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Non-Linear Pricing with Homogeneous Customers and Limited Unbundling

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ABSTRACT

This paper presents a model in which a firm conducts non-linear pricing though bundling. However some agents, ‘unbundlers’, find it profitable to unbundle output. Unbundlers have an increasing marginal cost of unbundling, which limits the extent of unbundling. Customers with identical demand can purchase either bundled or unbundled output. In equilibrium, some consumers purchase bundled output and others unbundled output. The analysis shows how the extent of unbundling and the optimal bundle size are related to the cost of unbundling. Failing to account for presence of unbundling could lead to a misinterpretation of market efficiency.

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Non-linear pricing with homogeneous customers and limited unbundling

Many goods are bundled when supplied. Bundling is profitable for suppliers, because it enables them to conduct non-linear pricing. When analysing the impact of such bundling, the economic literature almost universally assumes that once the good is bundled it cannot be, at some later time, unbundled and resold. [1] This paper presents a model in which some agents, ‘unbundlers’, find it profitable to unbundle output which is initially sold using non-linear pricing. Limited unbundling is present in the equilibrium of this model. Limited unbundling occurs when a fraction of the bundles sold are unbundled are the unbundled output resold. The extent of unbundling is limited because unbundlers have an increasing marginal cost of unbundling.

Many bundled goods are not inherently unable to be unbundled and resold. For example individual confectionaries are often bundled into larger packages. Bundling (or packaging in this example) can act to prevent resale. To sell individual confectionaries it is necessary to remove them from their packaging, which would involve some cost, and hence discourage unbundling. It is often observed that manufacturers attempt to discourage unbundling, for instance by making bundles unnecessarily difficult (hence costly) to unpack, or by including statements such as ‘not for sale separately’ on the individual confectionary packaging. (Such statements, by themselves, do not have the force of law on customers, but might be considered a discouragement to unbundling.) Alternatively manufacturers might enter into contractual arrangements that the goods were not to be unbundled, and then threaten legal actions against anyone who attempted to unbundle output.

Unbundling may also be unintentionally discouraged by various pieces of legislation. For instance legislation might requires that consumer information be included on each item sold.[2]

[1] The one exception I am aware of is Alger (1999) and McManus (2002), which are discussed below.
[2] In the US labelling requirements for food packages are governed by Title 21 of the Code of Federal Regulations. See:

Confectionary manufacturers can therefore discourage resale by including the required information on the bundle's packaging, but on individual wrappers. In addition food and safety laws can make it costly to unpackaged and then 'cut down' large blocks of chocolate into smaller blocks of chocolate. [3] However, there is not always this type of unintended legislative impediments to the unbundling of goods: for instance there would appear to be no equivalent legislative impediment to the unbundling of clothing bundles (e.g. socks).

Where unbundling breaches a law (or even when it is likely to simply aggravate the manufacturer), resale markets are necessarily informal. As such, evidence of such activity tends to be anecdotal. [4] Nonetheless some readers may have experienced some form of this behaviour. For instance, a vendor at a street market might sell unbundled confectionary. Alternatively, consumers may form coalitions which act as unbundlers. For example, a work place may have a convention in which employees take turns to purchase a large packet of confectionary, rather than each employee purchasing a separate confectionary. In effect, the employee who purchases the packet of confectionary acts as an 'unbundler', albeit in a barter market.

The model presented in this paper is designed to explore the implications of these considerations for the implementation of non-linear prices. It is assumed all customers have *identical* demand. A monopolist (who could be thought of as a manufacturer) bundles their output. Customers are able to purchase this bundle from the firm at a 'block' price. However customers are also able to purchase unbundled units of output from unbundlers: these are firms or individuals (such as the confectionary purchasing employees discussed above) which operate to unbundle output. Unbundlers have an increasing marginal cost of unbundling, which limits the extent of unbundling. Intuitively, one might think that in some workplaces

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http://www.fda.gov/Food/LabelingNutrition/FoodLabelingGuidanceRegulatoryInformation/default.htm
In Australia, Canada and the European Union nutrition facts label has to be placed on packages containing food.

[3] Where components of the bundle do not have their own packaging health and safety regulations may discourage resale. For instance breakfast cereal is usually packaged, but could, in principle, be unpacked and sold on a per unit basis. However, to comply with health and safety standards, this would require (costly) imposition of hygiene standards.

[4] For instance the following internet exchange: http://forums.macnn.com/89/macnn-lounge/239428/what-does-not-resale-mean-individually/ discusses a shop which we selling small candy bars marked "not for resale".

the opportunity cost of sharing packets of confectionaries is relatively low and in others it is relatively high. In equilibrium, a subset of consumers purchase bundled output and the remaining consumers purchase unbundled output.

The analysis shows that resale inhibits the use of non-linear prices. [5] Customers receive consumer surplus and firm profit is lower than when unbundling is impossible. The analysis shows how, in the presence of unbundling, the extent of unbundling and the optimal bundle size are related to the cost of unbundling. The impacts of legislation and manufacturers activities to discourage unbundling are analysed. The impact on efficiency of the presence of unbundling activities is analysed.

The discussion of non-linear pricing (in which firm does not use a linear (per unit) price to sell to a given customer) in the literature is usually conducted in conjunction with price discrimination (in which the firms sets different prices to different customers).[6] Apart from the papers discussed below, it is assumed by authors that unbundling and resale are impossible.[7] When unbundling and resale are indeed impossible, and when customers are identical (as assumed in this paper), firms can conduct first degree price discrimination either (i) by using a two part tariff with the per unit charge equal to marginal cost and an access price equal to the consumer surplus or (ii) by bundling the efficient level of output and charging a fee equal to total consumer benefit. It is often noted (though not formally modelled) that the use of a two part tariff is undermined if costless resale if possible. Under a two part tariff a customer can drive the average price of a unit of output to marginal cost if they make a sufficiently large purchase.[8] This customer can profit by reselling to other customers using linear pricing with some mark-up over marginal cost. These discussions leave the impression, though this is not explicitly stated, that bundling output (i.e. using block

[5] Hammond shows, in the context of a general equilibrium model with a continuum of agents that costless trading imposes a restriction on firms: namely they must use linear pricing.
[6] Maskin and Riley (1984) develop a model of price discrimination with non-linear pricing.
[7] However the discussion of resale is often perfunctory, aimed merely at establishing that resale is impossible, or unlikely, for a class of goods. Thus the literature has implicitly assumed that resale is either costless or infinitely costly. The implications of resale are not explored, either for price discrimination or non-linear pricing. Wilson (1993) pp. 11-12 however provides a thoughtful discussion of the impact of resale on the use of non-linear prices. However all of his examples involve price discrimination.
[8] See for example Carlton and Perloff (2005 p. 294). McManus (1998) discusses optimal consumer strategies in the presence of heterogeneous demand.

pricing) firms could overcome this problem. However the above examples indicate that this would not be the case if some customers can unbundle output at a modest cost. Thus the analysis in this paper also provides an explanation of how resale (unbundling in this case) limits the use of block pricing. [9]

Surprisingly relatively little attention has been given to unbundling or resale in the literature. And these papers consider unbundling in the presence of both nonlinear pricing and price discrimination. The closest paper to this one is Alger (1999). Alger assumes each of the two customer types can make within-type collations (joint purchases which enable unbundling) and (unlike this paper) face zero transaction costs when making and dividing up joint purchases. However, because incentive compatibility constraints always hold in Alger's model, no collations are present, and hence no unbundling occurs, in equilibrium. Alger shows that including the possibility of customers making joint purchases ensures each consumer type has strictly positive utility, the quantity in both type's bundle are downwardly distorted and firm profit is lower. The admittance of joint purchases in this model has an ambiguous effect on welfare, though Alger argues that it tends to lower welfare. [10]

McManus (2002) considers the impact of coalition formation a firm that utilises two part tariffs. Coalition size and composition is exogenous in these models. McManus assumes there is a transaction cost to the formation of coalitions. He finds that collation formation may improve firm profitability. In McManus's model when consumers are able to form collations the monopolist can set the fixed fee equal to the sum of the surpluses of coalition members. However when consumers can't form coalitions, the fee cannot exceed the surplus of the low demand type.

Gans and King (2007) consider whether costless consumer arbitrage necessarily undermines the use of perfect price discrimination. They present a model in which the firm may utilise

[9] The durable goods literature, drawing on Coase (1972), also considers the possibility of resale. This literature differs from this paper by (i) assuming either a linear price and/or unit demand and (ii) customer heterogeneity. See Waldman (2003) for a survey.

[10] Alger could not find a numerical model in which the admission of joint purchases raises welfare (1999 p. 751).

perfect price discrimination even when consumer arbitrage is possible. Their model's assumptions differ from the present one in that (i) consumers are of two types, (ii) each customer has only one unit of demand and (iii) the firm is uncertain as to the proportion of customers are of a given type. As these assumptions suggest, their model (like those of Alger and McManus) is designed to investigate the impact of consumer arbitrage on price discrimination rather than specifically on non-linear pricing.

Resale of a monopolist's output has been considered in different contexts. Aguirre and Paz Espinosa (2004) consider the impact of consumer arbitrage in a linear city model with convex transportation cost. Consumer arbitrage may lower total transportation costs, and thereby reduce variety in equilibrium. Aguirre and Espinosa's model, unlike that in this paper, utilises linear pricing (as is normal for linear city models). Calzolari and Pavan (2006) consider pricing mechanism for a monopolist who sells a durable good which can be resold in a secondary market. This paper considers only goods which can be used once, thus there does not exist a secondary market for the goods.

The paper proceeds in the following way. In the following section the model is introduced, and the profit maximising bundle size and fee are determined. In section 3 the implications of a change in the cost of unbundling is considered. In sections 2 and 3 it is assumed the unbundling cost is exogenous. In section 4 the model is extended to allow the firm to undertake costly activities which deter unbundling. Section 5 concludes the paper.

1. A model of non-linear pricing with unbundling

A monopolist producer sells output which is infinitely divisible. This firm sells output in L bundles with Z units per bundle. There are N final customers who have common benefit function, B(X), and thus common demand function $P(X) \equiv B'(X)$, where X is the quantity consumed by the customer and $P'(X)<0$. Each of these identical consumers receives the benefit B(Z) from consuming this bundle. The producer charges a fee (or block price), T, for the bundle. Define the linear component of this fee to be P(Z)Z and the non linear component of the fee, R, by R≡T-P(Z)Z.

Final customers purchase output from retailers. There are two types of retailers. One type, distributors, faces an infinite cost of unbundling output. The second type, unbundlers, have a total cost from unbundling activity G(M), where M is the number of bundles that are unbundled by these firms. Assume $G'(M)>0$ and $G''(M)>0$. There is assumed to be a given number of unbundlers. [11] All retailers have zero cost of distribution and the retail market is contestable. Final customers may purchase either bundled or unbundled output. Distributors sell bundles to final customers for the same price they were purchased from the monopolist and earn a zero profit.

There is assumed to be a competitive market for the unbundled output supplied by unbundlers. The producer cannot identify which retailers are unbundlers, so cannot exclude unbundlers from purchasing its output.[12] Under the above assumptions unbundled units would trade for a price:

$$P(Y) = [T+g(M)]/Z = R/Z + P(Z) + g(M)/Z \tag{1}$$

where $g(M) \equiv G'(M)$ is the cost of unbundling the marginal bundle. Call Y the unbundled quantity. It represents the amount of output a customer purchases, at the linear price of P(Y), if they choose to purchase unbundled output.

The above model could be given an alternative interpretation. Suppose that, instead of output being distributed by retailers, customers purchase bundled output directly from the producer. Unbundlers are interpreted as those customers, or groups of customers, who unbundle output

[11] This assumption can be weaken, without changing the qualitative results in this paper, so that the number of unbundlers increases with the price of unbundled output. For simplicity, this extension of the analysis is not included.

[12] This also means the producer cannot implement a two part tariff.

purchased from the producer to share amongst themselves or other customers. Equation (1) then represents the level of unbundling activity of this group of customers. Final customer then either consume bundled output purchased directly from retailers or unbundled output obtained through unbundlers. This interpretation does not alter the mathematical statement of the above model, but allows it to be applied to those examples considered by Alger (1999).

In equilibrium final customers must be indifferent between purchasing bundled and unbundled output. Thus

$$B(Z) - (R + P(Z)Z) = B(Y) - YP(Y) \geq 0 \tag{2}$$

By the implicit function theorem equation (2) implies R=R(Z,Y) and thus (1) implies that M=M(Z,Y). The producer's profit is given by:

$$\pi(Z,Y) = L(Z,Y)\Pi(Z,Y)\text{-}F = \left(N\text{-}\left(\frac{Z\text{-}Y}{Y}\right)M(Z,Y)\right)(R(Z,Y) + P(Z)Z\text{-}cZ) \text{ - } F \tag{3}$$

where L(Z,Y) is the number of bundles sold by the producer, Π(Z,Y) is the profit per bundle, F is the fixed cost and c is the constant marginal cost.[13] It is assumed that (in order to satisfy the second order conditions) that the benefit function is such that $\pi_{ZZ}(Z,Y)<0$ and $\pi_{YY}(Z,Y)<0$. The number of bundles sold is calculated by noting that each unbundled bundle supplies Z/Y customers. Thus the producer does not sell directly to ZM/Y customers. It does, however, sell M bundle to unbundlers. Thus the number of bundles sold is N- ZM/Y+M. The profit per bundle is made up of (P(Z)-c)Z, which called the linear component of profit per bundle and R the non-linear component of profit per bundle. The firm maximises its profit, π(Z,Y), subject to (2). In this event:

[13] Constant marginal cost is assumed for analytic convenience. However results are not qualitatively affected by assuming a more general form of cost.

Proposition 1: In equilibrium $Z>Y$. The firm chooses output per bundle according to:

$$P(Z)-c = (\Pi/L)\{M/Y+(Z-Y)M_Z/Y\} > 0 \tag{4}$$

where $M_Z>0$. The unbundled quantity is chosen according to:

$$\frac{P(Y)}{\varepsilon(Y)} = (\Pi/L)[ZM/Y^2-(Z-Y)M_Y/Y] > 0 \tag{5}$$

where $M_Y< 0$ and where $\varepsilon(Y)\equiv -P(Y)/YP'(Y)$ is the elasticity of demand.

The proof of proposition 1 is given in the appendix. This proof also contains expressions for M_Z and M_Y. Condition (4) indicates that $P(Z)>c$, and hence the size of the bundle sold by the firm is less than the efficient level. Intuitively, as in the case without unbundling, increasing the size of the bundle, Z, increases profit per bundle by P(Z)-c. However in the presence of unbundling, increasing the size of the bundle has two additional effects that are represented by the two terms on the RHS of (4). The first term on the RHS of (4) indicates that an increase in Z reduces the number of bundles need to supply a given level of unbundled output. Hence this effect increases the incentive to unbundle output. The second term on the RHS of (4) represents the effect that an increase in Z increases has on the benefit of unbundling, i.e. the RHS of (1). An increase in Z lowers increases the gap between P(Y) and P(Z) and increases the non-linear component. Thus there is an increases unbundling. Both effects on the RHS of (4) are positive. Hence the equilibrium bundle size is less than the efficient level of output.

Condition (5) identifies the profit maximising level of the unbundled output. The LHS of (5) represents the cost to the firm of raising Y. An increase in Y causes the price of unbundled output, P(Y), falls. Lowering the price of unbundled output (while maintaining Z) means that, by (2), the firm must lower the non-linear component, R. The RHS of (5) represents the benefit to the firm of increasing Y. An increase in Y means more bundles must be unbundled to supply a given number of customers, thus increasing the cost of unbundling. This effect is captured by the first term on the RHS of (5). By (1), an increase in Y lowers the price of unbundled output and reduces the non-linear component. There is thus less incentive to unbundle output as Y increases. Both effects on the RHS of (5) indicate that as Y increases the extent of unbundling reduces. These two effects raise total profit. The equilibrium value of Y occurs when the marginal cost of increasing Y (the LHS of (5)) is equal to the marginal benefit of increasing Y.

The equilibrium bundle satisfies both (4) and (5). Having determined the equilibrium values of Z and Y, (2) can be rearranged to give an expression for the share of profit per bundle due to the non-linear component. As proposition 1 shows Z>Y then:

Corollary 1: $0 < R < B(Z) – P(Z)Z$

Corollary 1 shows that the firm always enacts a non-linear component, but leaves customers with a non-zero consumer surplus. Thus, by (4), those customers who purchase bundles receive a smaller than optimal bundle, but by corollary 1 they also receive some consumer surplus. Those customers who purchase unbundled output acquire a smaller output than that in the bundle, but receive the same consumer surplus as provided by the bundle. Thus the presence of unbundling benefits consumers at the expense of both the firm's profit and efficiency.

The presence of unbundling lowers profits. It might be thought that the firm could thwart the unbundlers by (i) also selling output at the linear price P(Y) or (ii) lowering R infinitesimally to attempt to make the bundled output preferable to customers. However, because the marginal cost of unbundling is rising, this action would only 'crowd out' the marginal unbundler (or marginal bundle). All others unbundlers would strictly prefer to unbundle; in effect the price of unbundled output would drop marginally. The firm could attempt to reduce unbundling further by lowering the linear price of unbundled output. However, by (2), this would entail reducing the non-linear component of the fee, and would thus be unprofitable.

Note that the above argument is predicated on the assumption that the marginal cost of unbundling is strictly increasing (i.e. $g'(M)>0$). If the marginal cost of unbundling was constant (i.e. g(M) a constant), then P(Y) would represent the minimum price any unbundler would trade at. In such a case the firm could exclude unbundlers by ensuring that the bundle provided infinitesimally more consumer surplus than unbundled output could provide.

Now consider the efficient bundle size, assuming that it is not possible for a social planner to influence the level of unbundling. In this event, constraints (1) and (2) must both hold. The surplus is given by:

$$S = \left(N-\left(\frac{Z}{Y}\right)M(Z,Y)\right)(B(Z)-cZ) + \left(\frac{Z}{Y}\right)M(Z,Y)(B(Y)-cY)-G(M)-F \qquad (6)$$

It is readily seen that the efficient outcome is guaranteed by a linear price equal to marginal cost. In this case $Y=Z=X_E$ and $R=0$, where X_E is the efficient output level.

However suppose $F>0$, and it is also required that profit is non-negative. In this case the following constraint must also hold:

$$L(Z,Y)\Pi(Z,Y) \geq F \qquad (7)$$

The constrained efficient bundle maximises (6) subject to (7). A set of simultaneous equations describing the efficient bundle is provided in the appendix. The following is also shown:

> Proposition 2: The constrained efficient output per bundle is greater than the equilibrium output per bundle, and the constrained efficient unbundled quantity is greater than the equilibrium output per bundle.

Suppose the firm is producing the equilibrium bundle. If it increases Z by a small amount (while maintaining Y), its profit falls only infinitesimally (i.e. the fall in profit is second order effect). This is because the gain in profit from existing customers from the increase in bundle size is just outweighed by the fall in profit due to the loss of customers to unbundlers. However the net benefit to those customers who switch to unbundled output is included in the calculation of the surplus, but not in the calculation of profit. Hence the marginal profit from an increase in bundle size is less than the marginal surplus. Consequently the firm has an incentive to provide an inefficiently small bundle.

Suppose again that the firm is producing the equilibrium bundle, and then it increases Y by a small amount (while maintaining Z). The fall in profit from the resultant fall in the non-linear component to existing customers is just offset by increase in profit from the reduction in unbundling. In addition, the increase in Y reduces the deadweight loss from those customers who purchase unbundled output. This effect increases the surplus, but does not affect firm profit. Hence the marginal profit from an increase in Y is less than the marginal surplus. Consequently the firm has an incentive to choose an inefficiently small Y. Consequently, by (2), consumer surplus is also inefficiently small

2. Changes to the cost of unbundling

The cost of unbundling could be influence by either the intentional or unintentional consequences of legislation. It is thus of interest to understand how the cost of unbundling affects the equilibrium described by proposition 1. To this end, it is useful to assume that $g(M)=\alpha M$, where α represents the rate of increase of the cost of unbundling. The following proposition identifies two important 'extreme' special cases, and indicates how the monopolist's profits are influenced by changes in the cost of unbundling.

Proposition 3: Suppose that $g(M)=\alpha M$. Then:

(a) as $\alpha \to 0$, (i) $Z \to X_M$, (ii)$Y \to X_M$, (iii) $R \to 0$ and (iv) $L \to N$ where X_M is the linear pricing monopolist's output.

(b) as $\alpha \to \infty$, (i) $Z \to X_E$, (ii) $Y \to 0$ (iii) $R \to B(X_E)-cX_E$ and (iv) $L \to N$ where X_E is the efficient output level.

(c) increases in α increases profit.

In the case in which α =0 the firm is indifferent between linear and non-linear pricing. Specifically the firm is indifferent between selling each unit at the linear price $P(X_M)$ and selling X_M units for a fee of $P(X_M)X_M$. Proposition 3(a) parallels Hammond (1987). Hammond considers a continuum economy in which goods are costlessly exchangeable. In general equilibrium goods must be sold at linear prices: the exchangeability undermines non-linear pricing. Proposition 3(a) is a similar result in a polar opposite setting: there is only one customer type (rather than a continuum) and a partial equilibrium (rather than general equilibrium) is modelled.

In the case in which $\alpha\rightarrow\infty$ the monopolist can act as a first degree price discriminator. Thus proposition 3(a) and 2(b) show that the linear pricing monopoly (costless resale) and first degree price discriminator (infinitely costly resale) are nested in the model. In both cases the monopolist sells its output to all customers. Proposition 3(c) shows that the monopolist benefits from an increase in the cost of unbundling. An increase in α reduces the incentive of unbundlers to conduct resale, thus increasing the scope for the firm to extract consumer surplus from customers.

The most instructive method to further analyse the impact of a change in α on the equilibrium of the model is to consider an illustrative numerical example.[14] One such numerical example is now presented. In this numerical example it is assumed that the inverse demand function is linear, and specified by $P(X) = 10 - X$. Marginal cost is given by c=2 and fixed cost is zero. It is assumed there are 100 customers (i.e., N=100). With this specification a linear pricing monopoly would produce X_M=4 units per customer for a price of 6, thus generating a profit of 16. The linear pricing monopolist would cause a deadweight loss of 8 per customer and a benefit of 32 per customer (8 of which is consumer surplus). A first degree price discriminator would sell X_E=8 units per customer, and generate a profit of 32 per customer. The price of the bundle from the first degree price discriminator is thus 48, which equates to an average price of 6 per unit (which is the same as the linear pricing monopolist).

[14] Undertaking a comparative static analysis is not particularly illuminating, due to the multiplicity of effects that are present when there is a change to the cost of unbundling. Both Alger (1999) and McManus (2002) also find it useful to analyse their models using numerical examples.

Table 1 shows numerical solutions of the model with varying values of α. In this example the size of the bundle (Z) increases monotonically from the level produced by the linear pricing monopoly, 4, to the efficient level, 8, as α is increased. The value of Y decreases monotonically from the linear pricing level towards zero as α is increased. Consequently the resale price of output, P(Y), increases monotonically from the linear pricing level, 6, to the choke price, P(0)=10. Similarly the nonlinear component, R, rises monotonically from 0 to $B(X_E)-cX_E=32$. These results suggest that there is a monotonic transition from the linear pricing bundle to the first degree price discriminating bundle as α increases.

However, while these properties of the bundle are monotonic with respect to α, this is not the case for unbundled output. Note that there is an initial rise in the number of bundles that are unbundled, then this declines to zero as α increases. This is because there are two effects on M as a result of an increase in α: (i) an increase in α raises the price of unbundled output and hence encourages unbundling, but (ii) increases the marginal cost of unbundling and hence discourages unbundling. M increases when the first effect dominates and declines when the second effect dominates.

Note that for very low values of α, about 30 of the 100 bundles sold are unbundled. This figure is relatively high because, even for low α, the firm has a strong incentive to enact non-linear pricing rather than linear pricing: profit increases with the square of increases in Z. Because the marginal cost of unbundling is low, there is considerable scope for significant unbundling activity.

The number of bundles sold by the firm, L, is also non-monotonic in α. The firm sells 100 bundles for very low and very high values of α. However, as α increases from zero N declines, reaches a minimum (in the reported values) of $\alpha=1$, then increases as α increases. This is because the number of bundles sold is inversely related to the number of customers purchasing unbundled output. As α increases beyond 10^{-6} the number of bundles that are sold to unbundlers and unbundled declines. However the quantity purchased by customers consuming unbundled output also declines. The effect on L of an increase in α depends on which of these effects dominates. For high values of α $L \approx 100$, and the firm sells virtually all of these bundles directly to customers rather than unbundlers.

Table 1 also reports the value of T/Z, the average price per unit in a bundle. As noted above T/Z has a value of 6 for $\alpha=0$ and $\alpha=\infty$. However table 1 shows that T/Z has a higher value for intermediate values of α, with T/Z having a maximum (in the reported values) of $\alpha=1$. This pattern is the result of two effects: the average non linear component, R/Z, increases with α, however P(Z) declines with α. For low values of α R rises rapidly with α, hence this effect dominates.

The deadweight loss (DWL), and the DWL as a percentage of maximum surplus (D/B), are reported in table 1 for various values of α. DWL arises in two ways: the DWL associated with customers who purchase bundles is $B(X_E)-cX_E-[B(Z)-cZ]$, and the DWL associated with customers who purchase unbundled output is $B(X_E)-cX_E-[B(Y)-cY]$. Unsurprising the DWL is less than that associated with a linear pricing monopoly for positive values of α. The DWL decreases with α, reflecting that the bundle's output moves closer toward the efficient level as α increases. However observe that, as Y increases with α, the DWL associated with those customers purchasing unbundled output also increases with α. This, to some extent, offset the effect of increasing bundle size.

The size of the DWL could be underestimated in the unbundling market is informal in the way discussed in the introduction. In this event an investigator may not observe the resale market, and thus interpret L as the total number of customers. The DWL is assumed to arise because the bundle size is less than X_E. The DWL would therefore be incorrectly measured as $DI \equiv L\{B(X_E)-cX_E-[B(Z)-cZ]\}$. The value of this incorrectly measured DWL, DI, is reported in table 1. It can be seen that the difference between the actual DWL and DI rises with α. When $\alpha=1$ the incorrectly measured DWL is less than half the true DWL. Also reported in table 1 is the value of $DI/BI \equiv 1-[B(Z)-cZ]/[B(X_E)-cX_E]$. This figure represents an estimate of the DWL as a fraction of the maximum total surplus, based on the incorrect assumption that the total number of customers is L and not N (i.e. overlooking the resale market). It can be seen that this measure underestimates the efficiency loss, particularly for high values of α.

3. Endogenous unbundling cost

There are some products for which the firm may be able to influence the cost of unbundling.[15] The model considered in section 1 is now extended to allow the firm some scope to influence the cost of resale. Suppose the cost of resale is αM, and that the firm can ensure a particular marginal unbundling cost of α, through an expenditure of $\varsigma\alpha$. The marginal cost of influencing resale, ς, is exogenous to the firm. In this case equation (3) would be recast as:

$$\pi(Z,Y,a) = L(Z,Y,\alpha)\Pi(Z,Y,\alpha) = \left(N-\left(\frac{Z-Y}{Y}\right)M(Z,Y,\alpha)\right)(R(Z,Y) + P(Z)Z - cZ - \varsigma\alpha) \qquad (8)$$

where from (1):

[15] A recent dramatic case of unbundling, albeit with respect to the bundling of complementary rather than homogeneous goods, involved the unlocking of cell phones provided by TracFone. TracFone provided cheap cell phones which were locked to their network. The intention was that customers would purchase the phones, then pay a relatively high rate for calls. However traders purchased these phones from retail outlets and then unlocked them. The unlocked phones could be resold at a significant mark-up. See: http://www.msnbc.msn.com/id/25665617/ns/technology_and_science-wireless/#storyContinued

TracFone launched extensive (and thus costly) legal action to try to prevent the firms which were unbundling their output from conducting this arbitrage. Interestingly, while the unlocking (unbundling) is not in itself illegal, both terrorism laws and copyright/patent laws (i.e. laws not intended to restrict legal arbitrage) ere used in an attempt to curtail the unbundling.

$$M(Z,Y,\alpha) \equiv [(P(Y)-P(Z))Z - R(Z,Y)]/\alpha \qquad (9)$$

The conditions for the optimal bundle, (4), (5) and (2), are unaffected by this modification and the optimal level of α is given by:

$$E^{L}_{\alpha} \equiv \varsigma\alpha/\Pi \qquad (10)$$

where $E^{L}_{\alpha} \equiv \alpha L_{\alpha}/L = (Z-Y)M/YL > 0$. That is, the firm chooses the cost of resale so that the elasticity of bundles sold with respect to cost of resale is equal to the share of profit required to achieve that level of resale. In this extension it is to be expected that α will be higher for those products for which the marginal unbundling cost is sensitive to the firms' efforts to increase it.

Again, the most instructive method to further analyse the impact of a change in ς on the equilibrium of the model is to consider modifications to numerical example above. In particular the numerical model is modified so that the objective function is (8). In this event the profit maximising values of Z, R, Y and α vary with ς. Some simulations of the model are given in table 2. It can be seen from Table 2 that as ς falls, the equilibrium value of α increases.

Where comparable, the qualitative conclusions from the analysis in section 2 remain unchanged. This is unsurprising as conditions (4) and (5) have not been affected by the extension of the model. Table 2 reports total expenditure on discouraging unbundling, $\varsigma\alpha$. This has an inverse U-shape with respect to ς, reaching a maximum for reported values when ς=2.15 (or α=1). Total expenditure on discouraging unbundling a type of rent seeking activity, and is thus an additional component to deadweight loss. This expenditure increases deadweight loss, particularly for intermediate values. However as the relative magnitude of $\varsigma\alpha$ in this numerical model is not that great, the increase in DWL due to the inclusion of this rent seeking activity is not that great. However this need not be the case in general, and high efficiency losses resulting from expenditures on discouraging unbundling (as described by footnote 13) is possible.

4. Discussion

This paper has presented a parsimonious model of non-linear pricing in the presence of unbundling. The conditions for profit maximising bundle size and fee were found. It is shown that the bundles offered by the producer are smaller than the optimal. Consumers are endogenously divided into those who purchase bundled and unbundled output. In equilibrium consumers obtain positive consumer surplus (whether they purchases bundled or unbundled output). It is also shown that consumers who purchase unbundled output purchase less output than is in the bundle. As all customers are assumed identical, these results are not the result of price discrimination, but can be attributed entirely to the use of non-linear pricing when some unbundling is possible.

The impact of changes in the cost of unbundling are investigated using a numerical model. The cost of unbundling would be expected to vary from market to market. When unbundling is costless, the firm is indifferent between linear pricing and non-linear pricing. However the firm adopts non-linear pricing even for small resale cost. In this event the numerical model suggests that a considerable amount of unbundling is encouraged.

The numerical analysis indicates that the size of the bundle, the price of the bundle, and the deadweight loss are monotonically increasing with respect to the cost of unbundling. However the number of bundles sold is not monotonic with respect to the cost of unbundling. Importantly, in the numerical model, the number of bundles sold for moderate values of the cost of unbundling, is lower than for extreme values. When the unbundling market is informal, this possibility may not be recognised. This could lead to an under estimate of the number of customers of a product and thus the misspecification of demand. Similarly, the efficiency loss in a market would be underestimated if the customers purchasing unbundled output are not considered by investigators.

The price per unit is also not monotonic with respect to unbundling cost. Some studies (e.g. Nevo, 2001) use price per unit to approximate a linear price for the purpose of econometric estimation. This procedure could result in misspecification biases if conducted across industries which vary in the level of unbundling cost, if this variation is not accounted for. Furthermore, in the presence of non-linear pricing with unbundling, the ‘mark-up’ of price per unit over marginal cost cannot be used as a measure of market performance. As is indicated by Table 1, those industries with the highest price per unit are those with intermediate levels of unbundling cost, and hence intermediate values of DWL.

This analysis can be used to investigate the pricing effects of the unintended consequence of some types of legislation. Legislation may have predictable and intended effects; for example food safety laws may solve an adverse selection problem regarding food quality and safety, and thus raise demand. However, in addition to the benefit or cost of the legislation, the impact on pricing may also be important. To the extent that legislation raises the cost of unbundling, its effect is to raise profits and efficiency, and lower consumer surplus.

In some markets firms may be able to take actions that discourage unbundling. The effect of this on efficiency is twofold. Restricting unbundling promotes efficiency. However the actions taken to discourage unbundling is a type of rent seeking activity. The cost of this action reduces economic efficiency. The overall impact on economic efficiency depends on which of these two effects dominates. In the numerical calculations conduced in section 3 the former effect overwhelmingly dominated, so that the actions to restrict unbundling were efficient.

The obvious strategy for the firm to adopt to discourage unbundling is to write contracts with its customers that prohibits unbundling. Such contracts may be viewed as a form of vertical restraint, and thus raise a competition/antitrust law issue. In general, the extent to which vertical restraints are in the social interest is controversial. It is shown in this paper that, because the resale market is a constraint on achieving efficiency, contracts that prohibit resale are a type of vertical restraint that promotes efficiency. On the other had this type of vertical restraint does restrict competition (specifically in the resale market) and reduces consumer surplus. As is commonly noted competition/antitrust laws are often primarily directed at promoting competition and consumer interests, with economic efficiency as a secondary goal.

To the extent that this philosophy caused a ban on contracts prohibiting resale, it would be misconceived. Intuitively, it would seem better to promote competition between manufactures rather than resellers. However this remains to be demonstrated in future work.

The analysis in this paper was greatly simplified by the assumption that all customers are identical; this approach has the additional benefit that the impact of unbundling on non-linear pricing is considered in isolation from price discrimination. Of course, customer heterogeneity is ubiquitous, and accounting for this is important in many investigations. When customers differ the firm has an incentive to engage in both price discrimination and non-linear pricing. Sibly (2009) discusses unbundling with heterogeneous customers in the case in which customers purchase a small number of discrete units.

Appendix

Proof of Proposition 1

As R≥0, by (2) Z≥Y. The Lagrangian for the firm's optimisation problem is:

$$\mathcal{L}(Z,Y)=\left(N-\left(\frac{Z-Y}{Y}\right)M(Z,Y)\right)(R(Z,Y)+P(Z)Z-cZ)+\lambda[Z-Y] \qquad (A1)$$

where λ≥0 is the Lagrange multiplier. The first order condition of (3), $\mathcal{L}_Z(Z,Y) = 0$, yields:

$$\mathcal{L}_Z(Z,Y)= -[M/Y+(Z-Y)M_Z/Y]\Pi(Z,Y)+ L(Z,Y)[R_Z+P'(Z)Z+P(Z)-c]+\lambda = 0 \qquad (A2)$$

The first order condition of (3), $\mathcal{L}_Y(Z,Y) = 0$, yields:

$$\mathcal{L}_Y(Z,Y)= [ZM/Y^2-(Z-Y)M_Y/Y]\Pi(Z,Y)+ L(Z,Y)R_Y-\lambda = 0 \qquad (A3)$$

where from (2):

$$R_Z = -P'(Z)Z$$

and

$$R_Y = +P'(Y)Y$$

and from (1):

$$M_Z = \{[P(Y) - P(Z) - P'(Z)Z] - R_Z\}/g'(M) = [P(Y) - P(Z)]/g'(M) \geq 0$$

and

$$M_Y = \{P'(Y)Z - R_Y\}/g'(M) = P'(Y)[Z-Y]/g'(M) \leq 0$$

Hence (A2) becomes:

$$\mathcal{L}_Z(Z,Y)= -[M/Y+(Z-Y)M_Z/Y]\Pi(Z,Y)+ L(Z,Y)[P(Z)-c]+\lambda \qquad (A4)$$

and (A3) becomes:

$$\mathcal{L}_Y(Z,Y)= [ZM/Y^2 - (Z-Y)M_Y/Y]\Pi(Z,Y)+ L(Z,Y)P'(Y)Y-\lambda = 0 \qquad (A5)$$

First it is shown that Z>Y. This is done by contradiction. Suppose that Z=Y. In this case $\lambda>0$. Further from (A4)

$$\mathcal{L}_Z(Z,Y)= N[P(Z)-c]+\lambda > 0 \neq 0$$

and

$$\mathcal{L}_Y(Z,Y)= NP'(Z)Z-\lambda < 0 \neq 0.$$

Hence Z=Y cannot be an equilibrium.

Equations (4) and (4) follow from (A4) and (A5) respectively.

Proof of Proposition 2:

The Lagrangian is:

$$\mathcal{L}(Z,R,Y)= \left(N-\left(\frac{Z}{Y}\right)M(Z,Y)\right)(B(Z)-cZ) + \left(\frac{Z}{Y}\right)M(Z,Y)(B(Y)-cY) – G(M)$$

$$+ \mu(L(Z,Y)(R(Z,Y)+P(Z)Z-cZ)-F) \qquad (A6)$$

where $\mu\geq 0$ is the Lagrange multipliers. The Lagrangian may be rewritten:

$$\mathcal{L}(Z,R,Y)= (1+\mu)L(Z,Y)(R(Z,Y)+P(Z)Z-cZ)+N[B(Y)-YP(Y)]+Mg(M)$$

$$–G(M)- \mu F$$

$$= (1+\mu)\pi(Z,R) +N[B(Y)-YP(Y)]+Mg(M) –G(M)- \mu F \qquad (A7)$$

where (1) and (2) has been substituted in the rearrangement. The first order condition of (A7), $\mathcal{L}_Z(Z,Y) = 0$, yields:

$$(1+\mu)\{\pi_Z(Z,R)\}+Mg'(M)M_Z= 0 \qquad (A8)$$

Hence the efficient bundle satisfies:

$$\pi_Z(Z,Y) = -M[P(Y)-P(Z)]/(1+\mu) < 0. \qquad (A9)$$

The first order condition of (A6), $\mathcal{L}_Y(Z,Y) = 0$, and (A7) yields:

$$\mathcal{L}_Y(Z,R,Y) = (1+\mu)\pi_Y(Z,Y)-NYP'(Y)+Mg'(M)M_Y =0$$

Thus the efficient bundle satisfies:

$$\pi_Y(Z,Y) = [NYP'(Y) - Mg'(M)M_Y]/(1+\mu)$$

$$= YP'(Y)L/(1+\mu) < 0 \qquad \text{(A10)}$$

From (A9) and (A10) the efficient bundle must satisfy:

$$\frac{\pi_Z(Z,Y)}{\pi_Y(Z,Y)} = \frac{-M[P(Y)-P(Z)]}{YP'(Y)L} > 0 \qquad \text{(A11)}$$

When $\mu>0$ the efficient bundle solves the (A11) and (7).

From (A2):

$$\pi_{ZY}(Z,Y) = -[M_Y/Y-M/Y^2-ZM_Z/Y^2+(Z-Y)M_{ZY}/Y]\Pi(Z,Y)$$

$$-[M/Y+(Z-Y)M_Z/Y]R_Y(Z,Y)+ [ZM/Y^2-(Z-Y)M_Y/Y][P(Z)-c] > 0$$

as $M_{ZY}=P'(Y)/g'(M) < 0$. Then the slope for the curve $\pi_Z(Z,Y)=0$ is given by:

$$\left.\frac{dY}{dZ}\right|_{\pi_Z(Z,Y)=0} = \frac{-\pi_{ZZ}(Z,Y)}{\pi_{ZY}(Z,Y)} > 0$$

Hence the curve $\pi_Z(Z,Y)=0$ is upward sloping. Similarly the slope of the curve $\pi_Y(Z,Y)=0$ is given by:

$$\left.\frac{dY}{dZ}\right|_{\pi_Y(Z,Y)=0} = \frac{-\pi_{YY}(Z,Y)}{\pi_{ZY}(Z,Y)} > 0$$

so the curve is also upward sloping. The second order conditions require that $\pi_Z(Z,Y)=0$ is steeper than $\pi_Y(Z,Y)=0$. These curves are shown in figure A1.

As $\pi_{ZZ}(Z,Y)<0$, from (A9) the efficient bundle must lie below the $\pi_Z(Z,Y)=0$ curve. Similarly as $\pi_{YY}(Z,Y)<0$, from (A10) the efficient bundle must lie above the $\pi_Y(Z,Y)=0$ curve. Consequently the efficient bundle must lie in the region identified as $\pi_Z(Z,Y)<0$ $\pi_Y(Z,Y)<0$ in figure A1. As a result the efficient values of Z and Y must lie above the equilibrium values of Z and Y. ||

<u>Proof of proposition 3:</u>

(a) For sufficiently small α (1) gives

$$P(Y)Z - [R + P(Z)Z] \approx \alpha ZM \approx 0$$

or $R+P(Z)Z\approx P(Y)Z$. As $\alpha\to 0$, (2) gives:

$$B(Z)-P(Y)Z \approx B(Y)-P(Y)Y$$

or $X \approx Y$ and $R\approx 0$. In this case $L \approx N$, and hence:

$$\mathcal{L}(Z,T,Y) \approx N(P(Z)Z-cZ)$$

The optimal value of Z is thus $Z=X_M$.

(b) For sufficiently large α (1) gives $M\approx 0$ and hence $L(Z,T,Y) \approx N$.

$$\mathcal{L}(Z,T,Y) \approx N(R(Z,Y)+P(Z)-cZ)$$

Then:

$$\mathcal{L}_Y(Z,T,Y) \approx NYP'(Y) <0.$$

Hence $Y\approx 0$. Further $\mathcal{L}_Z(Z,T,Y) = 0$ yields:

$$P(Z) \approx c\lambda$$

Hence $Z=X_E$. Thus (2) implies $T=B(X_E)$.

(c) By the envelope theorem:

$$\partial\pi/\partial\alpha = \Pi(\partial L/\partial\alpha) = \left(\frac{Y\text{-}Z}{Y}\right)\left(\frac{\partial M}{\partial\alpha}\right) > 0$$

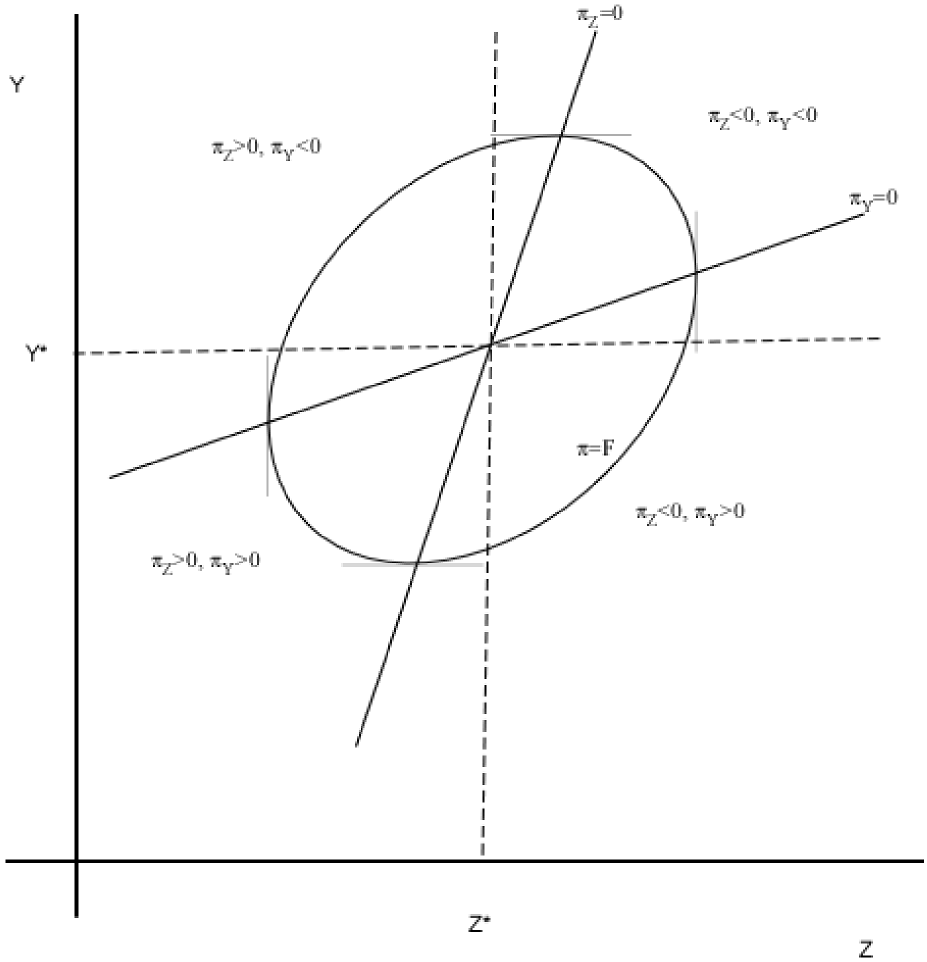

Figure A1: The firm's iso-profit curve

Table 1											
α	10^{-08}	10^{-06}	10^{-04}	0.01	0.1	1	10	100	1000	10^4	10^5
Z	4.001	4.005	4.053	4.446	5.078	6.114	7.171	7.747	7.937	7.986	7.997
Y	4	3.997	3.973	3.767	3.402	2.687	1.711	0.896	0.43	0.201	0.094
R	0.003	0.033	0.32	2.786	7.108	15.08	24.25	29.61	31.41	31.86	31.97
M	30.60	33.13	31.84	23.02	14.05	5.872	1.491	0.235	0.028	0.003	0.0003
Π	16.00	16.03	16.32	18.59	21.95	26.61	30.19	31.57	31.91	31.98	32.00
L	99.99	99.93	99.36	95.86	93.08	92.51	95.24	98.21	99.51	99.88	99.97
π	1600	1600	1620	1780	2040	2460	2880	3100	3180	3190	3200
T	24.00	24.04	24.42	27.47	32.10	38.84	44.54	47.06	47.78	47.95	47.99
T/Z	6	6.003	6.026	6.181	6.322	6.352	6.21	6.074	6.02	6.005	6.001
P(Y)	6	6.003	6.027	6.233	6.599	7.312	8.289	9.103	9.57	9.799	9.906
CS	7.999	7.989	7.894	7.097	5.786	3.610	1.463	0.402	0.093	0.020	0.004
DWL	799.9	798.9	789.2	703.3	559.1	342.6	155.8	54.32	15.10	3.669	0.835
DI	799.8	797.3	773.9	605.4	397.3	164.5	32.71	3.131	0.198	0.01	0.0005
D/B*	25.00	24.97	24.66	21.98	17.47	10.71	4.869	1.698	0.472	0.115	0.026
DI/BI	24.99	24.93	24.34	19.74	13.34	5.556	1.073	0.100	0.006	0.000	0.000

Table 2							
ς	1053.00	82.00	16.60	2.15	0.15	0.01	0.0002
α	10^{-4}	0.01	0.1	1	10	100	1000
Z	4.05	4.34	5.11	6.15	7.19	7.75	7.94
Y	3.97	3.76	3.39	2.66	1.69	0.89	0.43
R	0.32	2.33	7.30	15.40	24.41	29.63	31.41
M	32.03	16.36	14.79	6.11	1.51	0.24	0.03
Π	16.21	17.39	20.42	24.61	28.76	31.00	31.75
L	99.36	97.50	92.49	91.96	95.08	98.19	99.51
π	1611.00	1695.00	1889.00	2263.00	2734.00	3044.00	3159.00
T	24.43	26.89	32.29	39.07	44.62	47.07	47.78
T/Z	6.03	6.20	6.33	6.35	6.21	6.07	6.02
P(Y)	6.03	6.24	6.61	7.34	8.31	9.11	9.55
CS	7.89	7.08	5.73	3.53	1.43	0.40	0.09
ας	0.105	0.820	1.660	2.152	1.490	0.572	0.157
DWL	789.36	713.91	564.76	350.56	160.21	55.31	15.29
DI	773.81	653.50	387.55	156.84	31.28	3.07	0.20

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