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Intertemporal Household Demographic Models for Cross Sectional Data

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INTERTEMPORAL HOUSEHOLD DEMOGRAPHIC MODELS FOR CROSS SECTIONAL DATA

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ABSTRACT

This paper provides an intertemporal demographic model of household consumption that can be estimated using pooled cross-sectional data. By scaling within period utility by a demographic-discounting function allows the demographic effect on intertemporal allocations to be easily be examined. More specifically the demographic-discounting function can accommodate demographic expenditure shifts across time and/or the rate of time preference that can be dependent upon demographics. This is illustrated in the paper by using the presence and the number of children as the demographic variables that affect intertemporal expenditure. The model is estimated for Australian data and finds that households with a child have rates of time preference double or that they increase expenditure by 62%, than those without than those without .

Keywords: Equivalence Scales, Intertemporal Consumption, Demand Systems

JEL Classification: D1, D9, J1

I. INTRODUCTION

This paper provides an intertemporal demographic model of household expenditure that can be estimated using cross-sectional data. While allowance for demographic influences in the estimation of has been made in the past, it has frequently not been based in utility theory or when it has, has required the use of panel data. Browning, Deaton and Irish (1985), Blundell, Browning and Meaghir (1994) and the literature on intertemporal equivalence scales of Keen (1990), Pashardes (1991), Banks, Blundell and Preston (1994) have provided demographic intertemporal utility models. Of those that were empirically applicable, panel data or pseudo-panel data constructed from pooled cross-sections were required for estimation.

Intertemporal demographic models of consumption or expenditure can provide information on how demographic variables affect intertemporal expenditure over the lifetime. This information is in addition to the information from the interaction of demographics with allocation of the expenditure budget over goods and services. The inspiration for many such models is to identify the effect of children on lifetime expenditure to identify their "cost" to within period and lifetime utility for the construction of equivalence scales.

Banks, Blundell and Preston (1994) point out that full information on demographic preferences that enter lifetime utility function outside of intertemporal and atemporal expenditure behaviour can not be identified without unique information. Since the interaction of demographics with intertemporal and atemporal expenditure essentially identifies the cost of children to within period and lifetime utility, the unidentifiable component could be considered the benefit of children if children are planned. Rational household would only have children if the net effect on lifetime utility was positive. Unfortunately little information is available whether children are planned or not, the nonmaterial joy they bring coupled with expenditure behaviour, to help identify these preferences.

The use of equivalence scales has become common practice in order to make welfare or resource comparisons between households that differ in size and composition. Equivalence scales can be used to assess policy implications or compensation for households with children relative to those without. Using equivalence scales from static demand systems for welfare analysis ignores households' lifetime welfare and the allocation of their expenditure over their lifetime. For example when determining the appropriate level of government benefits for households with children relative to those without, the static analysis ignores that the household with children will eventually become a household without children.

Equivalence scales typically give the 'cost' of children relative to an adult or adult couple in terms of the additional expenditure required to keep the household at the level of welfare it would enjoy without children. Muellbauer (1974) was the first to advocate the estimation of equivalence scales in a utility theoretic framework, through the estimation static demand systems. This procedure has become a popular method of estimating equivalences amongst economists.

While the static analysis of household expenditure can provide evidence of the way household spending patterns respond to different demographics, it can not identify preferences over demographics, without making assumptions about those preferences, see Pollak and Wales (1979), Blackorby and Donaldson (1991) and Blundell and Lewbell (1991). Banks, Blundell and Preston (1994) show that in an intertemporal framework preferences over demographics independent of demands can be identified. This brings us much closer to establishing the true lifetime 'cost' of children on lifetime expenditure.

Pashardes (1991) was the first to explicitly examine the cost of children over the life-cycle and notes that households may reduce current consumption when children are not present saving for when children enter the household. Static comparisons of expenditure between demographically different households will be affected by the how willing and able parents are able to save and borrow for their child raising years. Pashardes terms an equivalence scale estimated in a static framework as an *equivalent expenditure scale* and an *equivalent income scale* as an equivalence scale developed in an intertemporal framework.

Banks, Blundell and Preston (1994) followed with a study on the intertemporal costs of children using pseudo-panel data constructed from the UK's FES from 1969 to 1988. Through simulations from the estimated parameters the authors constructed scales lifetime scales as the difference in total lifetime sum utility of a household with children and without, but found them too high without adding an arbitrary linear contribution based on the number of children.

With some simple but not unpalatable assumptions about household's expectations this paper and there effects allows an intertemporal demographic model of household consumption to be estimated from cross section data. The plan of this paper is as follows. The theoretical framework is presented and the estimating equations are derived in Section II. The data and estimation are briefly described in Section III. The results are presented and analysed in Section IV. The paper ends on the concluding note of Section V.

II. THEORETICAL FRAMEWORK

This section builds the demographic intertemporal model starting with the atemporal model in section 2.1, then the intertemporal model in 2.2 and considers the information on demographics that each can reveal. Section 2.3 specifies the within period utility function and lifetime utility which is scaled by a term that encompasses both demographic and discounting terms. The solution to optimal initial household consumption and its path are provided for general demographic-discounting term. Section 2.4 contains simplifying assumptions that make the estimation of optimal initial consumption. Section 2.5 contains a selection of specifications of the demographic-discounting term, the households' expectations about future demographics and the optimal initial consumption that each implies.

2.1 Traditional Atemporal Demographic Model of Demand

The standard atemporal or static model of the household's problem at any time *t* is to:

$$\text{Max } u_t^F = f\left(g\left(\mathbf{q}_t, \mathbf{z}_t\right), \mathbf{z}_t\right) \quad \text{subject to:} \qquad x_t = \mathbf{q}_t\,'\mathbf{p}_t \tag{1}$$

where $\mathbf{z}_t$ is a *Z* by 1 vector of *Z* demographic variables, in period *t*,

$\mathbf{q}_t$ is an *N* by 1 vector of the *N* quantity demands, in period *t*,

$\mathbf{p}_t$ is an *N* by 1 vector of the *N* price variables, in period *t*,

x_t is expenditure in period t ,

u_t^F full within period *t* utility that depends upon demographics, **z**, independently of its interaction with demands $g\left(\mathbf{q}_t, \mathbf{z}_t\right)$, and

$f(\ )$ is strictly monotonically increasing in $g\left(\mathbf{q}_t, \mathbf{z}_t\right)$ such that $u_t^F\left(\mathbf{q}_t, \mathbf{z}_t\right)$ is strictly concave in $\mathbf{q}_t$

If demographic variables directly affect utility, $u^F = f\left(g\left(\mathbf{q}_t, \mathbf{z}_t\right), \mathbf{z}_t\right)$ rather than through its interaction with demands, $\mathbf{q}_t$, then demand data can only identify preferences the

$g\left(\mathbf{q}_t, \mathbf{z}_t\right)$, which are conditional on the household's demographic vector[1]. Demand data can not provide information about $f\left(\ , \mathbf{z}_t\right)$ which is required for the construction of unconditional equivalence scales that give the true cost of demographic. This was first noted by Pollak and Wales (1979), and further investigated by Pollak and Wales (1979), Blackorby and Donaldson (1991) and Blundell and Lewbel (1991).

Atemporal demand data requires the maximisation of $u_t = g\left(\mathbf{q}_t, \mathbf{z}_t\right)$ rather than $u_t^F = f\left(u_t, \ \mathbf{z}_t\right)$ with the solution providing

the indirect utility function $u_t = v\left(x_t, \mathbf{p}_t, \mathbf{z}_t\right)$ from static demand data, but not $u_t^F = v^F\left(x_t, \mathbf{p}_t, \mathbf{z}_t\right) = f\left(v\left(x_t, \mathbf{p}_t, \mathbf{z}_t\right), \mathbf{z}_t\right)$. Other information is required in order to identify the preferences over demographics that are independent of static demand. By using information on intertemporal consumption behaviour, preferences over $f\left(g\left(\mathbf{q}_t, \mathbf{z}_t\right), \ \mathbf{z}_t\right)$ can be recovered, in particular preferences over demographics and intertemporal consumption.

2.2 Intertemporal Demographic Model of Consumption over time

Full lifetime utility, U^F can be considered function, $F[\ ]$ of within period utility, u_t^F and demographics through out life **z**, $U^F\left(\mathbf{q}, \mathbf{z}\right) = F\left[u_t^F, \mathbf{z}\right]$. Banks, Blundell and Preston (1994) point out that while information on intertemporal allocations can provide information on the preferences contained in $u_t^F = f\left(g\left(\mathbf{q}_t, \mathbf{z}_t\right), \mathbf{z}_t\right)$ it can not identify the preferences over demographic variables that enter the lifetime utility function outside of intertemporal and

[1] This is regardless of whether demographic variables, **z,** are an object of choice. If households do have control over demographic variables, conditional equivalence scales allow for excessive substitution, biasing the estimation of equivalence scales downwards.

atemporal consumption and expenditure behaviour. In which case the only information on how to restore $U(\mathbf{q},\mathbf{z}) = G\left[u_t^F\right]$ can be obtained, not $U^F(\mathbf{q},\mathbf{z}) = F\left[u_t^F, \mathbf{z}\right]$, the full information about lifetime demographic preferences.

Like previous studies this paper admits that restriction and ignores $F[\ ,\mathbf{z}]$ and focuses on the demographic influences on static and intertemporal consumption behaviour. It assumes additive separability of within period utility, $u(\mathbf{q}_t, \mathbf{z}_t)$, across time and specifies lifetime utility as the discounted-demographic adjusted sum of within period utility. Thus the household seeks to:

$$\text{Max} \quad U(\mathbf{q},\mathbf{z}) = \int_0^T u_t^F \, dt \qquad \text{subject to:} \qquad w_0 = \int_0^T e^{rt} \mathbf{q}_t{}' \mathbf{p}_t \, dt \qquad (2)$$

where

$u_t^F = f\left(g(\mathbf{q}_t, \mathbf{z}_t), \mathbf{z}_t\right)$ is the within period full utility function at period *t*,

$\mathbf{p}$ is an *N* by *T* matrix of current and future prices for the *N* goods through time *t*, $\mathbf{p} = [\mathbf{p}_0, .., \mathbf{p}_t, ..., \mathbf{p}_T]$ so that $\mathbf{p_t}$ is an *N* by 1 vector of prices at period *t*,

$\mathbf{z}$ is a *Z* by *T* matrix of current and future demographic variables through time *t*, $\mathbf{z} = [\mathbf{z}_0, .., \mathbf{z}_t, ..., \mathbf{z}_T]$ so that $\mathbf{z_t}$ is a *Z* by 1 vector of the *Z* demographic variables at period *t*, including time, *t*.

w_0 is wealth in period 0, and

r is the continuous interest rate for saving and borrowing.

Replacing $u(q_t, \mathbf{z}_t)$ with the indirect utility function $v(x_t, \mathbf{p}_t, \mathbf{z}_t)$ allows the intertemporal problem to be written:

$$\text{Max } U(w_0, \mathbf{p}, \mathbf{z}) = \int_0^T f\left(v(x_t, \mathbf{p}_t, \mathbf{z}_t), \mathbf{z}_t\right) dt \quad \text{subject to:} \qquad w_0 = \int_0^T e^{rt} x_t \, dt \quad (3)$$

The additive separable lifetime utility function allows the problem to be separated into to two stages, Banks, Blundell and Preston (1994). The first stage is the intertemporal allocation of expenditure over the life cycle (3) and the second the allocation of the given level of expenditure to the N goods, which is identical to the static demand model (1). The estimation of traditional static demand systems at the second stage can recover the parameters of $v\left(x_t, \mathbf{p}_t, \mathbf{z}_t\right)$ which can then be used in conjunction with information about intertemporal behaviour to recover the parameters of $f\left(v\left(x_t, \mathbf{p}_t, \mathbf{z}_t\right), \mathbf{z}_t\right)$.

Previous intertemporal demographic models of consumption have focussed on the evolution of consumption through time, given by the first order conditions of (3), see the appendix. This yields estimating equations that require the change in consumption, prices and demographics as variables. Such estimation requires panel data or pseudo panel data constructed by using cohort averages.

This paper instead solves the intertemporal problem for initial consumption as a function of lifetime wealth, prices and demographics. This provides estimating equations that require information about lifetime wealth, current consumption, and current and future expectations about demographics and prices. Some cross sectional data, such as consumer expenditure surveys, frequently contain this information in some respect with the exception of expectations about future demographics and prices. With the addition of assumptions of household expectations about future demographics and prices, optimal initial consumption can be estimated from cross sectional data.

2.3 Specification of Within Period and Intertemporal Utility Functions

Specifying $f\left(v\left(x_t, \mathbf{p}_t, \mathbf{z}_t\right), \mathbf{z}\right)$ as the product of within period utility $v\left(x_t, \mathbf{p}_t, \mathbf{z}_t\right)$ and by a demographic discounting term $d\left(\mathbf{z}_t\right)$ provides a convenient modification to intertemporal utility.

$$U\left(w_0,\mathbf{p},\mathbf{z}\right)=\int_0^T v\left(x_t,\mathbf{p}_t,\mathbf{z}_t\right)d\left(\mathbf{z}_t\right)dt \qquad (4)$$

Equating the demographically discounting scaled marginal utilities of expenditure $\frac{\partial v\left(x_t,\mathbf{p}_t,\mathbf{z}_t\right)}{\partial x_t}d\left(\mathbf{z}_t\right)=\frac{\partial v\left(x_s,\mathbf{p}_s,\mathbf{z}_s\right)}{\partial x_s}d\left(\mathbf{z}_s\right)$ for all *s* and *t*, >0 and < *T* provides the optimal path of consumption through time and with the wealth constraint the optimal initial consumption that maximises the household lifetime utility.

Specifying the within period utility function at period *t* as,

$$v\left(x_t,\mathbf{p}_t,\mathbf{z}_t\right)=\frac{\ln x_t-\ln a\left(\mathbf{p}_t,\mathbf{z}_t\right)}{b\left(\mathbf{p}_t,\mathbf{z}_t\right)} \qquad (5)$$

The price-demographic indices $a\left(\mathbf{p}_s,\mathbf{z}_s\right)$ and $b\left(\mathbf{p}_s,\mathbf{z}_s\right)$ characterise the shape of the Engel Curves for the *N* goods. $a\left(\mathbf{p}_s,\mathbf{z}_s\right)$ and $b\left(\mathbf{p}_s,\mathbf{z}_s\right)$ are homogenous of degree 1 and zero in prices, respectively.

The first order conditions of the Hamiltonian (see the Appendix for full details) provide the evolution of consumption through time

$$x_t=\left(\frac{d\left(\mathbf{z}_t\right)}{b\left(\mathbf{p}_t,\mathbf{z}_t\right)}\Bigg/\frac{d\left(\mathbf{z}_0\right)}{b\left(\mathbf{p}_0,\mathbf{z}_0\right)}\right)e^{rt}x_o\,. \qquad (6)$$

By inserting the optimal path into the lifetime wealth constraint gives optimal initial consumption as,

$$x_0=\left(\frac{d\left(\mathbf{z_0}\right)}{b\left(\mathbf{p}_0,\mathbf{z}_0\right)}\Bigg/\int_0^T\frac{d\left(\mathbf{z}_s\right)}{b\left(\mathbf{p}_s,\mathbf{z}_s\right)}ds\right)\tilde{w}_0\,. \qquad (7)$$

given initial lifetime wealth $\tilde{w}_0=w_0-w_Te^{-rT}+\int_0^T e^{-rs}y_s ds$.

2.4 Simplifying Assumptions

Time and Future Birth Expectations

Time at t = 0, can be considered the current point in time in which we observe a household. In which case T is remaining lifetime and is equal to life expectancy less age. It is assumed that households without children at time 0, do not plan on having any children. Essentially all children are surprises and there are no expectations of any future births.

To allow the demographic effect on intertemporal expenditure $d\left(\mathbf{z}_t\right)$ to be isolated from the effect of future demographic profiles and prices on atemporal expenditure allocation a simplification of the price-demographic indices is required. Thus in all time periods $s, b(\ )$ is specified as $b\left(\mathbf{p}_s, \mathbf{z}_s\right) = b\left(\mathbf{p}_s, \mathbf{z}_0\right)$ a function of prices through time $\boldsymbol{p}$ and only the current demographic profile, $\mathbf{z}_0$.

Demographic effects in this case effect the level and slope of the Engel Curves but expectations about demographic variables $\mathbf{z}$ are only allowed to effect within period behaviour via $a\left(\mathbf{p}_s, \mathbf{z}_s\right)$ and so restricted to scaling income and providing changes in the intercept of the Engel curves. If $a(\ )$ is also specified in the same way $a\left(\mathbf{p}_s, \mathbf{z}_s\right) = a\left(\mathbf{p}_s, \mathbf{z}_0\right)$, or if the marginal utility is independent of $a(\ )$, as is he case for logarithmic utility, then within period demographic effects can not impact on intertemporal consumption. Essentially the expectations of future demographic components of $a(\ )$ and $b(\ )$ are assumed to be independent of prices and subsumed into $d\left(\mathbf{z}_t\right)$.

Price Expectations

Households assume that relative prices are constant but that the price of all goods rise with the rate if inflation, π with certainty such that $\mathbf{p}_1 = e^{\pi t}\mathbf{p}_0$ in which case

$a(\mathbf{p}_t, \mathbf{z}_0) = e^{\pi t} a(\mathbf{p}_0, \mathbf{z}_0)$ and $b(\mathbf{p}_t, \mathbf{z}_0) = e^{\pi t} b(\mathbf{p}_0, \mathbf{z}_0)$ if $a(\mathbf{p}_t, \mathbf{z_0})$ and $b(\mathbf{p}_t, \mathbf{z_0})$ are homogenous of degree 1 and zero with respect to prices respectively.

$$x_t = x_o \frac{d(\mathbf{z}_t, t)}{d(\mathbf{z}_0, 0)} e^{(r-\pi)t} \qquad x_0 = \frac{d(\mathbf{z_0}, 0)}{\int_0^T e^{-\pi s} d(\mathbf{z}_s, s) ds} \tilde{w}_0 \tag{8}$$

Income Expectations

Households assume that incomes grow with the rate of inflation are constant but that the price of all goods rise with the rate if inflation, π such that $y_t = e^{\pi t} y_0$ in which case $\int_0^t e^{-rs} y_s ds = \int_0^t e^{-(r-\pi)s} y_0 ds$

Data on the growth household's income is generally not available with cross-sectional data and requires panel data. For this reason it was not included in the model.

For simplicity this paper will assume no bequest motive such that the household aims to run down its stock of wealth at terminal time T, $w_T = 0$. These two assumptions allow the household wealth constraint to be written

$$\tilde{w}_0 = w_0 + y \frac{1 - e^{-(r-\pi)T}}{(r-\pi)}. \tag{9}$$

Lifetime wealth can be constructed from cross sectional data on that provides information on current wealth and income. Alternatively this information can be used to estimate lifetime wealth over a range of demographic variables especially occupation, education.

2.5 Specification of d(z) and Demographic Expectations

Two Simple Non-Demographic Cases

To illustrate the simple mechanics of the model specifying $d(\mathbf{z}_t) = d(t) = 1$ so that there is no discounting and no demographic effects in which case optimal initial consumption $x_0 = w_0 \frac{1}{T}$. That is the household is simply dividing their lifetime wealth evenly among their remaining lifetime. Or with inflation $x_0 = \frac{\pi}{1-e^{-\pi T}} \tilde{w}_0$ and the household's initial optimal consumption is the PV in terms of inflation of initial lifetime wealth.

If $d(\mathbf{z}_t)$ is used to discount future utility by a rate of time preference, δ_0, such that $d(\mathbf{z}_t) = d(t) = e^{-\delta_0 t}$ then optimal initial consumption is $x_0 = \frac{\delta_0}{\left(1-e^{-\delta_0 T}\right)} w_0$ and $x_0 = \frac{\delta_0 + \pi}{\left(1-e^{-(\delta_0+\pi)T}\right)} w_0$ with inflation. These two results can be considered the optimal initial consumption for the reference household for which $d(\mathbf{z}_t)$ is normalised to unity for the current time period, such that $d\left(\mathbf{z}_0^R\right) = 1$.

Demographics and Time

If households believe that their current demographic profile will not change such that $\mathbf{z_s} = \mathbf{z}_0$ for all s periods, and that it does not interact with time then optimal initial consumption (and its path) are unaffected by demographics in $\mathbf{z}_0$. For example if $d(\mathbf{z_s}) = d(\mathbf{z}_0) e^{-\delta_0 t}$ then $x_0 = \frac{d(\mathbf{z_0}, 0)}{\int_0^T e^{-(\pi+\delta_0)s} d(\mathbf{z}_0, s)\, ds} \tilde{w}_0 = \frac{d(\mathbf{z_0}, 0)}{d(\mathbf{z_0}, 0)\int_0^T e^{-(\pi+\delta_0)s} ds} \tilde{w}_0$ thus,

$$x_0 = \frac{\delta_0 + \pi}{\left(1 - e^{-(\delta_0+\pi)T}\right)} w_0 \qquad (10)$$

or in the absence of inflation and discounting simply $x_0 = w_0 \frac{1}{T}$.

Thus in order for demographic effects to have an influence on intertemporal consumption the demographic component of must change over time $d(\mathbf{z}_t)$. This can be achieved through demographics affecting the rate of time preference, or via expectations about the changes in the household's demographic profile. Such expectations can be modelled as

$$\mathbf{z}_t = \boldsymbol{\omega}_t \mathbf{Z} \qquad (11)$$

Where $\mathbf{z_t}$ is a Z by 1 vector of the Z demographic variables for the household at t.

$\mathbf{Z}$ is a Z by 1 vector of the possible demographic variables for all households through out time,

$\boldsymbol{\omega}_t$ is a Z by Z matrix of variables indicating the households expectation of whether the demographic variables at time t are in effect, $\boldsymbol{\omega}_t = \begin{matrix} \omega_{1,t} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \omega_{Z,t} \end{matrix}$ where $\omega_{k,t} = 1$ if the k^{th} demographic variable is in effect in period t.

The first demographic variable in $\mathbf{Z}$ is the existence of the household, and $\omega_{1,t}$ is equal to unity in all periods t such that the first row of $\mathbf{z_t}$ is also 1 for all periods. This allows the expenditure behaviour of the reference household to be identified. In which case for the non-discounting and discounting model $d(\mathbf{z}_t)$ may be specified as $d(\mathbf{z}_t) = \boldsymbol{\kappa}'\mathbf{z}_t$ and $d(\mathbf{z}_t) = e^{-\boldsymbol{\delta}'\mathbf{z}_t t}$, respectively where $\boldsymbol{\kappa}$ and $\boldsymbol{\delta}$ are Z by 1 vectors of parameters on the Z demographic variables at time t. The two models can be combined as $d(\mathbf{z}_t) = \boldsymbol{\kappa}'\mathbf{z}_t e^{-\boldsymbol{\delta}'\mathbf{z}_t t}$.

Children as an example of intertemporal demographic effects

By specifying the 2nd to (nc+1)th row of **Z** to be the presence of 1 to nc children allows the demographic effect of children of children to be examined. By allowing $\boldsymbol{\omega}_t$ and thus $\mathbf{z_t}$ to change through time will result in demographic effects on intertemporal demand.

Demographic Intertemporal Shift Effects

Ignoring discounting for the moment allows a $d\left(\mathbf{z_s}\right)$ to be specified as a demographic intertemporal shift effect,

$$d\left(\mathbf{z_s}\right)=1+\sum_{k}^{nc}\kappa_k z_{k,s} \tag{12}$$

so that optimal expenditure is

$$x_0=\frac{1+\sum_{k}^{nc}\kappa_k z_{k,s}}{\int_0^T\left(1+\sum_{k}^{nc}\kappa_k z_{k,s}\right)ds}\tilde{w}_0=\frac{1+\sum_{k}^{nc}\kappa_k z_{k,s}}{T+\sum_{k}^{nc}\kappa_k\int_0^T z_{k,s}ds}\tilde{w}_0 \tag{13}$$

where $z_{k,s}$ is equal to 1 if the child is living in or dependent on the household in period s and zero otherwise.

If the effect of children on intertemporal consumption is permanent for the households remaining life then $z_{k,s}=1$ for all t periods. In which case the denominator simplifies to $T+\sum_{k}^{nc}\kappa_k T$ and optimal initial consumption is $x_0=\frac{1}{T}\tilde{w}_0$ and is unaffected by demographics (as is its path).

i) Temporary Demographic Intertemporal Shift Effects

If households with children expect to pay for their consumption until the age they leave home *la*, then they age $(la - ca_k)$ years remaining of supporting the k^{th} child, where ca_k is child *k*'s age. Then $z_{k,t} = 1$ for $s \le (la - ca_k)$ and $z_{k,t} = 0$ for $s > (la - ca_k)$ in which case initial expenditure is given by,

$$x_0 = \left(1 + \sum_k^{nc} \kappa_k z_{k,0} \middle/ \left(T + \sum_k^{nc} \kappa_k (la - ca_k)\right)\right) \tilde{w}_0 \qquad \text{(model 1)}$$

Estimates of κ_k and $d(\mathbf{z_s})$ can be recovered through estimating model 1 with knowledge of the ages of the children and specifying the age the they are no longer dependent, *la*, for example 18, 21 or 25.

ii Temporary Demographic Intertemporal Shift Effects with Discounting

Discounting can easily be introduced into the above model by scaling demographic shift effect by a discounting term $e^{-\delta_0 s}$ to give, $d(\mathbf{z_s}) = e^{-\delta_0 s}\left(1 + \sum_k^{nc} \kappa_k z_{k,s}\right)$. Optimal initial expenditure is given by

$$x_0 = \frac{1 + \sum_k^{nc} \kappa_k z_{k,0}}{\frac{1}{\delta_0}\left(1 - \text{Exp}[-\delta_0 T]\right) + \sum_k^{nc} \kappa_k z_{k,0} \frac{1}{\delta_0}\left(1 - \text{Exp}[-\delta_0 (la - ca_k)]\right)} \qquad \text{(model 2)}$$

Demographic Discounting

By allowing the rate of time preference to vary according to demographics, more specifically with the number of children, the demographic effect on intertemporal allocations can be easily be examined. One possible specification for $d(\mathbf{z}_t)$ is to allow demographics to adjust the discount rate such that

$$d(\mathbf{z}_t) = \text{Exp}\left[-(\delta_0 + \boldsymbol{\delta}'\mathbf{z}_t)t\right] = \text{Exp}\left[-\left(\delta_0 + \sum_k^{nc} \delta_k z_{k,t}\right)t\right]$$

Where $\boldsymbol{\delta}$ is a *Z* by 1 vector of parameters on the *Z* demographic variables at time *t*, $\mathbf{z}_t$

iii Permanent Demographic Discounting

If the effect on demographic on discounting is permanent so that $z_{k,s} = z_{k,0} = 1$ for all *s* then optimal initial consumption is

$$x_0 = \frac{\boldsymbol{\delta}'\mathbf{z}_t}{1 - \text{Exp}\left[(-\boldsymbol{\delta}'\mathbf{z}_t)T\right]} w_0 \qquad \text{(model 3)}$$

and its optimal path $x_t = x_o \text{Exp}\left[(r - \boldsymbol{\delta}'\mathbf{z}_t)t\right]$. Thus changes in the expected future demographic profile are not required for demographic effects on intertemporal expenditure so long the demographic profile interacts with time.

iv Temporary Demographic Discounting

If households with children expect to pay for their consumption until the age they leave home *la*, then they age $(la - ca_k)$ years remaining of supporting the k^{th} child, where ca_k is child *k*'s age. Then $z_{k,t} = 1$ for $s \le (la - ca_k)$ and $z_{k,t} = 0$ for $s > (la - ca_k)$ and optimal initial consumption is

$$x_0 = \frac{1}{\frac{1}{\delta_0}\left(1-\mathrm{Exp}\left[-\delta_0 T\right]\right)+\frac{1}{\delta_0+\sum_k^{nc}\delta_k z_{k,t}}\left(1-\mathrm{Exp}\left[-\left(\delta_0+\sum_k^{nc}\delta_k z_{k,t}\right)(la-ca_k)\right]\right)}$$ (mo

del 4)

v. Temporary Shift Effects with Permanent Demographic Discounting

$$x_0 = \frac{1+\sum_k^{nc}\kappa_k z_{k,0}}{\frac{1}{\delta_0}\left(1-\mathrm{Exp}\left[-\delta_0 T\right]\right)+\sum_k^{nc}\kappa_k z_{k,0}\frac{1}{\delta_0+\sum_k^{nc}\delta_k z_{k,t}}\left(1-\mathrm{Exp}\left[-\left(\delta_0+\sum_k^{nc}\delta_k z_{k,t}\right)(la-ca_k)\right]\right)}$$

(model 5)

vi. Temporary Shift Effects with Temporary Demographic Discounting

$$x_0 = \frac{1+\sum_k^{nc}\kappa_k z_{k,0}}{\frac{1}{\delta_0+\sum_k^{nc}\delta_k z_{k,t}}\left(1-\mathrm{Exp}\left[-\left(\delta_0+\sum_k^{nc}\delta_k z_{k,t}\right)T\right]\right)+\sum_k^{nc}\kappa_k z_{k,0}\frac{1}{\delta_0+\sum_k^{nc}\delta_k z_{k,t}}\left(1-\mathrm{Exp}\left[-\left(\delta_0+\sum_k^{nc}\delta_k z_{k,t}\right)(la\right.\right.}$$

(model 6)

III. DATA, ESTIMATION AND METHODOLOGY

The models are crudely estimated from Australian cross-sectional data to illustrate and investigate their performance. The Household Expenditure Survey (HES) confidentialised unit record files (CURFs) from the Australian Bureau of Statistics (ABS) 1993-94 is used to obtain household data on regular income, income from wealth and demographics. The sample was restricted to two adult households, for a sample of 4933 observations.

The HES datasets do not contain data on wealth but do contain property income, financial income (income from financial institutions) and capital income (income from investments in capital such as dividends, trusts, debentures). By dividing the income from an asset by the rate of return, an estimate of the level of assets can be obtained. Rates of return from the RBA were used to construct a weighted average rate of return for financial assets and capital assets, where the weights were taken from a supplement to the 1993-94 HES on the proportion of investment types held by households in 1993-94. The rate of return on property was assumed to be 5% for all surveys.

Table 1 Rates of Return for Wealth Estimation

Year	Nominal Rate of Return on Financial Assets	Nominal Rate of Return on Capital Assets	Nominal Rate of Return on Property Assets
1993 - 94	3.43%	4.48%	5.00%

The constant interest rate used to obtain human wealth was also chosen to be 5% and this is the figure used the calculation of the equivalence scales. Terminal time (estimated time of death) was specified as 90 years significantly high enough to ensure that all the sample were alive. Thus *T* remaining lifetime is equal to *death* = 90 minus the household head's age. The expected leaving age of dependents is set to 25, to recognised the fact that parents often support their children into their early 20s and the fact that sample contains dependents who are up to 25 years old.

An intercept and intercept dummies where included for state/territory (*STATE*) of residence and quarter of the year (*QTR*) in which the household was surveyed. Thus the equation to be estimated for each model was:

$$x_0 = \alpha_0 + \sum_{i=2}^{4} \alpha_i QTR_i + \sum_{j=2}^{8} \beta_j STATE_j + mpc \times \tilde{w}_0 + \varepsilon_t$$

where ε_t is the error term $N\left(0,\sigma^2\right)$, mpc is the *mpc* function for each model, which includes variables: lifetime remaining *T and* demographic variables $\mathbf{z}$**,** and parameters to be estimated, δ_0 and κ_k, δ_k for each k^{th} demographic effect. The models were estimated using Non Linear OLS using SAS v8.0.

IV. RESULTS

The tables below provide the $\bar{R}^2$, Log-likelihood score, (LL) and the parameters of interest, the intertemporal expenditure effect of children, κ_k and δ_k. The majority of the intercepts, state and quarter dummies were significant (available on request).

Table 1 Model 1 Parameter Estimates- Shift Model (temporary)

Model 1		κ_1	κ_2	κ_3
LL	Estimate	0.6159	0.4130	0.5493
-56503	SE	0.0551	0.1111	0.0936
$\bar{R}^2$	t-ratio	11.19	3.72	5.87
0.0199				

Table 2 Model 2 Parameter Estimates – Shift Model (temporary) with Discounting

Model 2		δ_0	κ_1	κ_2	κ_3
LL	Estimate	0.0116	0.3450	0.2964	0.3456
-56481	SE	0.0008	0.0451	0.0821	0.0669
$\bar{R}^2$	t-ratio	14.75	7.66	3.61	5.16
0.0284					

Model 1 estimate's suggest that a household with a single child raises its expenditure by 62% compared to when it does not have a child present. Expenditure is estimated to be 103% for a household with 2 children and additional 55% for each child after the 2nd. Allowing for standard discounting as in model 2, reduces the size of this demographic shift, to about 30% to 35% per child with the rate of time preference estimated to be 1.16%.

Model 3 estimates suggest that a household without children has a rate of time preference of 1.11%. But a household with a child has discount rate of 2.20% for the rest of its life. The effect of subsequent children on the rate of time preference is significant but smaller. A household with two children is estimated to have a rate of time preference of 2.75%.

Table 3 Model 3 Parameter Estimates –Discounting Model (permanent)

Model 3		δ_0	δ_1	δ_2	δ_3
LL	Estimate	0.0111	0.0109	0.0055	0.0022
-56484	SE	0.0008	0.0012	0.0013	0.0005
$\bar{R}^2$	t-ratio	13.77	9.30	4.23	4.13
0.0271					

Model 4, restricts the demographic effects of discounting to the years that children are dependent on the household. This as expected, increases the magnitude of the effect of children on the rate of time preference since the effects are over a shirt time period. The δ_0 parameter now, not only acts as the discount rate for households that never have children but also those that are no longer maintaining children. The estimate of 1.73% is higher than for the permanent demographic discounting model, where it was only applicable to households that never (or do not expect) to have children. This illustrates that there is some "lasting effect" on households' discount rate that persists after children are no longer dependent.

Table 4 Model 4 Parameter Estimates –Discounting Model (temporary)

Model 4		δ_0	δ_1	δ_2	δ_3
LL	Estimate	0.0173	0.0176	0.0625	0.0030
-56533	SE	0.0006	0.0074	0.0088	0.0010
$\bar{R}^2$	t-ratio	29.35	2.40	7.07	2.95
0.0076					

This model yields an estimate for a rate of term preference of 3.49% while the first child is present. It also provides are rather high estimate for the effect of second child while present with the first of 9.74%.

Combining the demographic intertemporal shift and demographic discounting models in model 5 and 6 improves the fit of the regression but the estimates of the demographic parameters become somewhat confused for model 5. Only the third child has a significant demographic shift effect and third child demographic discounting is significantly negative.

Table 5 Model 5 Parameter Estimates –
Shift (temporary) and (permanent) Discounting Model

Model 5		κ_1	κ_2	κ_3	δ_0	δ_1	δ_2	δ_3
LL	Estimate	0.0869	0.1231	0.7766	0.0106	0.0084	0.0016	-0.0052
-56474	SE	0.0986	0.1940	0.2208	0.0008	0.0031	0.0043	0.0019
$\bar{R}^2$	t-ratio	0.88	0.63	3.52	12.89	2.70	0.37	-2.65
0.0305								

The demographic discounting terms in Model 6 are insignificant and so the results are similar to model 2 – demographic shifts with standard discounting.

Table 6 Model 6 Parameter Estimates –
Shift (temporary) and (temporary) Discounting Model

Model 6		κ_1	κ_2	κ_3	δ_0	δ_1	δ_2	δ_3
LL	Estimate	0.3228	0.2410	0.3780	0.0113	0.0090	0.0054	0.0007
-56479	SE	0.0495	0.1069	0.1126	0.0008	0.0074	0.0081	0.0016
$\bar{R}^2$	t-ratio	6.52	2.26	3.36	14.11	1.21	0.67	0.43
0.0285								

V. CONCLUSION

This paper has proposed a method for estimating an intertemporal or lifetime equivalence scale without the need for panel data, by solving the optimal intertemporal allocations of expenditures as a function of initial lifetime wealth. Demographic variables affect the intertemporal allocations of expenditure by altering the rate of time preference, which is shown to be the marginal propensity to consume out of wealth. This allows the estimation of an intertemporal equivalence scale, as the ratio of lifetime expenditures of a particular household to the reference household's.

The presence of one child is estimated to raise expenditure by 62% in the period the child is dependent. While the demographic discounting model provides an estimate of a permanent discount rate of 2.20% for a household with one child compared to 1.11% for that never has a child.

The major limitation of the papers implementation of the model is the simplifying assumptions about price, income and demographics expectations made, principally due to the lack of any data on such information from cross section data. None the less sensible estimates are obtained from the two basic model types. More advanced modelling of such expectations can be made worthwhile if the information is available from data sets. The specification of the within period utility as AIDS allows the recovery of evolution of expenditure with ease but has linear Engel curves and no rich versus poor effects of non-linear models. Most of the improvements in the standard intertemporal utility maximising models such as liquidity constraints, finite lifetimes and uncertainty can be relatively easily incorporated into the paper's intertemporal demographic model.

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APPENDIX

The General Intertemporal Model of the Household

Maximise $U\left(w_0, \mathbf{p}, \mathbf{z}\right) = \int_0^T f\left(u\left(c_t, \mathbf{p}_t, \mathbf{z}_t\right), \mathbf{z}_t, t\right) dt$ (A1)

subject to $\dot{w}_t = rw_t + y_t - c_t$ (A2)

$w_T = 0$ (A3)

where $u(c_t, \mathbf{p}_t, \mathbf{z}_t)$ is the within period utility function at period *t*.

$\mathbf{p}$ is an *N* by T matrix of current and future prices for the *N* goods through time *t*, so that $\mathbf{p_t}$ is an *n* by 1 vector of prices at period *t*.

$\mathbf{z}$ is a *Z* by T matrix of current and future, *Z* demographic variables through time *t*, so that $\mathbf{z_t}$ is a *Z* by 1 vector of demographic variables at period *t*.

$\dot{w}_t$ is the change in financial wealth over time

w_t is financial wealth in period t,

c_t is consumption in period t,

y_t is labour income in period t,

r is the continuous interest rate for saving and borrowing ,

$$H = f\left(u\left(c_t, \mathbf{p}_t, \mathbf{z}_t\right), \mathbf{z}_t, t\right) + \lambda\left(rw_t + y_t - c_t\right) \quad \text{(A4)}$$

H1: $\frac{\partial H}{\partial c_t} = 0 \quad \Rightarrow \quad \lambda(t) = \frac{\partial f\left(u\left(c_t, \mathbf{p}_t, \mathbf{z}_t\right), \mathbf{z}_t, t\right)}{\partial c_t}$

H2: $\frac{\partial H}{\partial \lambda} = \dot{w} \quad \Rightarrow \quad \frac{dw_t}{dt} = r\ w_t + y_t - c_t$

$$\Rightarrow \quad w_t e^{-rt} = w_0 + \int_0^t e^{-rs} y_s ds - \int_0^t e^{-rs} c_s ds$$

H3: $\frac{\partial H}{\partial w_t} = -\dot{\lambda} \quad \Rightarrow \quad \frac{d\lambda_t}{dt} = -r\lambda_t$

$$\Rightarrow \quad \lambda_t = \lambda_0 e^{-rt}$$

With H1 and H3 providing the solution for the path of optimal consumption such that

$$\frac{\partial f\left(u\left(c_t,\mathbf{p}_t,\mathbf{z}_t\right),\mathbf{z}_t,t\right)}{\partial c_t}=\frac{\partial f\left(u\left(c_0,\mathbf{p}_0,\mathbf{z}_0\right),\mathbf{z}_0,0\right)}{\partial c_0}e^{-rt} \tag{A5}$$

can be used with H2 to find optimal initial consumption.

Specifying Utility

$$U\left(w_0,\mathbf{p},\mathbf{z}\right)=\int_0^T\left(\frac{\ln c_t-\ln a\left(\mathbf{p}_t,\mathbf{z}_t\right)}{b\left(\mathbf{p}_t,\mathbf{z}_t\right)}\right)d\left(\mathbf{z}_t\right)dt \tag{A6}$$

To illustrate specifying

$$f\left(u\left(c_t,\mathbf{p}_t,\mathbf{z}_t\right),\mathbf{z}_t,t\right)=f\left(c_t,\mathbf{p}_t,\mathbf{z}_t\right)d\left(\mathbf{z}_t\right)=\frac{1}{\theta}\left(\frac{c_t}{a\left(\mathbf{z}_t,\mathbf{p}_t\right)}\right)^{\theta}\frac{1}{b\left(\mathbf{z}_t,\mathbf{p}_t\right)}d\left(\mathbf{z}_t\right) \tag{A7i}$$

$$f\left(u\left(c_t,\mathbf{p}_t,\mathbf{z}_t\right),\mathbf{z}_t,t\right)=f\left(c_t,\mathbf{p}_t,\mathbf{z}_t\right)d\left(\mathbf{z}_t\right)=\frac{\ln c_t-\ln a\left(\mathbf{z}_t,\mathbf{p}_t\right)}{b\left(\mathbf{z}_t,\mathbf{p}_t\right)}d\left(\mathbf{z}_t\right) \tag{A7ii}$$

Then

$$H=\frac{\ln c_t-a\left(\mathbf{z}_t,\mathbf{p}_t\right)}{b\left(\mathbf{p}_t\right)}d\left(\mathbf{z}_t\right)+\lambda\left(rw_t+y_t-c_t\right) \tag{A8i}$$

$$H=\frac{1}{\theta}\left(\frac{c_t}{a\left(\mathbf{z}_t,\mathbf{p}_t\right)}\right)^{\theta}\frac{1}{b\left(\mathbf{z}_t,\mathbf{p}_t\right)}d\left(\mathbf{z}_t\right)+\lambda\left(rw_t+y_t-c_t\right) \tag{A8iI}$$

H1: $\frac{\partial H}{\partial c}=0 \quad \Rightarrow \quad \lambda\left(t\right)=\frac{d\left(\mathbf{z}_t,t\right)}{b\left(\mathbf{p}_t\right)c_t}$ (A9i)

$$\lambda\left(t\right)=\left(\frac{c_t}{a\left(\mathbf{p}_t,\mathbf{z}_t\right)}\right)^{\theta-1}\frac{d\left(\mathbf{z}_t\right)}{a\left(\mathbf{p}_t,\mathbf{z}_t\right)b\left(\mathbf{p}_t,\mathbf{z}_t\right)} \tag{A9ii}$$

From H1 when $t=0$ then $\lambda\left(t\right)$ is

$$\lambda_0 = \frac{d\left(\mathbf{z}_0\right)}{b\left(\mathbf{p}_0,\mathbf{z}_0\right)c_0} \tag{A10i}$$

$$\lambda_0 = \left(\frac{c_0}{a\left(\mathbf{p}_0,\mathbf{z}_0\right)}\right)^{\theta-1} \frac{d\left(\mathbf{z}_0\right)}{a\left(\mathbf{p}_0,\mathbf{z}_0\right)b\left(\mathbf{p}_0,\mathbf{z}_0\right)} \tag{A10ii}$$

Combing the above with H3 gives the optimal consumption path

$$\begin{aligned} \frac{d\left(\mathbf{z}_t,t\right)}{b\left(\mathbf{p}_t\right)c_t} &= \frac{d\left(\mathbf{z}_0,0\right)}{b\left(\mathbf{p}_0\right)c_0}e^{-rt} \\ c_t &= c_o\frac{d\left(\mathbf{z}_t,t\right)}{d\left(\mathbf{z}_0,0\right)}\frac{b\left(\mathbf{p}_0\right)}{b\left(\mathbf{p}_t\right)}e^{rt} \end{aligned} \tag{A11i}$$

$$\begin{aligned} \frac{d\left(\mathbf{z}_t,t\right)}{b\left(\mathbf{p}_t\right)c_t} &= \left(\frac{c_0}{a\left(\mathbf{p}_0,\mathbf{z}_0\right)}\right)^{\theta} \frac{d\left(\mathbf{z}_0\right)}{a\left(\mathbf{p}_0,\mathbf{z}_0\right)b\left(\mathbf{p}_0,\mathbf{z}_0\right)}e^{-rt} \\ \left(\frac{c_t}{a\left(\mathbf{p_t},\mathbf{z}_t\right)}\right)^{\theta} &= \left(\frac{c_0}{a\left(\mathbf{p}_0,\mathbf{z}_0\right)}\right)^{\theta} \frac{a\left(\mathbf{p_t},\mathbf{z}_t\right)b\left(\mathbf{p_t},\mathbf{z}_t\right)}{a\left(\mathbf{p}_0,\mathbf{z}_0\right)b\left(\mathbf{p}_0,\mathbf{z}_0\right)}\frac{d\left(\mathbf{z}_0\right)}{d\left(\mathbf{z}_t\right)}e^{-rt} \\ c_t &= \frac{a\left(\mathbf{p_t},\mathbf{z}_t\right)}{a\left(\mathbf{p}_0,\mathbf{z}_0\right)}\left(\frac{a\left(\mathbf{p_t},\mathbf{z}_t\right)b\left(\mathbf{p_t},\mathbf{z}_t\right)}{a\left(\mathbf{p}_0,\mathbf{z}_0\right)b\left(\mathbf{p}_0,\mathbf{z}_0\right)}\frac{d\left(\mathbf{z}_0\right)}{d\left(\mathbf{z}_t\right)}e^{-rt}\right)^{\frac{1}{\theta}}c_0 \end{aligned} \tag{A11ii}$$

$$c_t = \frac{1}{a\left(\mathbf{p}_0,\mathbf{z}_0\right)}\left(\frac{d\left(\mathbf{z}_0\right)}{a\left(\mathbf{p}_0,\mathbf{z}_0\right)b\left(\mathbf{p}_0,\mathbf{z}_0\right)}\right)^{\frac{1}{\theta}} a\left(\mathbf{p_t},\mathbf{z}_t\right)\left(\frac{a\left(\mathbf{p_t},\mathbf{z}_t\right)b\left(\mathbf{p_t},\mathbf{z}_t\right)}{d\left(\mathbf{z}_t\right)}\right)^{\frac{1}{\theta}} e^{-rt\left(1+\frac{1}{\theta}\right)}c_0$$

Inserting the above equation into the equation of motion for wealth H2 gives

$$w_t e^{-rt} = w_0 + \int_0^t e^{-rs}y_s ds - c_0\frac{b\left(\mathbf{p}_0,\mathbf{z}_0\right)}{d\left(\mathbf{z}_0,0\right)}\int_0^t \frac{d\left(\mathbf{z}_s,s\right)}{b\left(\mathbf{p}_s,\mathbf{z}_s\right)}ds\,. \tag{A12i}$$

$$w_t e^{-rt} = w_0 + \int_0^t e^{-rs}y_s ds - c_0\frac{1}{a\left(\mathbf{p}_0,\mathbf{z}_0\right)}\left(\frac{d\left(\mathbf{z}_0,0\right)}{a\left(\mathbf{p}_0,\mathbf{z}_0\right)b\left(\mathbf{p}_0,\mathbf{z}_0\right)}\right)^{\frac{1}{\theta}}\int_0^t a\left(\mathbf{p}_s,\mathbf{z}_s\right)\left(\frac{a\left(\mathbf{p}_s,\mathbf{z}_s\right)b\left(\mathbf{p}_s,\mathbf{z}_s\right)}{d\left(\mathbf{z}_s,s\right)}\right)^{\frac{1}{\theta}}e^{-rs\left(1+\frac{1}{\theta}\right)}ds$$

.. (A12ii)

Setting t=T to find c_0.

$$w_T e^{-rT} = w_0 + \int_0^T e^{-rs}y_s ds - c_0\frac{b\left(\mathbf{p}_0,\mathbf{z}_0\right)}{d\left(\mathbf{z}_0,0\right)}\int_0^T \frac{d\left(\mathbf{z}_s,s\right)}{b\left(\mathbf{p}_s,\mathbf{z}_s\right)}ds$$

Defining $\tilde{w}_0 = w_0 - w_T e^{-rT} + \int_0^T e^{-rs} y_s ds$ allows optimal initial consumption to be written

$$c_0 = \tilde{w}_0 \frac{d(\mathbf{z_0}, 0)}{b(\mathbf{p}_0)} \frac{1}{\int_0^T \frac{d(\mathbf{z}_s, s)}{b(\mathbf{p}_s)} ds} \tag{A13ii}$$

$$c_0 = w_0\, a(\mathbf{p}_0, \mathbf{z}_0) \left(\frac{a(\mathbf{p}_0, \mathbf{z}_0) b(\mathbf{p}_0, \mathbf{z}_0)}{d(\mathbf{z}_0)} \right)^{\frac{1}{\theta}} \Bigg/ \int_0^T a(\mathbf{p_t}, \mathbf{z}_t) \left(\frac{a(\mathbf{p_t}, \mathbf{z}_t) b(\mathbf{p_t}, \mathbf{z}_t)}{d(\mathbf{z}_t, t)} \right)^{\frac{1}{\theta}} e^{-rt\left(1+\frac{1}{\theta}\right)} dt$$

(A13ii)

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