality than the monopoly and efficient level of durability.

Bulow (1986) shows (using a 2 period model) that an oligopolist who sells its output has an incentive to over provide durability to reduce its competitor's market share in the second period.[4] The static model presented in this paper does not allow for dynamic aspects in the competition for customers. Georing and Boyce (1999) consider an oligopoly that creates emissions as a by-product of production, and analyse the impact of an emissions tax on durability. They show that market structure affects durability in the presence of an emission tax, and that an emissions tax increases durability.

The market structures and externality types discussed in the above literature are special cases of those considered in this paper. The approach of this paper provides a structure in which the results of the literature can be interpreted.

Section 1 of the paper provides the foundation for the analysis by introducing the technology used by the firm. Section 1.1 indicates how customer perceptions of the firm's qpi is related to the effort the firm undertakes to raise the qpi (technical quality) and the number of items produced (via the c-c externality). Section 1.2 defines the firm's cost function. Cost is a function of quantity and qpi but it is useful to write it as a function of uoq and qpi. This formulation of cost allows straightforward identification of the level of qpi that minimizes the firm's private cost for a given number of uoq. The total cost of producing market output (market cost) is defined in section 1.3. Market cost differs from private cost because the former internalizes the c-c externality whereas the latter does not. The case in which all firms have identical cost functions is considered in section 1.4.

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[4] However Georing (1992) demonstrates that an oligopolist produces the efficient level of durability when it rents its output, faces linear demand, and has constant returns to scale technology.

Section 2 considers the market outcome. In section 2.1 the quality-quantity choice of a Cournot oligopolist is derived. Monopoly and perfect competition are special cases of this model. Section 2.2 derives market equilibrium qpi under the assumption that all firms have identical cost functions. The role of the cost function and impact of increased competition on equilibrium qpi is considered.

Section 3 considers the efficient outcome Section 3.1 derives the efficient outcome. The efficient level of quality is chosen to minimize the social cost (the sum of market cost and the cost of the n-c externality) of producing the efficient number of uoq. Section 3.2 compares the efficient outcome to the market outcome. Section 4 considers corrective policies. Section 5 concludes the paper

1. The Model

1.1 Quality per item

Suppose there are j firms, where $j \in [1,2,..n]$. It is assumed that firm j's perceived qpi is given by $q(X,y^j)$, where X is the total noi produced, y^j the technical qpi produced by firm j, $q_1 \leq 0$ and $q_2 \geq 0$. Technical qpi is an objective measure of the firm's effort to set qpi of its good. Allowing for a distinction between technical and perceived qpi admits the possibility that consumers' perception of qpi exhibit diminishing returns to the firm's efforts to raise quality. The c-c externality is captured by allowing perceived qpi to be adversely affected by an increase in total noi, given the level of technical qpi. Firm j's perceived qpi is common to all consumers and is exogenous from the point of view of individual consumers.

The firm j's production of uoq, x^j, is defined by:

$$x^j = X^j q(X,y^j) \qquad (1)$$

where X^j is the noi produced by firm j and $X = \sum_{i=1}^{n} X^i$. Using the implicit function theorem,

(1) yields $X^j(x^j,y^j,X^{-j})$ where $X^{-j} = \sum_{i \neq j} X^i$. Taking the total derivate of (1) yields the following partial derivates:

$$X_1^j(x^j,y^j,X^{-j}) = \frac{1}{q(X,y^j)+Xq_1(X,y^j)} = \frac{1}{q(X,y^j)(1+(X^j/X)\varepsilon_{qX}^j)} \qquad (2)$$

and

$$X_2^j(x^j,y^j,X^{-j}) = \frac{-X^j q_2(X,y^j)}{q(X,y^j)+X^j q_1(X,y^j)} = \frac{-X^j q_2(X,y^j)}{q(X,y^j)(1+(X^j/X)\varepsilon_{qX}^j)} \qquad (3)$$

where $\varepsilon_{qX}^j = Xq_1(X,y^j)/q(X,y^j)$ is the elasticity of the perceived qpi of firm j's product with respect to the total noi. Observe that $X_2^j < 0$ as an increase in qpi must be accompanied by a decrease in the noi to hold uoq fixed.

The analysis below is conducted in terms of technical qpi (rather than perceived qpi) so, for brevity, technical qpi is referred to simply as qpi. At times it is useful to assume that qpi is multiplicatively separable:

$$q(X,y) = \rho(X)\theta(y) \qquad (4)$$

where $\rho'(X)\leq 0$ and $\theta'(y)>0$. Then $\varepsilon_{qX}=\varepsilon_{\rho X}(X)\equiv X\rho'(X)/\rho(X)$ and $\varepsilon_{qy}=\varepsilon_{\theta y}(y)\equiv y\theta'(y)/\theta(y)$. The following condition proves particularly useful:

Isoelastic qpi condition: Suppose qpi is multiplicatively separable and $\varepsilon_{\rho X}(X) = -\rho$ and $\varepsilon_{\theta y}(y) = \theta$ with $\rho>0$ and $\theta>0$.

1.2 Private Costs

Firm j's technology is summarized by the cost function $C^j(X^j,y^j)$, ie the total cost of production is a function of the firm's production of noi and qpi. It is assumed that marginal cost, $C^j_1(X^j,y^j)$, and the marginal cost of qpi, $C^j_2(X^j,y^j)$, are non-decreasing, ie $C^j_1 \geq 0$ and $C^j_2 \geq 0$. The cost function can be expressed as a function of uoq and qpi, $c^j(x^j,y^j,X^{-j})$, in the following way:

$$c^j(x^j,y^j,X^{-j}) \equiv C^j(X^j(x^j,y^j,X^{-j}),y^j) \qquad (5)$$

If the uoq produced by firm j (and the noi and qpi of other firms) are held constant, $c^j(x^j,y^j,X^{-j})$ represents as the cost to firm j as a function of its qpi. Assume that there exists a level of qpi that minimizes the cost of production given the value of x^j and X^{-j} . The cost minimization problem is:

$$\min_{y^j} \ c^j(x^j,y^j,X^{-j}) \Leftrightarrow \min_{y^j} \ C^j(X^j(x^j,y^j,X^{-j}),y^j) \qquad (6)$$

To ensure that c^j has a unique minimum in y^j (when x^j and X^{-j} are assumed fixed) assume that $c^j_{22}(x^j,y^j,X^{-j})>0$. The first order condition of (6) is:

$$c_2^j(x^j,y^j) = \frac{-X^j C_1^j\, q_2}{q(1+(x^j/x)\varepsilon_{qX}^j)} + C_2^j = 0 \qquad (7)$$

or:

$$\frac{y^j C_2^j(X^j,y^j)}{\varepsilon_{qy}^j} = \frac{X^j C_1^j(X^j,y^j)}{1+(x^j/x)\varepsilon_{qX}^j} \qquad (8)$$

The cost function summaries not only the technology of the firm, but also the technology of use. For example, the qpi of a good exhibits either non-rivalness or rivalness. Examples of the first type of good are roads (indeed infrastructure in general) or advertising. A unit improvement in the qpi of road (or advertising) automatically improves each item, irrespective of use level. A car is an example of the second type of good. An improvement in qpi of one item does not automatically raise the qpi of other items.

The cost of qpi is dependent on the number of items produced for goods in which qpi has a rival nature. This is often modeled by the assumption of 'constant returns to scale'; that is, the total cost of qpi is linearly related to the number of items. In general constant returns to scale can be written:

$$C^j(X^j,y^j) = X^j(\omega^j(X^j) + \zeta^j(y^j)) \qquad (9)$$

where $\omega^j(X^j)$ is the production cost of the item and $\zeta^j(y^j)$ is the cost of qpi. The special case in which there is constant marginal cost is captured by the assumption that $\omega^j(X^j)$ is independent of noi. Increasing marginal cost occurs when $\omega^{j\prime\prime}(X^j)>0$.

Goods in which qpi has a rival nature might also satisfy the multiplicatively separable cost function:

$$C^j(X^j,y^j) = \chi^j(X^j)\psi^j(y^j) \qquad (10)$$

where $\chi^{j\prime}(X^j)>0$ and $\psi^{j\prime}(y^j)>0$. The multiplicatively separable form (10) encompasses the special case of constant returns to scale with constant marginal cost by assuming

$\chi^j(X^j)=X^j$ and $\psi^j(y^j) = \omega^j+\zeta^j(y^j)$, where ω^j represents constant marginal cost and $\zeta^j(y^j)$ the cost of qpi. If (10) holds, the private cost minimizing qpi (16) satisfies:

$$\frac{\varepsilon^j_{\psi y}(y^j)}{\varepsilon_{\theta y}(y^j)} = \frac{\varepsilon^j_{\chi X}(X^j)}{(1+ (X^j/X)\varepsilon_{\rho X}(X^j))} \tag{11}$$

where $\varepsilon^j_{\chi X}(X^j)\equiv X^j\chi^{j\prime}(X^j)/\chi(X^j)$ and $\varepsilon^j_{\psi y}(y^j)\equiv y^j\psi^{j\prime}(y^j)/\psi(y^j)$. Observe that the cost minimizing qpi, y*, is independent of the number of items if the RHS of (11) is independent X^j. Note that if $\varepsilon^j_{\rho X}(X^j)=0$ (the c-c externality is not present) that this can only occur if $\varepsilon^j_{\chi X}(X^j)$ is a constant.

Goods that exhibit non-rivalness in qpi may be modeled by assuming the cost function is additively separable:

$$C^j(X^j,y^j) = \Phi^j(X^j) + \Psi^j(y^j) \tag{12}$$

where $\Phi^{j\prime}(X^j)>0$, $\Psi^{j\prime}(y^j)>0$, $\Phi^{j\prime\prime}(X^j)>0$ and $\Psi^{j\prime\prime}(y^j)>0$. The cost associated with qpi is independent of noi. In this case, using (16) the cost minimizing qpi satisfies:

$$\frac{y^j\Psi^{j\prime}(y)}{\varepsilon_{\theta y}(y^j)} = \frac{X^j\Phi^{j\prime}(X^j)}{(1+(X^j/X)\varepsilon^j_{\rho X}(X^j))} \tag{13}$$

where it has been assumed qpi is multiplicatively separable as in (4). If $\varepsilon^j_{\theta y}(y^j)$ and $\varepsilon^j_{\rho X}(X^j)$ are non-increasing functions, then differentiation of (13) demonstrates that an increase in the number of items increases the cost minimizing qpi. Note that if the cost minimizing qpi is increasing with X^j, then it is also increasing with x^j.

1.3 Market Costs

Define market cost by $C(X,y) = \sum_{i=1}^{n} C^j(X^j,y)$ such that $C_1^j(X^j,y)= C_1^k(X^k,y)$ for all $j,k\in[1,..n]$ and $X = \sum_{j=1}^{n} X^j$. Market cost represents the most efficient manner of producing X noi which have technical qpi y for a given market structure (ie a given n).[5] Total uoq, x, is given by:

$$x = \sum_{j=1}^{n} x^j = \sum_{j=1}^{n} X^j q(X,y) = Xq(X,y) \tag{14}$$

By the implicit function theorem the noi may be written as X(x,y), and thus market cost may be written as:

$$c(x,y) = C(X(x,y),y) \tag{15}$$

Market cost is minimized when:

$$\frac{yC_2}{\varepsilon_{qy}} = \frac{XC_1}{(1+\varepsilon_{qX})} \tag{16}$$

This yields:

> <u>Proposition 1</u>: Suppose a firm supplies less than the whole market ($X^i<X$). Then the qpi that minimizes its private cost is less than the qpi that minimizes market cost, for a given number of uoq.

Proof: By definition the market cost minimizing level of X^j, y^j minimize the cost of producing $\hat{x}$, a given number of uoq.

$$L = C(X,y) + \mu(\hat{x} - Xq(X,y))= \sum_{i=1}^{n}[C^j(X^j,y)- \mu X^j q(X,y^j)] + \mu\hat{x} \tag{17}$$

[5] In general it may be more efficient to allow different firms produce different qpi. However allowing for this possibility introduces additional technical complications that are tangential to the analysis in this paper. Consequently this possibility is not admitted into the analysis.

The first order conditions for cost minimization, $\partial L/\partial X^j = 0$ and $\partial L/\partial y^j = 0$ yield:

$$C_2^j - \frac{\varepsilon_{qy}^j}{y}\left(\frac{X^j C_1^j}{(1+\varepsilon_{qX}^j)}\right) = 0 \tag{18}$$

From (7) private cost satisfies:

$$c_2^j(x^j,y^j) = C_2^j - \frac{X^j C_1^j\, q_2}{q(1+(x^j/x)\varepsilon_{qX}^j)} = C_2^j - \frac{X^j C_1^j\, q_2}{q(1+\varepsilon_{qX}^j)} - \frac{X^j q_2 C_1^j (1-x^j/x)\,\varepsilon_{qX}^j}{q(1+(x^j/x)\varepsilon_{qX}^j)\,(1+(x^j/x)\varepsilon_{qX}^j)} \tag{19}$$

Thus, when market cost is minimized, the private cost minimizing qpi satisfies:

$$c_2^j(x^j,y^j) = -\frac{\varepsilon_{qy}^j}{y}\left(\frac{X^j C_1^j (1-X^j/X)\,\varepsilon_{qX}^j}{(1+\varepsilon_{qX}^j)\,(1+(X^j/X)\varepsilon_{qX}^j)}\right) \tag{20}$$

where $\varepsilon_{qy}^j = y^j q_2(X,y^j)/q(X,y^j)$. The RHS of (20) is positive provided $X^j/X<1$. As it is assumed that $c_{22}^j(x^j,y^j)>0$, the market cost minimizing qpi is greater than the private cost minimizing qpi. However if the firm is a monopoly, so that $X^j/X = 1$, then the market cost minimizing qpi coincides with the private cost minimizing qpi. ||

Intuitively, market cost internalizes the presence of the c-c externality. Marginal cost in (16) is therefore fully discounted, by ε_{qX}, to account for its presence. However the private cost of firm j only partially internalizes the c-c externality. Hence marginal cost in (8) is only partially discounted, by $(X^j/X)\varepsilon_{qX}$, as a result of its presence.

When the market is supplied by a monopoly, there is obviously no distinction between private and market cost. A monopoly therefore fully internalizes the c-c externality.

1.4 Identical private costs

When discussing market equilibrium it is useful to assume that all firms have identical private cost functions, ie $c^j(x^j,y^j) = \bar{c}(x^j,y^j)$ or, equivalently, $C^j(X^j,y^j) = \bar{C}(X^j,y^j)$ for all j. Assume all firms produce an identical noi $X^j = \bar{X} = X/n$, identical uoq $x^j = \bar{x} = x/n$ and identical qpi $y^j = y$. Let market cost be related to private cost by $C(X,y) = n\bar{C}(\bar{X},y) = n\bar{C}(X/n,y)$, and $c(x,y) = C(X(x,y),y)$. Thus $C_1(X,y) = \bar{C}_1(\bar{X},y)$ and $C_2(X,y) = n\bar{C}_2(\bar{X},y)$.

To interpret the results of this paper's analysis it is necessary to be able to describe how the cost minimizing qpi relates to changes in both qpi and noi.

<u>Proposition 2</u>: (i) The private and market cost minimizing qpi is constant (increasing) with uoq if it is constant (increasing) with noi. (ii) The private and market cost minimizing qpi is constant (decreasing) with noi if it is constant (decreasing) with uoq.

Proof: The market cost minimizing qpi, y^m, satisfies $c_2(x,y^m) = c_2(Xq(X,y^m),y^m) = 0$. Thus, taking total derivatives along the market cost minimizing qpi curve:

$$dy^m/dx = -c_{21}/c_{22} \qquad (21)$$

and

$$dy^m/dX = -c_{21}(q+Xq_1)/[c_{21}Xq_2+c_{22}] \qquad (22)$$

Thus: (i) If $dy^m/dX = (>) 0$, then $c_{21} = (<) 0$, in which case $dy^m/dx = (>) 0$. (ii) If $dy^m/dx = (<) 0$ when $c_{21} = (>) 0$, in which case $dy^m/dX = (<) 0$. The proof for private cost minimizing qpi is similar. ||

The following condition describes a useful class of cost functions.

<u>Invariance condition</u>: Firms have identical multiplicatively separable cost functions with $\varepsilon_{\chi X}(\bar{X})$ is constant (i.e. $\varepsilon_{\chi X}(\bar{X}) = \chi$ where $\chi>0$) and the isoelastic qpi condition holds.

Note that the constant returns to scale cost function with constant marginal cost satisfies the invariance condition with $\chi = 1$. The following proposition indicates the motivation for the naming of the invariance condition.

Proposition 3: Under the invariance condition (i) private cost minimizing qpi is independent of the uoq produced when firms have equal market share; and, (ii) the market cost minimizing qpi are independent of the uoq produced.[6]

Proof: (i) Under the invariance condition (11) becomes:

$$\frac{\varepsilon_{\psi y}(y)}{\varepsilon_{\theta y}(y)} = \frac{\chi}{1-\rho/n} \qquad (23)$$

where each firm's market share $X^j/X = 1/n$. Equation (23) does not involve the number of items, so the private cost minimizing qpi is independent of X.

(ii) Under the invariance condition (16) becomes:

$$\frac{\varepsilon_{\psi y}(y)}{\varepsilon_{\theta y}(y)} = \frac{\chi}{1-\rho} \qquad (24)$$

so the market cost minimizing qpi is independent of X. ||

The following condition specifies a second useful class of cost functions:

Co-variance condition: Suppose firms have identical private cost functions that either satisfy constant returns to scale with increasing marginal cost or are additively separable. Further suppose isoelastic qpi condition holds.

The following proposition indicates the motivation for the naming of the co-variance condition.

Proposition 4: Under the co-variance condition (i) the market cost minimizing qpi is positively related to the noi produced and (ii) the private cost minimizing qpi is positively related to the noi produced when firms have equal market share.

[6] It is readily shown that the condition that $\varepsilon_{\theta y}(y)$ be constant is not necessary for proposition 4 to hold.

Proof: (i) Follows from substituting (9) into (8) (constant returns) or by substituting (13) into (8) (additively separable) and taking the total derivative. (ii) Follows by substitution of relevant cost function into (16) and taking the total derivative.||

The market and private cost minimizing qpi when firms have identical cost functions can be depicted graphically using figures 1 and 2. Proposition 1 can be illustrated using figure 1. In figure 1 the private marginal cost of qpi curve is labeled $\overline{c}_2(\overline{x}.y)$. The market marginal cost of qpi curve, $c_2(x,y)$ lies below private marginal cost of qpi curve when $\overline{c}_2(\overline{x},y) = 0$ because the firm does not fully internalize the c-c externality.[7] The private cost minimizing qpi, $y^*(\overline{x})$, is the point at which the private marginal cost of qpi cuts the horizontal axis, which is to the left of $y^m(\overline{x})$, the market cost minimizing qpi.

The relationship between private cost minimizing qpi, $y^*(\overline{x})$ market cost minimizing qpi, $y^m(\overline{x})$, and uoq is depicted in figure 2. Proposition 1 indicates that $y^*(\overline{x})$ must lie to the left of $y^m(\overline{x})$, as is depicted in figure 2.. It is assumed the cost function depicted in figure 2 satisfies the co-variance condition so, by propositions 2 and 4, the curves are upward sloping. If, instead, the invariance condition held, both $y^*(\overline{x})$ and $y^m(\overline{x})$ would be vertical lines.

[7] This is shown formally in the proof to proposition 8.

2. Cournot Oligopoly

Consumers gain utility from the number of uoq (the product of perceived qpi and noi) consumed. Consumers view a unit of quality from each firm as perfect substitutes, and purchases from the firm that has the lowest price of quality.

In this way a single market price of quality is established. It is shown in the appendix that a market demand for uoq, x(p), is a function of p, the price of quality. Integrating the market demand for uoq yields the consumer surplus, v(p).[8]
Note that $v'(p) = -x$ and therefore $\partial v/\partial P = -X$. Total consumer surplus may also be written as a function of uoq consumed, V(x), by substitution of the inverse uoq demand function:

$$V(x) \equiv v(p(x)) \qquad (25)$$

Note that $V'(x) = v'(p).p'(x) = -p'(x)x = p/\varepsilon_x$.

2.1 The firm's choice of uoq and qpi

In this section, the firm's choice of uoq and qpi are considered under Cournot equilibrium. Under Cournot oligopoly each firm chooses the number of uoq to supply to the market, and then the price of quality moves to clear the market. Thus the price of quality under oligopoly is p(x). The profit of firm j is:

$$\pi^j(x^j,y^j,X^{-j},x^{-j}) = p(x)x^j - c^j(x^j,y^j) = p(x)x^j - C^j(X^j(x^j,y^j,X^{-j}),y^j) \qquad (26)$$

where $x^{-j} = \sum_{i\neq j} x^i$. Under Cournot equilibrium firms choose uoq and qpi to maximize profit given the noi and qpi produced by other firms. The first order condition for profit maximization, $\pi^j_1 = -0$, yields:

[8] The surplus provides a good approximation of welfare when income effects are negligible, in particular when PX^i is a small fraction of the consumer's income (see Tirole, 1988, p. 11).

$$p = \frac{c^j_1 .\ \varepsilon_x}{(\varepsilon_x - (x^j/x))} \Leftrightarrow P^j = \frac{C^j_1\ \varepsilon_x}{(\varepsilon_x - (X^j/X))(1 + (X^j/X)\varepsilon^j_{qX})} \qquad (27)$$

where $\varepsilon^j_{qX} = xq_1(x,y^j)/q(x,y^j)$. The oligopolist chooses uoq as a textbook oligopolist would choose output: the price of quality is mark-up on private marginal cost (of uoq). Note that, as the oligopolist sells only to a fraction, x^j/x, of the market, the private cost function only partially internalizes the c-c externality.

Monopoly is the special case of (27) when $x^j/x = 1$. In this case

$$p = \frac{c_1 .\ \varepsilon_x}{(\varepsilon_x - 1)} \Leftrightarrow P = \frac{C_1 .\ \varepsilon_x}{(\varepsilon_x - 1)(1 + \varepsilon_{qX})} \qquad (28)$$

Because the monopolist sells to the entire market it fully internalizes the c-c externality in its calculation of marginal cost. Perfect competition is the special case when $x^j/x = 0$. In this case

$$p = c^j_1(x^j,y^j) \Leftrightarrow P^j = C^j_1 \qquad (29)$$

The perfectly competitive firm sets noi such that price of quality equals it private marginal cost (of uoq), and thus takes no account c-c externality in setting noi.

The following proposition describes the firm's qpi choice.

<u>Proposition 5</u>: Each firm's profit maximizing qpi is it's private cost minimizing qpi.

Proof: The first order condition, $\pi^j_2 = 0$ is equivalent to:

$$\min_{y^j} c^j(x^j,y^j) \Leftrightarrow \min_{y^j} C^j(X^j(x^j,y^j),y^j) \qquad (30)\ \|$$

A monopolist (for whom $x^j = x$) fully internalizes the c-c externality and thus, as demonstrated by proposition 1, produces the qpi that minimizes market cost. As shown by (8) the monopolist fully discounts its private marginal cost for the presence of the c-c externality when choosing qpi. An oligopolist, however, only partially internalize the c-c externality. It is only partially discounts its private marginal cost for the presence of the c-c externality when choosing qpi. Therefore, as indicated by proposition 1, the oligopolist has an incentive to produce an inefficiently low qpi. Firms in perfectly competitive markets do not take account of the presence of the c-c externality. Private marginal cost in (8) is not discounted to allow for the presence of the c-c externality.

The firm's profit maximizing uoq and qpi are jointly determined by (27) and (8). However an alternative characterization of the firm's profit maximizing choice of qpi and quantity is given by the following proposition.

Proposition 6: Firm i's cost revenue ratio is given by:

$$\frac{C^j}{P^j X^j} = \frac{\left(1 - \frac{x^j}{x\varepsilon_x}\right)\varepsilon^j_{qy}}{\varepsilon^j_{cy}} \qquad (31)$$

where $P^j = p(x)/q(X,y^j)$, $\varepsilon^j_{qy} = y^j q_2(X,y^j)/q(X,y^j)$ and $\varepsilon^j_{cy} = y^j C^j_2/C^j$.

Proof: (31) is obtained by substituting (8) into (27). ||

When the firm is a monopoly (31) represents the Dorfman-Stiener condition. Observe that the cost revenue ratio is unaffected by the presence of the c-c externality. Thus proposition 6 provides an alternative characterization of the market qpi and uoq choice. Specifically, the firm chooses its profit maximizing uoq according to (27). The internalized portion of the c-c externality is a factor in this decision. However qpi is chose to yield the cost revenue ratio (31). This ratio increases with (i) increases in the elasticity of the firm's residual demand curve, $x\varepsilon_x/x^j$, and (ii) decreases in the elasticity of cost with respect to qpi.

2.2 Equilibrium qpi

To analyse the market equilibrium, it is now assumed, following section 1.4, that all firms have identical private cost functions. In this case, by (28), the profit maximizing uoq and noi satisfy:

$$p(x) = \frac{\varepsilon_x . \bar{c}_1(\bar{x},\bar{y})}{(\varepsilon_x-(1/n))} \Leftrightarrow P(X,y) = \frac{\varepsilon_x . \bar{C}_1(\bar{X},\bar{y})}{(\varepsilon_x-(1/n))(1+(\varepsilon_{qX}/n))} \qquad (32)$$

Profit maximizing uoq, $\bar{x}^*(y)$, for a given qpi, is defined by (32). The following proposition indicates when the profit maximizing uoq increases with uoq.

<u>Proposition 7</u>: $\bar{x}^{*\prime}(y) > (=,<)\ 0$ if $c_{12} < (=,>)\ 0$.

Proof: Follows from the FOC $\pi_1(\bar{x},y) = 0$ (which is equivalent to (32)) and by assuming the SOC, $\pi_{11}(\bar{x},y)<0$, holds. ||

Profit maximizing qpi satisfies the following:

<u>Proposition 8</u>: When the market is not monopolistic ($n \geq 2$) then, for a given number of uoq, the profit maximizing qpi, $y^*(x)$, is less than $y^m(x)$, the market cost minimizing qpi. When the market is monopolistic ($n = 1$) the profit maximizing qpi is equal to the market cost minimizing qpi.

Proof: By (30), the profit maximizing qpi satisfies $\bar{c}_2(\bar{x},\bar{y}) = 0$, where:

$$\bar{c}_2(\bar{x},y) = \frac{1}{n}\left(c_2(x,y) - \frac{(n-1)q_2\bar{C}_1\bar{X}\varepsilon_{qX}}{q(1+(\varepsilon_{qX}/n)\ (1+\varepsilon_{qX}))} \right) \qquad (33)$$

When the market is supplied by a monopoly ($n = 1$), the market cost is the private cost, and hence $c_2(x,y) = \bar{c}_2(\bar{x},y)$.When, however, the market is oligopolistic ($n \geq 2$), $c_2(x,y) < 0$ when $\bar{c}_2(\bar{x},y) = 0$. ||

Proposition 8 holds because a firm that faces competitors only partially internalizes the c-c externality and, by proposition 5, produces the private cost minimizing qpi. By proposition 1, the market cost minimizing qpi is therefore greater than the profit maximizing qpi. This is shown in figure 1.

It is of interest to know how increased competition affects qpi. However, as proposition 8 assumes a given number of uoq, it cannot be used to draw conclusions concerning the impact of increased competition. Increased competition increases the total uoq produced, which may alter the cost minimizing qpi. However:

> <u>Proposition 9</u>: Under the invariance condition increased competition (i) does not vary qpi if the c-c externality is absent; and, (ii) decreases qpi when the c-c externality is present.

Proof: Follows from (24) and by observing the second order conditions requires:

$$\frac{d}{dy}\left(\frac{\varepsilon_{\psi y}(y)}{\varepsilon_{\theta y}(y)}\right) > 0 \qquad (34) \qquad \|$$

Under the invariance condition the cost minimizing qpi is independent of the scale of the firm. This implies firm's qpi is independent of demand shocks; in particular the shock to demand that arises with additional competition. When the c-c externality is present private cost minimizing qpi is independent of scale. However increased competition reduces the extent to which the firm internalizes the c-c externality, thus reduces the private cost minimizing qpi. The increase in competition, arising from an increase in the number of firms from n to n + 1, can be represented in figure 3. The increased competition is reduces the private cost minimizing qpi from y*(n) to y*(n + 1).

The profit maximizing uoq is depicted by the curve $\overline{x}^*(y,n)$ in figure 3. Using proposition 7 it can be shown the profit maximizing uoq curves have the 'inverted U-shape' shown in figure 3. The increased number of firms can either increase or reduce the firm's production of uoq (for given qpi): market output is increased by increased competition, however the output must be shared between more firms. The impact on a firm's uoq depends on which effect is dominant. In figure 3 it is assumed that an increase in

competition results in a decrease in uoq from $\overline{x}^*(y,n)$ to $\overline{x}^*(y,n+1)$. Clearly the impact of increased competition on qpi is determined by exclusively by changes to the private cost minimizing qpi.

Proposition 9 is elaborated in the following proposition:

Proposition 10: Assume cost takes the isoelastic functional form,

$C(\overline{X},y) = C\overline{X}^{\chi}(\omega+Zy^{\zeta})$, and the isoelastic qpi condition holds. Then the profit maximising qpi level is given by:

$$y^{*} = \left(\frac{\theta\chi\omega}{Z(\zeta(1-\rho/n)-\theta\chi)}\right)^{\frac{1}{\zeta}} \qquad (35)$$

The market cost minimizing qpi is given by (35) with n = 1.

The proof of proposition 10 follows directly from substituting the assumed functional forms into (11). This result starkly represents the interaction between the c-c externality and the level of competition. In particular the impact on qpi of a increase in competition is the inverse of the impact of an increase in the strength of the c-c externality.

If the co-variance condition holds, the impact of an increase in competition is ambiguous. This ambiguity is demonstrated in appendix B by the use of mathematical example. However the intuition behind this ambiguity can be illustrated graphically by figure 4. Initially the private cost minimizing qpi curve is y*(x,n), and the profit maximizing uoq curve is $\overline{x}$*(y,n). The profit maximizing uoq curves are upward sloping when (i) cost is additively separable, and (ii) 'near' the intersection with the private cost minimizing qpi curve when cost exhibits constant returns to scale with increasing marginal cost. In figure 4 the number of firms is shown to increase by one from n to n + 1. The private cost minimizing qpi shifts leftward to y*(x,n + 1). As noted above, the profit maximizing uoq curve may shift either up or down. If the profit maximizing uoq curve shifts downward, it is easily seen that the equilibrium qpi must increase.

However the case in which the profit maximizing uoq curve shifts upward is depicted in figure 4. In this case the impact on equilibrium qpi is ambiguous. The overall impact on equilibrium qpi depends on (i) the relative slope of the private cost minimizing curve and the equilibrium uoq curve and (ii) the extent of the shift of the private cost minimizing curve. The slope of the curve depends on the nature of technology. The shift is a result of two effects which raise c_2, the marginal cost of qpi: (i) the decrease in the noi produced by each firm as a result of an expansion of the number of firms and (ii) the increased number of firms reduces the extent to which the c-c externality is internalized. The extent to which the marginal cost of qpi is increased determines the extent to which the profit maximizing qpi falls. If these effects are strong there is a significant rightward shift in $y^*(\bar{x},n)$ to (say) $y^{*1}(\bar{x},n + 1)$. In this case the increased level of competition has lowered qpi from $y^*(n)$ to $y^{*1}(n + 1)$. However if the shift is not great, the increased competition leads to a small rightward shift in $y^*(\bar{x},n)$ to (say) $y^{*2}(\bar{x},n + 1)$, and thus qpi rises with the consequent increase in qpi from $y^*(n)$ to $y^{*2}(n + 1)$.

In a similar fashion, if the private cost minimizing curves are sufficiently steep (so the invariance condition 'almost' holds) the increased competition reduces qpi. However if technology is such that the private cost minimizing curves are not steep qpi may increase as a result of increased competition.

It should be noted that the ambiguity in the impact of increased competition on profit maximizing qpi is not a result of the presence of the c-c externality. This may be illustrated in figure 5, which shows the choice of qpi of an individual firm in the absence of the c-c externality. An increase in the level of competition does not shift $y^*(\bar{x})$, the profit maximizing qpi curve. However, for the usual reasons, the profit maximizing uoq may be either increased or decreased by the increased level of competition. An increase in the uoq produced is illustrated by the shift of the profit maximizing uoq curve from $\bar{x}^*(y,n)$ to $\bar{x}^{*1}(y,n+1)$, with the consequent increase in qpi from $y^*(n)$ to $y^{*1}(n + 1)$. On the other hand, a reduction in profit maximizing uoq from $\bar{x}^*(y,n)$ to $\bar{x}^{*2}(y,n+1)$ results in a reduction of qpi from $y^*(n)$ to $y^{*2}(n + 1)$.

3. Efficient uoq and qpi

Assumed that there is an external social cost $G(X)$, with $G'(X) > 0$, which depends on the noi produced. Total social cost of production may therefore be written:

$$c(x,y)+ g(x,y) \tag{36}$$

where $g(x,y) = G(X(x,y))$ and thus $g_1 > 0$ and $g_2 < 0$.

3.1 Conditions for Efficiency

The social surplus from consumption, $S(x,y)$, is given by the sum of consumer surplus, profit and the external social benefit:

$$S(x,y) = V(x)+ p(x)x - [c(x,y)+ g(x,y)] \tag{37}$$

The first order condition, $S_1 = 0$, indicates the efficient (surplus maximizing) uoq (given the level of qpi). It yields:

$$p = c_1(x,y) + g_1(x,y) \Leftrightarrow P = (C_1+G')/(1+\varepsilon_{qX}) \tag{38}$$

Thus efficiency occurs when price of quality equals the marginal social cost of a uoq. Note that $C_1 + G'$ represents the marginal social cost of an item. Thus (38) states that noi is efficient when price per item equals marginal social cost, discounted for the presence of the c-c externality. The efficient qpi is given by $S_2 = 0$, which is also given by the qpi that minimizes social cost (for a given number of uoq):

$$\min_y \; c(x,y)+g(x,y) \Leftrightarrow \min_y C(X(x,y),y)+G(X(x,y)) \tag{39}$$

The first order condition for social cost minimization is:

$c_2(x,y) = -g_2(x,y) > 0$ (40)

By the implicit function theorem expression (40) yields $y^e(x)$, the efficient qpi for a given number of uoq. Expression (40) yields:

<u>Proposition 11</u>: For each number of uoq, the efficient qpi is (i) equal to the market cost minimizing qpi when the n-c externality is not present (ie G = 0); and, (ii) is more than the market cost minimizing qpi when the n-c externality is present (ie G' > 0).

Proof: Expression (40) yields:

$C_2 - q_2C_1/(q+Xq_1) = G'q_2/(q+Xq_1)<0$ (41)

As the LHS of (41) is the derivative of the market cost function, $c_2(x,y)$, and it is assumed $c_{22}(x,y) > 0$.Therefore (41) indicates that the efficient qpi is more than the cost minimizing level when G'>0. ||

The relationship between the efficient and the cost minimizing qpi is depicted in figures 1 and 2. In figure 1 the marginal market cost of qpi, $c_2(x,y)$, is depicted for a given number of uoq. The minimum market cost of qpi is $y^m(x)$.The marginal external benefit of qpi, $-g_2(x,y)$, is depicted for a given number of uoq. The marginal external benefit of qpi curve is shown to be positive because, as given by (3), an increase in qpi reduces the required noi to achieve a given number of uoq, and thus decreases external cost. The marginal external benefit of qpi need not be downward sloping, however the second order conditions require that its slope is less than c_2. The efficient level of qpi for a given number of uoq, $y^e(x)$, is given by the qpi level where the marginal cost of qpi curve cuts the marginal external benefit of qpi curve. Thus, $y^e(x)$ is greater than $y^m(x)$ when $g_2 < 0$. The social cost minimizing curve, $y^e(x)$, can then be shown in figure 2 as lying to the right of the market cost minimizing curve.

Condition (41) may be re-written as:

$yC_2/\varepsilon_{qy} = X.(C_1+G')/(1+\varepsilon_{qX})$ (42)

Note that in determining the efficient qpi, the marginal social cost of noi is fully discounted for the presence of the c-c externality. Equation (42) enables the following alternative characterization of the efficient qpi and quantity.

Proposition 12: The efficient cost revenue ratio is given by:

$$C/PX = \varepsilon_{qy}/\varepsilon_{cy} \tag{43}$$

where $\varepsilon_{cy} = yC_2(X,y)/C(X,y)$.

Expression (43) is obtained by substitution of (42) into (38).

Proposition 12 indicates that the cost revenue ratio is independent of the existence or magnitude of both the c-c and n-c externality, as well as the elasticity of demand. Thus uoq is chosen according to (38), which takes into account the presence of the c-c externality and the n-c externality. However qpi is chosen to yield the cost revenue ratio (43).

Observe that the cost revenue ratio under perfect competition has the same functional form as (31) with $\varepsilon_x = \infty$. Thus perfect competition can be thought of as choosing qpi by a rule that produces the efficient cost revenue ratio. However perfect competition yields an inefficient number of uoq due to the presence of the c-c and n-c externalities.

3.2 Market qpi vs. efficient qpi.

From figure 1 it is readily observed that the efficient qpi is greater than the profit maximizing qpi for a given number of uoq. Assume the diagram in figure 1 is drawn for the efficient number of uoq. Then the qpi given by the intersection of the $c_2(x^e,y)$ and $g_2(x^e,y)$ represents the efficient qpi. The gap between the efficient and market qpi depends on how $y^*(x)$ (the private cost minimizing qpi) varies with uoq. However:

Proposition 13: Under the invariance condition the profit maximizing qpi is less than the efficient qpi when the n-c externality is present ($G'(X) > 0$).

Proof: Under the invariance condition the cost minimizing qpi is independent of uoq produced. By proposition 11 the efficient qpi is less than the market cost minimizing qpi. By proposition 9 the firm therefore produces a lower qpi than is efficient. ||

In general little can be said about the relationship between the efficient and market qpi when the invariance condition does not hold. For example suppose the cost function satisfies the co-variance condition, so the private cost minimizing qpi, $y^*(\overline{x})$, is upward sloping as shown in figure 2. By proposition 9, social cost minimizing qpi curve, $y^e(\overline{x})$, lies to the right of $y^m(\overline{x})$, the market cost minimizing qpi curve. From figure 2 it is readily seen that the efficient qpi is greater than the profit maximizing qpi when the efficient uoq is greater than the profit maximizing uoq. For example in figure 2 the profit maximizing uoq $\overline{x}^*$ is less than the assumed efficient uoq $\overline{x}^{e2}$. In this case the efficient qpi, $y^e(\overline{x}^{e2})$ is greater than the profit maximizing qpi $y^*(\overline{x}^*)$. When the efficient uoq is less than the profit maximizing uoq, qpi may be greater or less than the efficient level. For example in figure 2, when the efficient uoq is $\overline{x}^{e2}$, the efficient qpi, $y^e(\overline{x}^{e2})$ is seen to be less than the profit maximizing qpi .

4. Corrective Policies

4.1 Corrective taxes on uoq and qpi

Suppose the government imposes taxes t_x on a uoq and t_y on qpi. Firms choose uoq according to:

$$p = \frac{\varepsilon_x(\overline{c}_1(\overline{x},y) + t_x)}{\varepsilon_x-(1/n)} \qquad (44)$$

and qpi according to:

$$\overline{c}_2(\overline{x},y) = - t_y \qquad (45)$$

The efficient tax on uoq satisfies:

$$t_x = c_1 + g_1 - \frac{\varepsilon_x\overline{c}_1(\overline{x},y)}{\varepsilon_x-(1/n)}$$

$$= \left(1 - \frac{\varepsilon_x(1+\varepsilon_{qX})}{(\varepsilon_x-(1/n))(1+(\varepsilon_{qX}/n))}\right)\frac{C_1(\overline{X},y)}{q(1+\varepsilon_{qX})} + \left(\frac{G'(X)}{q(1+\varepsilon_{qX})}\right) \qquad (46)$$

The sign of the efficient tax is the result of three effects. The presence of market power encourages the representative firm to under produce, and this causes the efficient tax to be negative. However the presence of the n-c externality and the c-c externality (when $n > 1$) creates a tendency toward overproduction, and causes the efficient tax to be positive. Thus when the sign of the efficient tax on uoq is indeterminate. However when the market is competitive ($n = \infty$), there is no need to compensate for the market power of firms, and the efficient tax is unambiguously positive.

The efficient tax on qpi satisfies:

$$t_y = \frac{q_2}{nq(1+\varepsilon_{qX})}\left(\frac{(n-1)\varepsilon_{qX}\overline{C}_1}{1+(\varepsilon_{qX}/n)} - G'(X)\right) \tag{47}$$

Observe the sign of the efficient tax on qpi is negative. The reason is given by reference to figure 1. The tax on uoq ensures its efficient level, $\overline{x}^e$, is produced. Without a tax on qpi the qpi produced is $\overline{y}^*(\overline{x}^e)$. The efficient tax on qpi must be negative to induce firms to increase qpi to $y^e(\overline{x}^e)$, the efficient qpi level.

The two taxes specified by (46) and (47) are usually both required to achieve efficiency. The only instances where a single tax could be used to achieve efficiency are when the n-c externality is not present and either (i) there is a monopoly or (ii) the c-c externality is also not present.

4.2 Corrective taxes on noi and qpi

Suppose the government imposes taxes τ_X the noi and τ_y on qpi. Firms choose noi according to:

$$P = \frac{\varepsilon_x.(\overline{C}_1(\overline{X},y) + \tau_X)}{(\varepsilon_x-(1/n))(1+(\varepsilon_{qX}/n))} \tag{48}$$

In order to achieve the efficient price per item it is necessary to set the tax on noi as:

$$\tau_X = \left(\frac{(\varepsilon_x-(1/n))(1+(\varepsilon_{qX}/n))}{\varepsilon_x(1+\varepsilon_{qX})} - 1\right)C_1(\overline{X},y) + \left(\frac{(\varepsilon_x-(1/n))(1+(\varepsilon_{qX}/n))}{\varepsilon_x(1+\varepsilon_{qX})}\right)G'(X)$$

$$= \left(\frac{q(\varepsilon_x-(1/n))(1+(\varepsilon_{qX}/n))}{\varepsilon_x}\right)t_x \tag{49}$$

Thus the sign of the efficient tax on noi is the same as the efficient tax on uoq. However, the magnitudes of the taxes differ, as a tax on noi has an affect on the firm's choice of qpi.

Firms choose qpi according to:

$$\frac{y(\overline{C}_2(\overline{X},y)+\tau_y)}{\varepsilon_{qy}} = \frac{\overline{X}(\overline{C}_1(\overline{X},y)+\tau_X)}{1+\varepsilon_{qX}/n} \tag{50}$$

Thus the efficient qpi is achieved by setting a tax:

$$\tau_y = \frac{q_2}{nq(1+\varepsilon_{qX})}\left(\frac{n(1+\varepsilon_{qX})\tau_X}{1+(\varepsilon_{qX}/n)} - G'(X) + \frac{(n-1)\varepsilon_{qX}\overline{C}_1}{1+(\varepsilon_{qX}/n)}\right) \tag{51}$$

$$= \frac{q_2\tau_X}{nq(1+(\varepsilon_{qX}/n))} + t_y \tag{52}$$

The tax on qpi, when noi is taxed, differs from the tax on qpi when uoq is taxed. A tax on noi affects the firm's choice of qpi, and this must be compensated for in the tax on qpi. For example, if a positive tax is imposed on the noi, the firm's noi is reduced but its qpi is increased. The efficient tax on qpi must be increased by to compensate for this increase.

From (52), there exists, in principle, a market structure (ie value of n) for which $\tau_y = 0$. However it is extremely unlikely the market structure is such that the effect of the c-c externality and n-c externality exactly offsets the effect of the tax on noi. So in practice, efficiency can only be achieved by a combination of both a tax on noi and a tax on qpi.

4.3 Price ceilings

Kihlstrom and Levahi (1977) show that, in the absence of externalities, if a ceiling on price per item is imposed, a monopoly does not produce the efficient qpi. However the monopoly can be induced to produce the efficient qpi and quantity by setting a maximum price of quality rather than a maximum price per item.

In this section we consider the impact of a price ceiling, $\overline{p}$, on the price of quality. Firm j's profit is:

$$\pi^j(x^j,y^j) = \overline{p}x^j - \overline{c}(x^j,y^j) \tag{53}$$

Profit is maximized when:

$$\overline{p} = \overline{c}_1(x^j,y^j) \text{ and } \overline{c}_2(x^j,y^j)=0 \qquad (54)$$

The ceiling on the price of quality can be used to achieve the efficient uoq. However the firm produces the efficient qpi only when the marginal private cost of qpi equals to the marginal social cost of qpi. As noted above, this occurs only if the n-c externality is not present and either (i) there is a monopoly or (ii) the c-c externality is also not present.

In general, a ceiling on the price of quality needs to be accompanied by another policy instrument in order to achieve efficiency. One possible form of control is qpi regulations. It is shown above that the efficient qpi is greater than the profit maximizing qpi. Therefore minimum qpi standards would need to be combined with price of quality controls to achieve efficiency.

5. Conclusion

This paper has compared the profit maximizing and efficient qpi in an oligopolistic market in which two types of externality are present. It is shown firms choose qpi to minimize private cost. Variation in the firm's choice of qpi can be traced to movements along or shifts in the firm's private cost minimizing qpi curve. The slope of the private cost minimizing qpi curve is determined by the technology of the firm. When the invariance condition holds it is vertical however when the co-variance condition holds it is upward sloping. In the absence of the c-c externality a change in competition causes movement along, but no shift in, the private cost minimizing qpi curve. Thus under the invariance condition competition causes no change in qpi, whereas under the co-variance condition qpi moves in the same direction as the firm's production of uoq.

Both externalities considered in this paper are potentially detrimental to welfare. However the c-c and the n-c externalities have different effects on the choices of the firm. The n-c externality affects activities not related to firm's output. It is therefore is not internalized by the firm, and the firm under provides qpi when it is present. As shown by figure 2, the social cost minimizing qpi curve lies to the right of the market cost minimizing qpi curve. On the other hand the c-c externality affects market participants. The greater the firms' markets share the greater extent the c-c externality is internalized. A monopoly fully internalizes the c-c externality, so the private cost minimizing qpi curve coincides with the market cost minimizing qpi curve. As the degree of competition increases the extent of internalization of the c-c externality decreases. Thus, in the presence of the c-c externality, the private cost minimizing qpi curve shifts to the left as the degree of competition increases. Thus, as shown in figure 2, the private cost minimizing qpi curve lies to the left of the market cost minimizing curve for markets consisting of two or more firms

The nature of technology determines the impact on qpi of increased competition. Under the invariance condition an increase in competition decreases qpi when the c-c externality is present. Under the co-variance condition an increase in competition also decreases qpi if the profit maximizing uoq is simultaneously reduced. If however the profit maximizing uoq is increased by competition the effect on qpi is ambiguous.

The nature of technology also determines the relationship between the efficient and profit maximizing qpi. Under the invariance condition the efficient qpi is greater than the profit maximizing qpi. However Under the co-variance condition the efficient qpi is greater than the profit maximizing qpi when the efficient uoq is greater than the profit maximizing uoq. This would occur if the n-c externality is not too strong. If the profit maximizing uoq is greater than the efficient uoq the relationship between the efficient and profit maximizing qpi is ambiguous.

The firm's profit maximizing cost revenue ratio was derived. The expression is a generalization of the Dorfman-Stiener condition. The cost revenue ratio depends on the elasticity of the firms residual demand curve and the elasticity of cost with respect to qpi. Thus an increase in the number of firms increases the cost revenue ratio as it raises the elasticity of the firm's residual demand curve. The firm's cost revenue ration is shown to be independent of the strength of the c-c externality. Furthermore, it is shown that the efficient cost revenue ratio is independent of the strength of both externalities. Thus the efficient cost revenue ratio is given by the same formula as that for perfect competition: the ratio of the elasticity of perceived qpi with respect to qpi and the elasticity of cost with respect to qpi.

The taxes require to achieve efficiency (for a given market structure) were derived. Two types of tax regimes were considered: one in which taxes are levied on uoq and qpi, and the other in which taxes are levied on noi and qpi. It was demonstrated that, when the n-c externality is absent, a single tax on uoq can cause a monopoly to achieve efficiency even when the c-c externality is present. This is because the monopolist internalizes the c-c externality, and produces the market (and social) cost minimizing qpi. The tax on uoq is sufficient to ensure the efficient uoq and qpi is produced. However when n-c externality is present, or there is not a monopoly (so the c-c externality is not internalized), the firm does not produce the social cost minimizing level of qpi. In this event a tax on qpi is also required to produce efficiency. Similarly a tax on noi distorts the firm's choice of qpi and, when such a tax is imposed, a tax on qpi is also required to achieve efficiency.

Two issues stand out as not having been addressed in this paper and warranting further investigation. First, the definition of efficiency used in this paper was made for a given market structure. The impact of free entry, and the efficiency of market structures, has not been considered. The importance of this issue can be seen by noting that an economy of scale in the production of noi exists when cost is additively separable (because qpi is non-rival). Under such circumstance a market with free entry need not be efficient. Second, in many applications one or both of the externalities may be positive rather than negative externalities. Reversing the direction of one of the externalities would be expected to cause additional ambiguity in the results. It would be useful to identify the impact that assuming a positive rather than negative externality has on qpi. The techniques used in this paper could be used to conduct such an investigation.

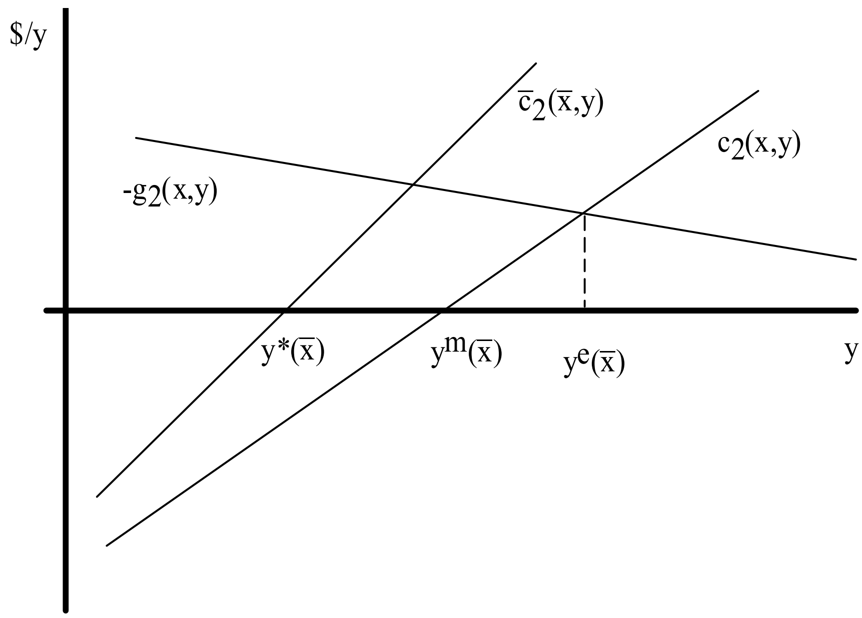

Figure 1:Private cost minimising, market cost minimising and efficient qpi for a given number of uoq.

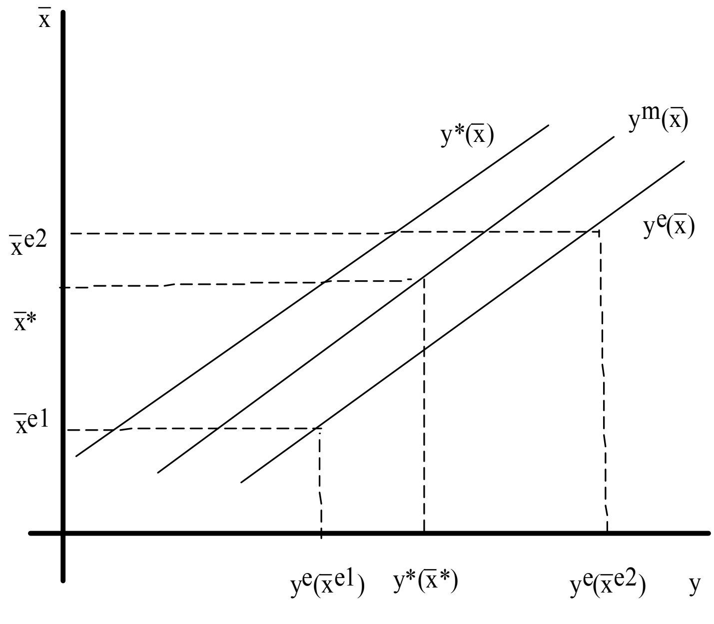

Figure 2: Private cost minimsing, market cost minimising and efficient qpi under the co-variaince condition.

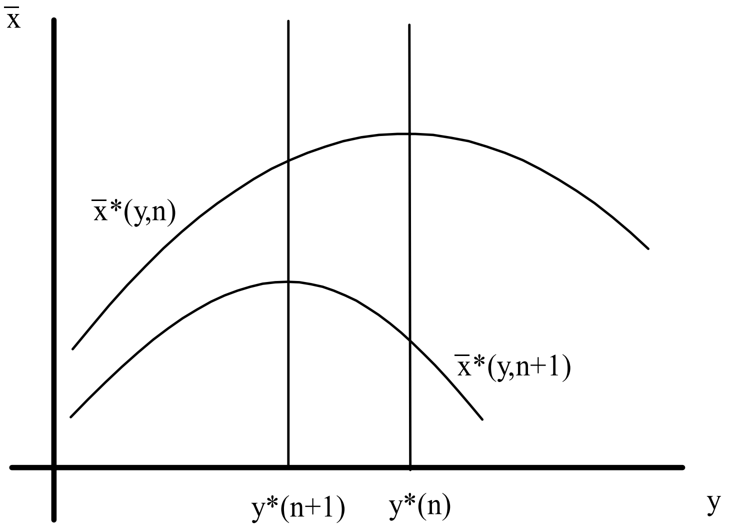

Figure 3: An increase in competition
under the invariance condition

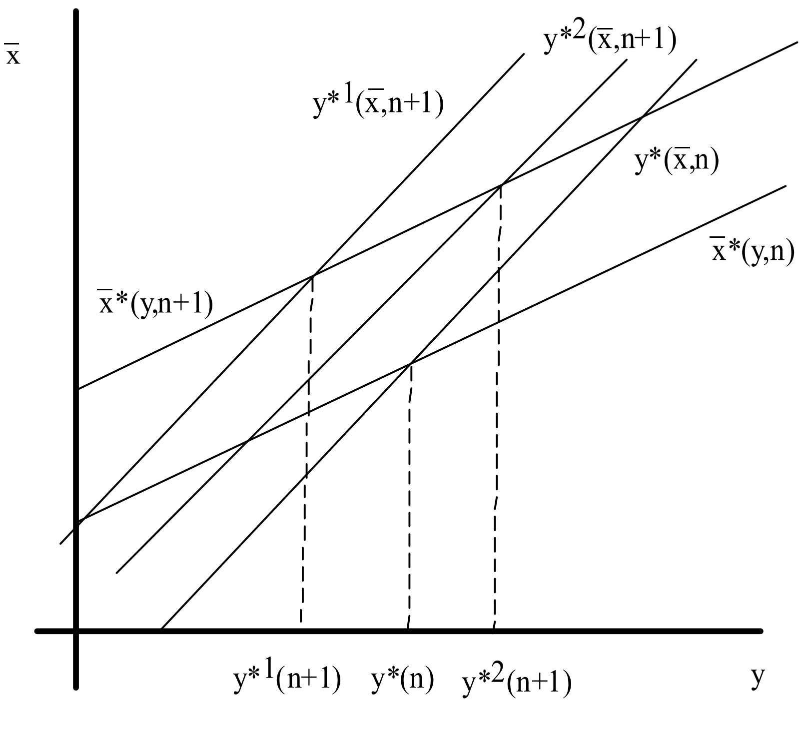

Figure 4: An increase in competition
under the co-variance condition.

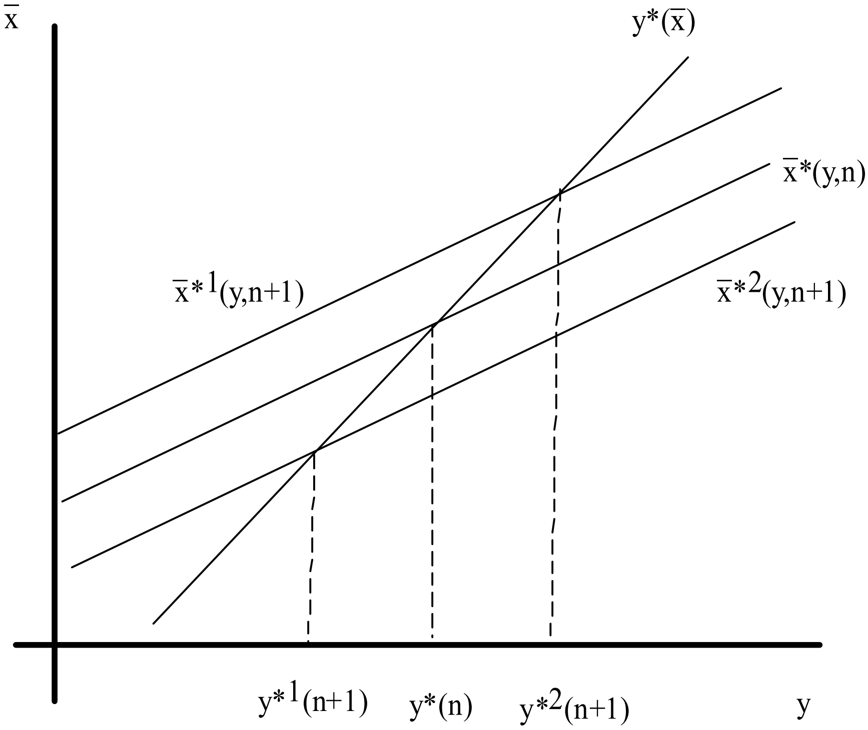

Figure 5: Firm's response to an increase in competition without c-c externality

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Appendix A: Derivation of the demand function

Following the argument in the introduction, assume consumer i's utility function is given by $U^i(qX^i, \underline{\mathbf{x}}^i)$, where i = 1…k, X^i is consumer i's noi purchased, $\underline{\mathbf{x}}^i$ is a vector of the consumption of other goods by consumer i and q is perceived qpi.

Define x^i, consumer i's demand for uoq, by $x^i \equiv qX^i$. The consumer's budget constraint is:

$$PX^i + \underline{\mathbf{p}}.\underline{\mathbf{x}}^i = I \qquad \text{(A1)}$$

where P is the price of an item of the good, and $\underline{\mathbf{p}}$ it the vector of prices of other goods. Define p the price of quality by p = P/q. Then the consumer i's optimization problem becomes:

$$\text{Max } U^i(x^i, \underline{\mathbf{x}}^i) \text{ subject to } px^i + \underline{\mathbf{p}}.\underline{\mathbf{x}}^i = I \qquad \text{(A2)}$$

Using standard consumer theory demand for uoq is given by:

$$x^i = x^i\,(p, \underline{\mathbf{p}}, I) \qquad \text{(A3)}$$

From (A3) demand for noi may be written as:

$$X^i(P, q, \underline{\mathbf{p}}, I) = x^i\,(P/q, \underline{\mathbf{p}}, I)/q \qquad \text{(A4)}$$

As both $\underline{\mathbf{p}}$ and I are exogenous to the analysis below, for brevity reference to them is suppressed in the discussion below. Define market demand for uoq, x(p), as:

$$x(p) = \sum_{i=1}^{k} x^i(p) \qquad \text{(A5)}$$

and market demand for noi, X(P,q), as:

$$X(P,q) = \sum_{i=1}^{k} X^i(P,q) \qquad \text{(A6)}$$

Integrating consumer i's demand for uoq yields their consumer surplus[9], CS^i, where:

$$CS^i = v^i(p) \qquad \text{(A7)}$$

Total consumer surplus, CS, is given by:

$$v(p) = \sum_{k=1}^{n} v^i(p) \qquad \text{(A8)}$$

Note that v'(p) = -x and therefore $\partial v/\partial P = -X$.

The inverse market uoq demand, p(x), can be obtained by inverting (A5). Then total consumer surplus may also be written as a function of uoq in the following way:

$$CS = V(x) \equiv v(p(x)) \qquad \text{(A9)}$$

Note that $V'(x) = v'(p).p'(x) = -p'(x)x = p/\varepsilon_X$.

Appendix B: Additive Isoelastic Costs

In this appendix an example is given of the ambiguity of the impact on qpi of an increase in competition when costs are additively separable. Assume each firms' costs are as given by (12) and $\Phi(X) = \mathit{\Phi} X^{\Phi}$ and $\Psi(y) = \mathit{\Psi} y^{\Psi}$ where $\mathit{\Phi}$, Φ, $\mathit{\Psi}$ and Ψ are positive parameters. Substituting into (8) yields:

$$y = \left(\frac{\mathit{\Phi}\Phi\theta}{\mathit{\Psi}\Psi(1-\rho/n)}\right)^{\frac{1}{\Psi}} X^{\frac{\Phi}{\Psi}} \qquad \text{(B1)}$$

[9] The surplus provides a good approximation of welfare when income effects are negligible, in particular when PX^i is a small fraction of the consumer's income (see Tirole, 1988, p. 11).

From (32), the profit maximizing choice of noi satisfies:

$$P(n\bar{X},y) = \frac{\varepsilon_x . \Phi'(\bar{X})}{(\varepsilon_x-(1/n))(1+(1+\varepsilon_{\rho X}/n))} \qquad \text{(B2)}$$

Suppose consumer i has a CES utility function:

$$\text{Max } U^i(x^i, \underline{\mathbf{x}}^i) = \left((x^i)^{1-\varepsilon} + (\tilde{\underline{\mathbf{x}}}^i)^{1-\varepsilon}\right)^{1/1-\varepsilon} \qquad \text{(B3)}$$

where $\tilde{\underline{\mathbf{x}}}^i$ is the CES utility derived from consumption of other goods. The goods are assumed substitutes, so $\varepsilon > 1$. The user's optimal uoq is given by:

$$x^i(p,\underline{\mathbf{p}},I) = Ap^{-\varepsilon} \qquad \text{(B4)}$$

where A is given by:

$$A = \frac{I}{p^{1-\varepsilon} + \tilde{\underline{\mathbf{p}}}^{i\,1-\varepsilon}} \qquad \text{(B5)}$$

where $\tilde{\underline{\mathbf{p}}}^i$ is the index of other good's prices. When the number of other goods is large, A is unaffected by changes in p.

If $\rho(X)$ and $\theta(y)$ are isoelastic, perceived qpi may be written as:

$$q(X,y) = Q\, X^{-\rho} y^{\theta} \qquad \text{(B6)}$$

where $\rho > 0$ and $\theta > 0$ are the elasticities of perceived qpi and technical qpi respectively, and $Q > 0$ is the level of qpi. In this case, when demand is given by (B4), the inverse demand curve is given by;

$$P = BX^{-\left(\frac{1+\rho(\varepsilon-1)}{\varepsilon}\right)} y^{\frac{\theta(\varepsilon-1)}{\varepsilon}} \qquad \text{(B7)}$$

where $B = QA^{\frac{1}{\varepsilon}}$. If demand is given by (B7), (45) becomes:

$$\bar{X} = \left(\frac{B(1-\rho/n)(\varepsilon-1/n)}{\varepsilon \mathit{\Phi}\Phi}\right)^{\frac{\varepsilon}{1+\rho(\varepsilon-1)+\varepsilon(\Phi-1)}} y^{\frac{\theta(\varepsilon-1)}{1+\rho(\varepsilon-1)+\varepsilon(\Phi-1)}} n^{\frac{-(1+\rho(\varepsilon-1))}{1+\rho(\varepsilon-1)+\varepsilon(\Phi-1)}} \qquad \text{(B8)}$$

Substituting (B8) into (B1) yields:

$$y = \left(\frac{B\,(\varepsilon-1/n)}{\varepsilon}\right)^{\frac{\varepsilon\Phi}{\Theta}} \left(\frac{(1-\rho/n)}{\mathit{\Phi}\Phi}\right)^{\frac{(1-\rho)(\varepsilon-1)}{\Theta}} \left(\frac{\mathit{\Psi}\Psi}{\theta}\right)^{\frac{(1-\rho)(\varepsilon-1)-\,\varepsilon\Phi}{\Theta}} n^{\frac{-\Phi(1+\rho(\varepsilon-1))}{\Theta}} \qquad \text{(B9)}$$

where $\Theta = \Psi(1+\rho(\varepsilon-1)+\varepsilon(\Phi-1)) - \Phi\theta(\varepsilon-1)$. It is readily observed by differentiation of (B7) that the impact of an increase in competition (ie n) has an ambiguous effect on qpi (y).

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