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Abstract

Recent literature has addressed how product creation amplifies economic fluctuations via the love of variety. Yet, the empirical evidence on variety effects is sparse. The current paper demonstrates that decreasing returns in the variety-level production technology, which leads to increasing marginal costs, can similarly amplify business cycles. An expansion of the firm's product scope reduces marginal costs and gives an incentive to produce multiple products even if the variety effects are entirely absent. The efficiency gains from adjusting product scopes makes the economy more susceptible to sunspot equilibria. The model is estimated via Bayesian methods and data favours mild decreasing returns with animal spirits explaining a significant fraction of U.S. business cycles.

Keywords: Indeterminacy, sunspot equilibria, multi-product firms, business cycles, Bayesian estimation.
JEL Classification: E32, L11.

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1 Introduction

An important line of research has demonstrated how product creation via the entry of firms can amplify shocks and be a source of sunspot equilibria that leads to fluctuations driven by self-fulfilling beliefs. The two central mechanisms that produce these results are countercyclical markups and the love of variety.[1] More recently, Minniti and Turino (2013) and Pavlov and Weder (2017) extend these entry models by utilizing variety effects as an incentive for firms to produce multiple products. Yet, the empirical evidence on the size of these effects is sparse - casting doubt on the ability of this mechanism to explain the contribution of product creation on the business cycle. The current paper addresses this issue by laying out a model where intra-firm product creation amplifies business cycles and makes the economy more susceptible to sunspot equilibria even in the complete absence of the love of variety.

Specifically, it investigates the role of increasing marginal costs in a general equilibrium model with endogenous entry and oligopolistic multi-product firms.[2] When the production technology of intermediate good firms has decreasing returns, marginal costs increase with output per variety. This gives firms an incentive to produce multiple products even in the absence of variety effects.[3] The efficiency gains of adjusting product scopes amplifies economic fluctuations and creates sunspot equilibria at more realistic situations, which are not attainable with mono-product firms. Hence, technological decreasing returns and increasing marginal costs provide a novel mechanism for product creation within firms and for generating indeterminacy. This is in stark contrast to Benhabib and Farmer (1994) and Farmer and Guo (1994), where indeterminacy is a result of technological increasing returns for mono-product firms in the absence of entry. Finally, the model is estimated by Bayesian methods and the results support recent findings that belief shocks (i.e. animal spirits, sunspots) explain a significant fraction of U.S. business cycles.

The way indeterminacy arises is most easily understood in terms of the equilibrium wage-hours locus. Product creation and countercyclical markups generate an endogenous efficiency

[1] For example, under the love of variety, Devereux et al. (1996) assess the effect of technology shocks, while Pavlov and Weder (2012) examine the conditions for local indeterminacy. Jaimovich (2007) investigates how indeterminacy can be generated by oligopolistic firms with countercyclical markups.

[2] Bils (1987) and Shea (1993) show evidence of procyclical short-run marginal cost. Findings on U.S. returns to scale by Burnside (1996) and Basu and Fernald (1997) point against decreasing marginal costs.

[3] There are other channels for product creation to affect marginal costs. For example, an alternative modelling approach would be to have spillovers resulting from shared inputs among different products.

wedge which makes this locus upwardly sloping. If the locus is steeper than the labor supply curve, then sunspots can act as self-fulfilling expectation shocks. For example, if people become optimistic about the future path of income, then the wealth effect shifts the labor supply curve upwards, raising employment and output - thereby confirming the initial belief. More precisely, when the labor supply curve shifts up due to optimistic expectations, the higher demand for output and profit opportunities induce firm entry. Greater competition pushes markups downwards and causes firms to expand output. Since marginal costs would increase with production, firms choose to expand their product scopes rather than ramp up the production of existing varieties. The fall in output per variety due to the cannibalization effect (new varieties reducing the demand for existing varieties) then leads to falling marginal costs. Together, the efficiency gains of product creation and falling markups shift out the labor demand curve far enough to allow the initial belief about higher income to become self-fulfilling.

On the theoretical side, the paper is most closely related to Minniti and Turino (2013) and Pavlov and Weder (2017). The former investigates the role of the product scope in magnifying fundamental shocks, while the latter shows how the multi-product structure makes the economy more susceptible to sunspot equilibria. Feenstra and Ma (2009) use a similar model in the context of international trade. In contrast, the current paper does not utilize variety effects. Instead, increasing marginal costs are an incentive for firms to expand their product scopes.

The empirical approach and results are closely connected to Pavlov and Weder (2017) and Dai et al. (2019), with the latter employing financial frictions to generate indeterminacy. Both use Bayesian methods to estimate indeterminate models and their results on the importance of self-fulfilling beliefs parallel the findings of the current paper. In a related work, Lubik (2016) estimates a real business cycle model with productive externalities and finds evidence for close to constant returns to scale in aggregate U.S. data.

The focus on multi-product firms is motivated by recent empirical work. Bernard et al. (2010) find that about 90 percent of total sales in the manufacturing sector are made by multi-product firms. Broda and Weinstein (2010) report that over 90 percent of product creation and destruction occurs within firms and that the contribution of product scope adjustments within firms is at least as important to the evolution of aggregate output as firm entry and exit.

The paper proceeds as follows. Section 2 outlines the model. Section 3 analyzes the local dynamics. Capital utilization is introduced in Section 4 and the model is estimated in Section 5. Section 6 concludes.

2 Model

The artificial economy is based on the multi-product model of Minniti and Turino (2013). There are two key differences. First, the love of variety effects are entirely absent in aggregating intermediate goods into final goods. Second, eventual decreasing returns to scale in the variety-level production technology imply that marginal costs increase with production. Firms therefore take into account the effect of their product scope decision on marginal costs. Time evolves continuously to make comparisons with previous studies on indeterminacy straightforward.

2.1 Final goods

Final output, Y_t, is produced under perfect competition using the range of intermediate inputs supplied by M_t multi-product firms. This is done via two nested CES aggregators. The first combines the varieties from an individual firm

$$Y_t(i) = N_t(i)^{\frac{1}{1-\theta}} \left(\int_0^{N_t(i)} y_t(i,j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}} \qquad \theta > 1 \tag{1}$$

where $N_t(i)$ is firm i's product scope, $y_t(i,j)$ is the amount of the unique intermediate good j produced by firm i, and θ is the elasticity of substitution. The firm-composite goods are then aggregated to form the final good

$$Y_t = M_t^{\frac{1}{1-\theta}} \left(\sum_{i=1}^{M_t} Y_t(i)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} . \tag{2}$$

Note that the CES aggregators have been formed to eliminate the love of variety.[4] Hence, unlike in Minniti and Turino (2013) where variety effects were necessary for the existence

[4] As we will see later, an intermediate good firm will charge the same price for all of its varieties and produce them in equal quantities. Together with the elimination of the love of variety, this implies that differences in intra-firm and inter-firm elasticities of substitution are irrelevant for dynamics.

of multi-product firms, the current model has no increasing returns in the aggregation of products.

The profit maximization problem yields

$$y_t(i,j) = \left(\frac{p_t(i,j)}{P_t}\right)^{-\theta} \frac{Y_t}{M_t N_t(i)} \tag{3}$$

where the aggregate price index satisfies

$$P_t = M_t^{\frac{1}{\theta-1}} \left(\sum_{i=1}^{M_t} P_t(i)^{1-\theta}\right)^{\frac{1}{1-\theta}}. \tag{4}$$

In addition, we have the price index for firm i's goods

$$P_t(i) = N_t(i)^{\frac{1}{\theta-1}} \left(\int_0^{N_t(i)} p_t(i,j)^{1-\theta} dj\right)^{\frac{1}{1-\theta}}. \tag{5}$$

2.2 Intermediate good firms

Each firm chooses the number of different products to produce and the prices to sell them at. This task is solved in two stages. In the first, firms decide their product scopes. In the second stage, firms act as Bertrand competitors in the product market and set their prices. The model is solved by backward induction using the subgame Nash perfect equilibrium concept. The number of active firms is determined by a zero-profit condition each period. Since all firms have the same technology and behavior is governed by identical optimality conditions, a symmetric Nash equilibrium emerges.

Intermediate goods are produced using capital, $k_t(i,j)$, and labor, $h_t(i,j)$, that are supplied on perfectly competitive factor markets. The production technology has decreasing returns and involves two fixed costs. The variety-level fixed cost, ϕ, restricts the amount of varieties a firm will produce. The firm-level fixed cost, ϕ_f, provides economies of scope and determines the number of active firms via a zero-profit condition. Hence, a firm's output is given by

$$\int_0^{N_t(i)} y_t(i,j)dj = \int_0^{N_t(i)} \left[(k_t(i,j)^{\alpha} h_t(i,j)^{1-\alpha})^{\eta} - \phi\right] dj - \phi_f \qquad \alpha, \eta \in (0,1). \tag{6}$$

The presence of decreasing returns via $\eta < 1$ makes it profitable for firms to produce multiple products.[5] Each firm sets prices to maximizes profits

$$\pi_t(i) = \int_0^{N_t(i)} p_t(i,j)y_t(i,j) - w_t h_t(i,j) - r_t k_t(i,j)dj \tag{7}$$

where w_t and r_t are the labor and capital rental rates. As in Yang and Heijdra (1993), intermediate good firms are large enough to take the aggregate price index into consideration when making their pricing decision.[6] Appendix A.1 shows that a firm charges the same price, $p_t(i)$, for all of its varieties and the markup becomes

$$\mu_t(i) \equiv \frac{p_t(i)}{mc_t(i)} = \frac{\theta[1-\epsilon_t(i)]}{\theta[1-\epsilon_t(i)]-1}$$

where $mc_t(i)$ is the marginal cost and $\epsilon_t(i) \equiv P_t(i)Y_t(i)/(P_tY_t)$ is firm i's market share which increases in the number of goods $N_t(i)$. In contrast to Minniti and Turino (2013) and others, this does not arise from the love of variety but instead is due to the effect of the product scope on the firm's marginal costs. If the production technology had constant returns to scale (minus the fixed costs), the marginal cost of producing an additional unit of a variety would be independent of the scale of production. Firms would take the marginal cost as given when making their product scope decisions. Profits would be decreasing in $N_t(i)$ because of the variety-level fixed cost, ϕ, and firms would only produce a single product. However, since the production technology has decreasing returns, the marginal cost is increasing in the scale of production:

$$mc_t(i) = \left(y_t(i) + \phi + \frac{\phi_f}{N_t(i)}\right)^{\frac{1-\eta}{\eta}} \frac{w_t^{1-\alpha}r_t^{\alpha}}{\eta(1-\alpha)^{1-\alpha}\alpha^{\alpha}}.$$

Firms then need to take into account how an introduction of a new product affects marginal costs through the demand for their varieties.

Firms determine their optimal number of products by maximizing profits with respect to $N_t(i)$ by taking into account the effect on its own and other firms' pricing decisions (see Appendix A.2). The first-order condition is

$$\begin{aligned} mc_t(i)\eta\phi &= \left(1-\eta+\eta\theta\left(\frac{p_t(i)-mc_t(i)}{p_t(i)}\right)^2\right)P_tY_t\frac{\partial\epsilon_t(i)}{\partial N_t(i)} \\ &\quad -\eta[N_t(i)\phi+\phi_f]\frac{\partial mc_t(i)}{\partial N_t(i)} + Y_t\epsilon_t(i)\left(\frac{p_t(i)-mc_t(i)\eta}{p_t(i)}\right)\frac{\partial P_t}{\partial N_t(i)}. \end{aligned} \tag{8}$$

[5] More precisely, returns at the variety level are initially increasing due to the fixed cost ϕ but eventually decreasing due to $\eta < 1$.

[6] Under monopolistic competition where firms take the aggregate price index as given, the markup and product scope are constant over the business cycle (see Appendix A.3).

Firms equate the cost of producing a new variety (the left-hand side) with the gains on the right-hand side. The first two terms on the right-hand side are due to the presence of decreasing returns: introducing a new product increases the firm's market share, $\partial \epsilon_t(i)/\partial N_t(i) > 0$, due to the effect on marginal costs, $\partial mc_t(i)/\partial N_t(i) < 0$. The last term represents that introducing a new product reduces the aggregate price index, $\partial P_t/\partial N_t(i) < 0$, which from (3) leads to a lower demand for firm i's products.

2.3 Symmetric equilibrium

In the symmetric equilibrium, each firm produces the same number of varieties, $N_t(i) = N_t$, charges the same price, $p_t(i) = p_t = 1$, and has the same market share $\epsilon_t(i) = 1/M_t$. Using (1) and (2), output per variety is

$$y_t = \frac{Y_t}{N_t M_t}. \tag{9}$$

The markup simplifies to

$$\mu_t = \frac{\theta(M_t - 1)}{\theta(M_t - 1) - M_t}. \tag{10}$$

Since new entrants reduce firms' market shares, the markup is countercyclical. Note that as the number of firms becomes large, the steady state markup converges to its monopolistic competition level of $\mu = \theta/(\theta - 1)$. Furthermore, the steady state version of this equation can be written as

$$M = 1 + \frac{\mu}{\mu(\theta - 1) - \theta}$$

and calibrating the steady state markup μ and elasticity θ pins down the number of firms.

An increase in the firm's product scope reduces its own price and the prices of other firms: to lower price competition, firms under-expand their product scopes in comparison to the case of monopolistic competition where such strategic linkages are absent. The extent of this under-expansion can be seen by substituting $\partial \epsilon_t(i)/\partial N_t(i)$, $\partial mc_t(i)/\partial N_t(i)$ and $\partial P_t/\partial N_t(i)$ into (8) and rearranging for the product scope:

$$N_t = \frac{1-\eta}{\eta} \frac{\mu_t}{\phi} \frac{Y_t}{M_t} \Theta_t.$$

The function Θ_t (see Appendix A.2) is less than one and is increasing in M_t: the strategic effect of the product scope decision becomes less important as the number of firms increases and this gives an incentive to introduce new varieties. When M_t becomes very large this term approaches unity and the markup converges to its monopolistic competition level of $\theta/(\theta-1)$. Intuitively, as the number of firms grows, the impact on the market share of adding an additional variety becomes smaller, which has then a smaller impact on the price of the variety. The dynamics of the product scope are thus similar to Minniti and Turino (2013), but instead of the love of variety being the incentive for product creation, it is the efficiency gains of reducing marginal costs. When firms want to expand their output it is efficient to introduce new products, rather than ramp up the production of existing varieties whose production technology is subject to diminishing returns.

The number of firms can be determined from the zero profit condition:

$$M_t = \frac{\mu_t - \eta}{\eta} \frac{Y_t}{\phi N_t + \phi_f}. \tag{11}$$

To obtain aggregate output, first note that (6) can be written as

$$y_t = k_t^{\eta\alpha} h_t^{\eta(1-\alpha)} - \phi - \frac{\phi_f}{N_t}.$$

Using this together with (9) and (11) gives

$$Y_t = \eta \frac{M_t^{1-\eta} N_t^{1-\eta}}{\mu_t} K_t^{\eta\alpha} H_t^{\eta(1-\alpha)} \tag{12}$$

where $K_t = M_t N_t k_t$ and $H_t = M_t N_t k_t$. The term $\frac{M_t^{1-\eta} N_t^{1-\eta}}{\mu_t}$ can be interpreted as an endogenous efficiency wedge. Combining (11) and (12), output can be written as

$$Y_t = \frac{\eta}{\mu_t} K_t^{\alpha} H_t^{1-\alpha} \left(1 - \frac{\eta}{\mu_t}\right)^{\frac{1-\eta}{\eta}} \left(\frac{N_t}{\phi N_t + \phi_f}\right)^{\frac{1-\eta}{\eta}}. \tag{13}$$

Since markups are countercyclical, the first term in brackets implies that decreasing returns ($\eta < 1$) have a contractionary effect on the efficiency wedge. However, the second term in brackets implies that an increase in product scopes has an expansionary effect. This latter effect outweighs the former and as we will see in Section 3, procyclical product scope makes the economy more susceptible to sunspot equilibria.

Finally, the equilibrium real wage and rental rate are given by

$$w_t = (1-\alpha)\frac{Y_t}{H_t} \qquad \text{and} \qquad r_t = \alpha \frac{Y_t}{K_t}.$$

2.4 Agents

The representative agent derives lifetime utility from the function

$$U = \int_0^\infty e^{-\rho t} u(C_t, H_t) dt \qquad \rho > 0.$$

Here, ρ denotes the subjective rate of time preference and period utility takes the functional form

$$u(C_t, H_t) = \ln C_t - \upsilon \frac{H_t^{1+\chi}}{1+\chi} \qquad \upsilon > 0,\ \chi \geq 0$$

where χ is the inverse of the Frisch labor supply elasticity. The agents own the capital stock and sell labor and capital services. The period budget is constrained by

$$w_t H_t + r_t K_t + \Pi_t \geq X_t + C_t$$

where Π_t denotes potential profits and investment, X_t, is added to the capital stock such that:

$$\dot{K}_t = X_t - \delta K_t \qquad 0 < \delta < 1.$$

Time derivatives are denoted by dots and δ stands for the constant rate of physical depreciation of the capital stock. The solution to the maximization problem gives

$$\upsilon H_t^\chi = \frac{w_t}{C_t}$$

and

$$\frac{\dot{C}_t}{C_t} = r_t - \delta - \rho.$$

which represent the agents' leisure-consumption trade-off and the intertemporal Euler equation. In addition the transversality condition must hold.

3 Dynamics

This section analyzes the local dynamic properties of the multi-product model and compares it to the mono-product model. The equilibrium conditions are log-linearized and the dynamical system is arranged to

$$\begin{bmatrix} \dot{K}_t/K_t \\ \dot{C}_t/C_t \end{bmatrix} = \mathbf{J} \begin{bmatrix} \hat{K}_t \\ \hat{C}_t \end{bmatrix}.$$

Hatted variables denote percent deviations from their steady-state values and $\mathbf{J}$ is the 2×2 Jacobian matrix of partial derivatives. Note that C_t is a non-predetermined variable and that K_t is predetermined. Indeterminacy requires both roots of $\mathbf{J}$ to be negative, that is Det$\mathbf{J}$>0>Tr$\mathbf{J}$. For easier comparison to previous studies, the parameters are calibrated at a quarterly frequency as $\alpha = 0.3$, $\rho = 0.01$, $\delta = 0.025$ and $\chi = 0$.

As explained in the previous section, the current model requires decreasing returns, $\eta < 1$, for firms to have an incentive to produce multiple products. Figure 1 presents the indeterminacy zones for the mono-product model with $\eta = 1$ and the multi-product model with $\eta = 0.8$ and 0.9. Lower variety-level returns to scale increase the size of the indeterminacy zone and sunspot equilibria occur for lower levels of market power. The mono-product model is virtually identical to Jaimovich (2007) if, in his paper, the intersectoral elasticity of substitution is set to unity. Indeterminacy is driven entirely by the effect of the countercyclical markup on the efficiency wedge. While not shown in the figure, decreasing returns in the mono-product model reduce the plausibility of indeterminacy. The reason is that falling markups increase the output of firms and then the diminishing returns have a negative effect on the efficiency wedge (recall the first bracketed term in equation 13). As θ falls towards $\mu/(\mu-1)$ the number of firms approaches infinity and the markup converges to its monopolistic competition level of $\theta/(\theta-1)$. In this case, the markup is constant, the dynamics converge to that of the model with monopolistic competition, and indeterminacy cannot exist. Higher θ, on the other hand, increases the cyclicality of the markup and indeterminacy is thus possible for a lower level of market power.[7]

When decreasing returns are present, marginal costs increase with output per variety and firms have an incentive to produce multiple products. Here, decreasing returns amplify fluctuations and reduce the level of market power required for indeterminacy. This stands completely in contrast to how decreasing returns affect the mono-product model. Why is this the case? First, note that if markups were constant, the product scope, output per firm and output per variety would also be constant: η would have no effect on local dynamics (see Appendix A.3). Under oligopolistic competition, however, the entry of new firms reduces existing firms' market shares and encourages them to expand their product scopes. Due

[7] The markup elasticity $(\partial\mu/\partial M)(M/\mu) = (\mu-1)(\mu(1-\theta)+\theta)/\mu < 0$ is increasing (in absolute value) in μ and θ, and approaches zero as μ approaches $\theta/(\theta-1)$.

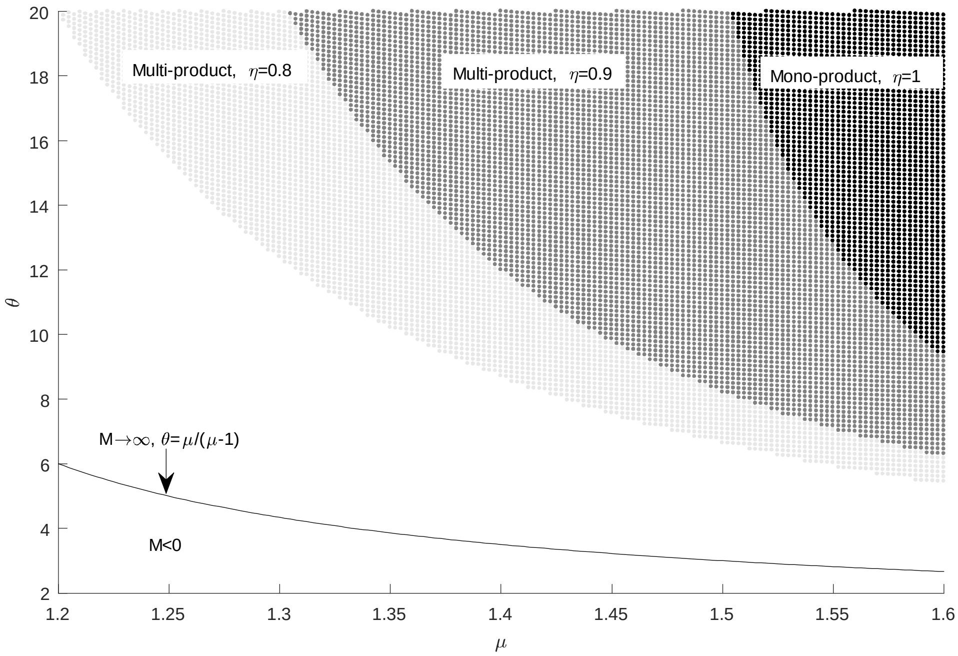

Figure 1: Mono and multi-product models. Shaded regions represent areas of indeterminacy.

to the cannibalization effect (new products reducing demand for existing products), firms reduce production for each of their varieties, which then reduces their marginal costs. This efficiency gain - firms do not run into diminishing returns as quickly as their mono-product counterparts - acts as an additional mechanism that amplifies business cycles. That is, a low number of firms leads to two inefficiencies: high markups and low product scopes (with high output per variety). Firm entry reduces these inefficiencies and expands production possibilities.

4 Capital utilization

The last section has demonstrated that when marginal costs increase with the level of production, the possibility of sunspot equilibria increases when firms can choose their product scopes. However, it could be argued that the level of market power required for indeterminacy is on the higher end of empirical estimates. This section addresses the issue by showing that the levels of market power can be reduced substantially by introducing variable capital

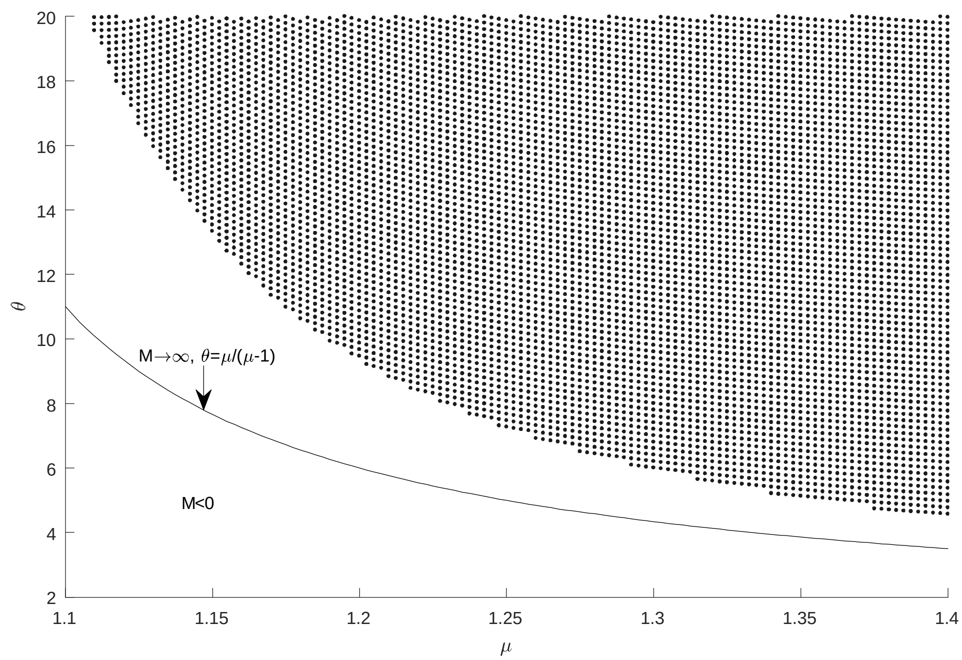

Figure 2: Multi-product model with variable capital utilization, $\eta = 0.9$.

utilization. Each intermediate good firm i now operates the production technology

$$\int_0^{N_t(i)} y_t(i,j)dj = \int_0^{N_t(i)} \left[(U_t^{\alpha} k_t(i,j)^{\alpha} h_t(i,j)^{1-\alpha})^{\eta} - \phi \right] dj - \phi_f$$

where U_t stands for the utilization rate of capital set by its owners. Capital evolves according to

$$\dot{K}_t = X_t - \delta_t K_t = X_t - \frac{1}{\varrho} U_t^{\varrho} K_t \qquad \varrho > 1$$

and the optimal rate of utilization follows

$$r_t = U_t^{\varrho - 1}.$$

The calibration remains the same, and as in Wen (1998), the steady state first-order conditions pin down $\varrho = (\rho + \delta)/\delta = 1.4$. Figure 2 demonstrates how the introduction of variable capital utilization significantly reduces the level of market power and the elasticity of substitution that are required for indeterminacy. This occurs because higher utilization, like lower markups, increases the demand for labor.

To gain further understanding about the effect of sunspots and the dynamics of the model, the impulse responses of the main variables are plotted in Figure 3. The sunspot shock is modelled as an expectation shock to consumption that raises it one percent above its steady state level. This discrete-time version of the model is calibrated as $\alpha = 0.3$, $\delta = 0.025$, $\chi = 0$, $\eta = 0.9$, and a discount factor at $\beta \equiv (1+\rho)^{-1} = 0.99$. The steady markup is set to $\mu = 1.3$, which lies in the middle of value-added markup estimates for the US (see Jaimovich, 2007). Finally, as in Minniti and Turino (2013), the elasticity of substitution is set to $\theta = 7.5$. The impulse response functions reveal that both net product creation and net business formation positively comove with output, with the former being more volatile than the latter. We can also observe the cannibalization effect: an introduction of a new variety reduces the demand for existing varieties, that is, output per variety drops. The countercyclically fluctuating markup, together with the efficiency gains of product creation on marginal costs leads to an upwardly sloping wage-hours locus that enables the propagation of self-fulfilling beliefs described earlier.

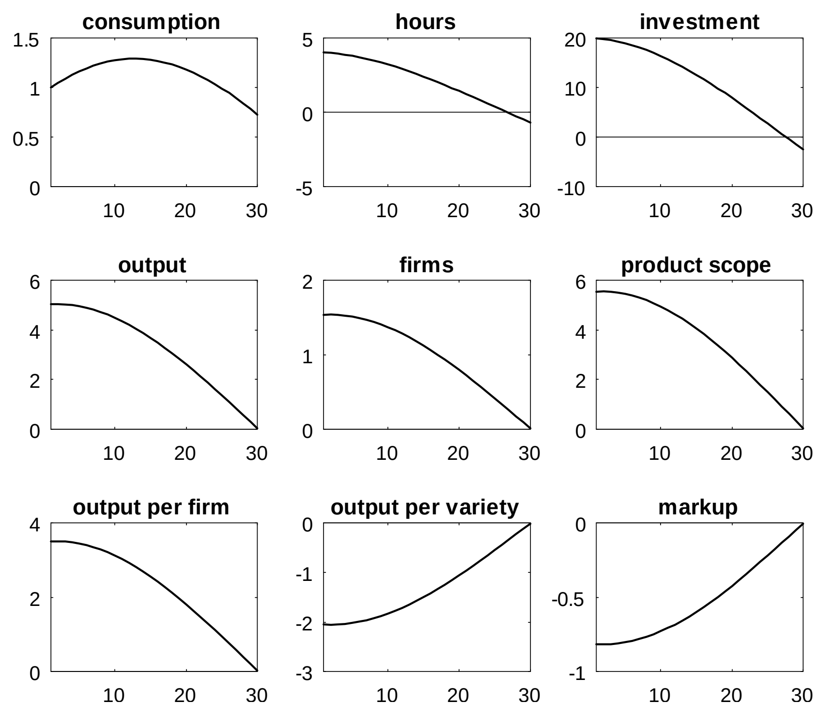

Figure 3: Impulse responses to a consumption (sunspot) shock (percent deviations from the steady state).

5 Estimation

So far, this paper has shown that under increasing marginal costs, intra-firm product creation can generate indeterminacy under more plausible situations. The current section estimates the model in log-linear form to see if it can replicate the basic business cycle facts by comparing its second moments to the US quarterly time series counterparts (see Appendix A.4 for the data sources). The Bayesian estimation procedure is based on Farmer et al. (2015) and largely follows Pavlov and Weder (2017). Finally, a comparison is made to the mono-product model where marginal costs can be decreasing.

5.1 The model

The discrete-time model builds upon the capital utilization economy from Section 4. In addition to sunspots, fundamental demand and supply disturbances are sources of uncertainty.

On the supply side, exogenous labor augmenting technological progress, A_t, affects all firms equally and implies that aggregate output is given by

$$Y_t = \frac{(U_t K_t)^{\alpha\eta}(A_t H_t)^{(1-\alpha)\eta}}{\mu_t}.$$

It is non-stationary and follows the process

$$\ln A_t = \ln A_{t-1} + \ln a_t$$

where

$$\ln a_t = (1-\psi_A)\ln a + \psi_A \ln a_{t-1} + \varepsilon_t^A \qquad 0 \leq \psi_A < 1.$$

Here $\ln a$ is the average growth rate of technology and ε_t^A is an i.i.d. disturbance with variance σ_A^2.[8]

The first demand disturbance is a preference shock to the agent's utility of consumption that leads to an urge to consume as in Baxter and King (1991). Period utility takes the form

$$u(C_t, H_t) = \ln(C_t - \Delta_t) - \upsilon \frac{H_t^{1+\chi}}{1+\chi}$$

where a positive shock to Δ_t increases the marginal utility of consumption and causes agents to reduce leisure for higher consumption. It follows the process

$$\Delta_t = \psi_\Delta \Delta_{t-1} + \varepsilon_t^\Delta \qquad 0 \leq \psi_\Delta < 1$$

[8] Detrended output is given by $\tilde{Y}_t = Y_t / A_t^\eta$ and $\hat{Y}_t = \ln \tilde{Y}_t - \ln \tilde{Y}$, where $\tilde{Y}$ is the steady state value.

with the shock variance σ_Δ^2. The shock drives the economy's labor wedge and can also be interpreted as a reduced form way of capturing changes to monetary policy, taxes, or labour market frictions.

The second demand disturbance is a shock to government expenditures, G_t, financed by lump sum taxes. Government spending follows a stochastic trend

$$A_t^g = (A_{t-1}^g)^{\psi_{ag}} (A_{t-1})^{1-\psi_{ag}}$$

where ψ_{ag} governs the smoothness of the trend relative to the trend in output. Detrended government spending is $g_t \equiv G_t/(A_t^g)^\eta$ and follows the process

$$\ln g_t = \psi_g \ln g_{t-1} + \varepsilon_t^g \qquad 0 \leq \psi_g < 1$$

with the shock variance σ_g^2.

Finally, as in Pavlov and Weder (2017), the non-fundamental sunspot shock is modelled as an expectation error to output that is unrelated to any fundamental changes in the economy.[9] Since the economy's response to fundamental shocks is not uniquely determined (see Lubik and Schorfheide, 2003 and 2004), the behavior of output is then

$$\hat{Y}_t = E_{t-1}\hat{Y}_t + \Omega_A \varepsilon_t^A + \Omega_\Delta \varepsilon_t^\Delta + \Omega_g \varepsilon_t^g + \varepsilon_t^s$$

where parameters Ω_A, Ω_Δ and Ω_g determine the effect of technology, preference and government shocks on output and ε_t^s is an i.i.d. sunspot shock with variance σ_s^2.

5.2 Bayesian estimation

The model is estimated via Bayesian methods using the quarterly real per capita growth rates of output, consumption, investment, government spending and the logarithm of per capita hours worked from 1955:I-2007:IV as observables.[10] Since the model is very small scale and lacks financial frictions, the series are truncated right before the Great Recession. The measurement equation is thus

$$\begin{bmatrix} \ln Y_t - \ln Y_{t-1} \\ \ln C_t - \ln C_{t-1} \\ \ln X_t - \ln X_{t-1} \\ \ln G_t - \ln G_{t-1} \\ \ln H_t - \ln H \end{bmatrix} = \begin{bmatrix} \hat{Y}_t - \hat{Y}_{t-1} + \eta \hat{a}_t \\ \hat{C}_t - \hat{C}_{t-1} + \eta \hat{a}_t \\ \hat{X}_t - \hat{X}_{t-1} + \eta \hat{a}_t \\ \hat{G}_t - \hat{G}_{t-1} + \eta(\hat{a}_t^g - \hat{a}_{t-1}^g + \hat{a}_t) \\ \hat{H}_t \end{bmatrix} + \begin{bmatrix} g_y \\ g_y \\ g_y \\ g_y \\ 0 \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{m.e.} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

[9] Results are robust to the choice of expectation error (Farmer et al. 2015).

[10] Unfortunately, no (long) time series are available for the number of firms and the product scope.

where $g_y = 0.0046$ is the average quarterly growth rate of per capita real GDP, $a_t^g \equiv A_t^g/A_t = (a_{t-1}^g)^{\psi_{ag}} a_t^{-1}$ is the ratio between the government spending and technology trends, $\varepsilon_t^{m.e.}$ is a measurement error restricted to account for not more than ten percent of output growth in the data and $\ln H$ is the logarithm of the average hours worked over the sample period. The share of government expenditures in output, G/Y, is set to 0.21, which is consistent with the data sample. The parameters that are calibrated remain the same as in the previous section: $\alpha = 0.3$, $\delta = 0.025$, $\chi = 0$, $\beta = 0.99$, and $\mu = 1.3$.[11]

The estimated parameters include the degree of variety-level decreasing returns η, the elasticity of substitution θ and the parameters for the stochastic processes: ψ_A, ψ_Δ, ψ_g, ψ_{ag}, σ_s, σ_A, σ_Δ, σ_g, Ω_A, Ω_Δ, Ω_g, $\sigma^{m.e.}$. Endogenous priors are used to prevent overly high estimated model variances (Christiano et al. 2011). Table 1 presents the initial prior and posterior distributions. The returns to scale parameter η is bounded by unity at the upper end since increasing returns at the variety-level would lead to firms producing only one good (that is, this restriction is necessary to keep the product scope strictly positive). The mean is centered at 0.95 as studies such as Burnside (1996) and Basu and Fernald (1997) have shown evidence for close to constant returns to scale. A normal distribution with a mean of 10 is assumed for θ (approximately the halfway point between the two elasticities estimated by Broda and Weinsten, 2010). Given the calibrated steady state markup, this elasticity is bounded by $\mu/(\mu-1) = 4.33$ at the lower end to keep the number of firms strictly positive.[12] A wide uniform distribution is employed for the expectation error parameters Ω_A, Ω_Δ, and Ω_g. Finally, the shock processes follow the standard inverse gamma distribution. The Metropolis-Hastings algorithm is employed to obtain 500,000 draws from the posterior mean for each of the five chains. Half of the draws are discarded and the scale in the jumping distribution is adjusted to achieve a 25-30 percent acceptance rate for each chain.

Table 1 shows that the parameters are precisely estimated with data favouring mild decreasing returns. As discussed in previous sections, this results in a substantial amplification mechanism coming from product scope variations. The relatively high value for θ indicates an elastic markup that serves as another shock amplification channel. The remaining esti-

[11] This parameterization is standard in the sunspot literature. The steady state markup is set around the middle of value-added markup estimates for the U.S. (see Jaimovich, 2007). Appendix A.5 presents robustness checks for alternative markup calibrations.

[12] Since θ and μ jointly determine the markup elasticity, identification issues prevented the estimation of both parameters.

Table 1: Prior and posterior distributions

Prior					Posterior	
Name	Range	Density	Mean	Std. Dev.	Mean	90% Interval
η	(0,1)	Normal	0.95	0.05	0.931	[0.917,0.945]
θ	[4.34,+∞]	Normal	10	2	17.679	[15.895,19.450]
ψ_A	[0,1)	Beta	0.5	0.2	0.007	[0.001,0.012]
ψ_Δ	[0,1)	Beta	0.5	0.2	0.989	[0.984,0.993]
ψ_g	[0,1)	Beta	0.5	0.2	0.991	[0.987,0.995]
ψ_{ag}	[0,1)	Beta	0.5	0.2	0.975	[0.961,0.990]
σ_s	R^+	Inverse Gamma	0.1	Inf	0.529	[0.501,0.556]
σ_A	R^+	Inverse Gamma	0.1	Inf	0.739	[0.695,0.783]
σ_Δ	R^+	Inverse Gamma	0.1	Inf	0.457	[0.440,0.475]
σ_g	R^+	Inverse Gamma	0.1	Inf	1.112	[1.047,1.179]
$\sigma^{m.e.}$	$[0, 0.28]$	Uniform	0.14	0.081	0.280	[0.279,0.280]
Ω_A	[-3,3]	Uniform	0	1.732	-0.607	[-0.684,-0.528]
Ω_Δ	[-3,3]	Uniform	0	1.732	0.774	[0.657,0.890]
Ω_g	[-3,3]	Uniform	0	1.732	0.333	[0.285,0.381]

This table presents the prior and posterior distributions for model parameters and shocks. Inf implies two degrees of freedom for the inverse gamma distribution. Standard deviations are in percent terms.

mates are consistent with previous studies. Preference and governments shocks are highly persistent and cause an increase in output. The persistence of the permanent technology shock is close to zero with a resulting fall in detrended output consistent with the determinate plain real business cycle model.

Table 2 reports the second moments of the main macroeconomic aggregates and reveals that the model fits the data well. When considering growth rates, the model reproduces the empirical volatility of output growth but slightly overpredicts the variance of the other series. When HP filtered, the model slightly underpredicts all series except for government spending which it matches perfectly.[13] The relative volatilities and correlations are consistent with the data. Due to the rich internal propagation mechanism of the indeterminate model, the autocorrelation functions (ACF) show persistence in the growth rates despite the lack of the many real frictions employed in the literature.

Table 3 displays the variance decomposition which reveals the relative contribution of each of the four shocks to the macroeconomic aggregates. Consistent with the findings of Pavlov and Weder (2017) and Dai et al. (2019), sunspots explain a significant fraction of U.S. business cycle: over 40 percent of output fluctuations when considering growth rates

[13] A Hodrick-Prescott filter with a smoothing parameter of 1600 was applied.

Table 2: Business cycle dynamics

	Data			Model		
x	σ_x	$\rho(x, \ln(Y_t/Y_{t-1}))$	ACF	σ_x	$\rho(x, \ln(Y_t/Y_{t-1}))$	ACF
$\ln(Y_t/Y_{t-1})$	0.89	1	0.29	0.91	1	0.16
$\ln(C_t/C_{t-1})$	0.50	0.51	0.22	0.73	0.62	0.03
$\ln(X_t/X_{t-1})$	2.10	0.71	0.53	2.68	0.80	0.31
$\ln(G_t/G_{t-1})$	1.08	0.26	0.07	1.12	0.41	0.00
$\ln(H_t/H)$	4.05	0.05	0.98	4.90	0.12	0.99
		$\rho(x, Y)$			$\rho(x, Y)$	
Y_t	1.49	1	0.84	1.32	1	0.80
C_t	0.81	0.80	0.84	0.76	0.71	0.73
X_t	4.37	0.89	0.89	3.92	0.91	0.82
G_t	1.44	0.10	0.76	1.44	0.36	0.72
H_t	1.73	0.86	0.90	0.99	0.99	0.80

Business cycle statistics for the artificial economy are calculated at the posterior mean. σ_x denotes the standard deviation of variable x, $\rho(x, Y)$ is the correlation of variable x and output, and ACF is the first order autocorrelation coefficient. The last five rows are from HP filtered series.

Table 3: Unconditional variance decomposition (in percent)

	$\ln\left(\frac{Y_t}{Y_{t-1}}\right)$	$\ln\left(\frac{C_t}{C_{t-1}}\right)$	$\ln\left(\frac{X_t}{X_{t-1}}\right)$	$\ln\left(\frac{G_t}{G_{t-1}}\right)$	$\ln\left(\frac{H_t}{H}\right)$	Y_t	C_t	X_t	G_t	H_t
ε_t^s	42.86	2.71	71.84	0	20.68	33.83	4.40	60.96	0	39.10
ε_t^A	15.21	42.33	12.91	0.49	14.99	36.72	16.43	26.97	0	25.81
ε_t^Δ	20.66	53.54	7.62	0	33.61	14.29	77.52	5.99	0	17.13
ε_t^g	21.27	1.42	7.63	99.51	30.73	15.16	1.65	6.07	100	17.96

Variance decompositions are performed at the posterior mean. The last five columns are calculated from HP filtered series.

and one third when the series are HP filtered. Investment is even more sunspot driven while the fundamental shocks better explain the behavior of consumption and hours worked.

5.3 Mono-product model comparison

The previous section has shown that a multi-product model with mild variety-level decreasing returns to scale does a good job in replicating the regularities of the U.S. business cycle. Due to the small scale nature of the model, the returns to scale parameter η was restricted to be less than unity in order to preserve the multi-product structure and a strictly positive product scope. The current section compares the performance of the model against a mono-product model with potentially increasing returns to scale. The mono-product economy is effectively the model of Jaimovich (2007) with variable technological returns to scale. Here, increasing returns to scale make the labour demand curve flatter and make indeterminacy more plausible

Table 4: Posterior distributions for the mono-product model

Name	Mean	90% Interval
η	1.261	[1.252,1.268]
θ	14.224	[11.759,16.593]
ψ_A	0.130	[0.118,0.143]
ψ_Δ	0.989	[0.985,0.993]
ψ_g	0.992	[0.988,0.995]
ψ_{ag}	0.958	[0.940,0.978]
σ_s	0.536	[0.509,0.561]
σ_A	0.444	[0.424,0.463]
σ_Δ	0.463	[0.446,0.481]
σ_g	1.114	[1.047,1.179]
$\sigma^{m.e.}$	0.280	[0.279,0.280]
Ω_A	-0.624	[-0.738,-0.504]
Ω_Δ	0.897	[0.795,1.000]
Ω_g	0.338	[0.291,0.385]

Table 5: Model Comparison

	Multi-product	Mono-product
Prior Model Probability	0.5	0.5
Log-data density	3026.28	3010.97
Posterior Model Probability	1.00	0.00

This table compares the empirical fit of the multi-product and mono-product models. Posterior probabilities have been calculated based on the output of the Metropolis-Hastings algorithm (log marginal densities based on the modified harmonic mean).

as in Benhabib and Farmer (1994). Table 4 presents the posterior estimates. Apart from η, which no longer has the upper bound of unity, all prior distributions are identical to Table 1. As expected, to compensate for the lack of the product scope amplification mechanism, the estimated model has moderate increasing returns at the variety-level, implying decreasing marginal costs. However, the other parameter estimates are not substantially different. Second moments and variance decompositions (including the role of sunspots) are also very similar to the multi-product model (and are not presented here to conserve space). Table 5 presents a comparison using log-data densities between the two models. While the log-data densities are not too far apart, data clearly favours the multi-product economy.

6 Conclusion

Previous studies have shown that product creation within firms can be a source of business cycle amplification and sunspot equilibria. Yet, this result and the existence of multi-product firms relies on the love of variety, for which empirical evidence is limited. The current paper addresses this issue. It investigates the role of increasing marginal costs in a dynamic general equilibrium model without the love of variety. Marginal costs increase with output per variety due to decreasing returns in the production technology. Product scope expansions then reduce marginal costs and firms have an incentive to produce multiple products. The efficiency gains of adjusting product scopes provides an amplification mechanism that creates sunspot equilibria at more realistic situations, which are not attainable with mono-product firms. Hence, increasing marginal costs provide an additional and novel mechanism for product creation that makes it easier for indeterminacy to occur. The estimated indeterminate model generates artificial cycles that closely resemble empirically observed fluctuations. The estimation supports recent findings that non-fundamental belief shocks (animal spirits) explain a significant portion of U.S. business cycles.

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A Appendix

A.1 Markups

This Appendix derives the intermediate good firm's optimal markup. Taking logs of (3) gives

$$\ln y_t(i,j) = -\theta \ln p_t(i,j) + \theta \ln P_t + \ln Y_t - \ln N_t(i) - \ln M_t.$$

Then using (4) and (5), the price elasticity of demand is

$$\frac{\partial \ln y_t(i,k)}{\partial \ln p_t(i,j)} = \underbrace{-\theta}_{\text{absent for } k \neq j} + \frac{\theta}{N_t(i) M_t} \left(\frac{p_t(i,j)}{P_t} \right)^{1-\theta}. \tag{A.1}$$

Firm i maximizes profit (7) subject to the constraint (6):

$$\begin{aligned} \mathcal{L} &= \int_0^{N_t(i)} p_t(i,j) y_t(i,j) - w_t h_t(i,j) - r_t k_t(i,j) dj \\ &+ \Lambda_t \left(\int_0^{N_t(i)} \left[z_t (k_t(i,j)^\alpha h_t(i,j)^{1-\alpha})^\eta - \phi \right] dj - \phi_f - \int_0^{N_t(i)} y_t(i,j) dj \right). \end{aligned}$$

Optimality gives

$$\frac{\partial \mathcal{L}}{\partial p_t(i,j)} = y_t(i,j) + \int_0^{N_t(i)} [p_t(i,j) - \Lambda_t] \frac{\partial y_t(i,j)}{\partial p_t(i,j)} dj = 0 \tag{A.2}$$

$$\frac{\partial \mathcal{L}}{\partial h_t(i,j)} = -w_t + \Lambda_t \eta (1-\alpha) z_t k_t(i,j)^{\alpha\eta} h_t(i,j)^{(1-\alpha)\eta-1} = 0 \tag{A.3}$$

$$\frac{\partial \mathcal{L}}{\partial k_t(i,j)} = -r_t + \Lambda_t \eta \alpha z_t k_t(i,j)^{\alpha\eta-1} h_t(i,j)^{(1-\alpha)\eta} = 0. \tag{A.4}$$

The Lagrange multiplier, Λ_t, is obtained by combining (A.3) and (A.4) then applying Shephard's lemma, and amounts to the marginal cost, $mc_t(i,j)$, of producing one more variety:

$$\begin{aligned} mc_t(i,j) &= \left(z_t k_t(i,j)^{\alpha\eta} h_t(i,j)^{(1-\alpha)\eta}\right)^{\frac{1-\eta}{\eta}} \frac{w_t^{1-\alpha} r_t^{\alpha}}{z_t^{\frac{1}{\eta}} \eta (1-\alpha)^{1-\alpha} \alpha^{\alpha}} \\ &= \left(y_t(i,j) + \phi + \frac{\phi_f}{N_t(i)}\right)^{\frac{1-\eta}{\eta}} \frac{w_t^{1-\alpha} r_t^{\alpha}}{z_t^{\frac{1}{\eta}} \eta (1-\alpha)^{1-\alpha} \alpha^{\alpha}}. \end{aligned} \tag{A.5}$$

Substituting (A.1) into (A.2) and some algebra yields

$$y_t(i,j) - \theta \frac{y_t(i,j)}{p_t(i,j)} \left[p_t(i,j) - mc_t(i,j)\right] + \int_0^{N_t(i)} \frac{y_t(i,k)}{p_t(i,j)} \left[p_t(i,k) - mc_t(i,k)\right] dk \frac{\theta}{N_t(i) M_t} \left(\frac{p_t(i,j)}{P_t}\right)^{1-\theta} = 0.$$

Substituting (3) for $y_t(i,j)$, the above equation simplifies to

$$P_t Y_t \left[1 - \theta \frac{p_t(i,j) - mc_t(i,j)}{p_t(i,j)}\right] + \theta \int_0^{N_t(i)} y_t(i,k) \left[p_t(i,k) - mc_t(i,k)\right] dk = 0.$$

As the second term of this equation is the same for all $j \in [0, N_t(i)]$, this implies that firm i will charge the same price for all of its varieties.[14] Hence, $p_t(i,j) = p_t(i,k) = p_t(i) = P_t(i)$ and $mc_t(i,j) = mc_t(i)$. Some algebra gives

$$\mu_t(i) \equiv \frac{p_t(i)}{mc_t(i)} = \frac{\theta[1 - \epsilon_t(i)]}{\theta[1 - \epsilon_t(i)] - 1}. \tag{A.6}$$

where

$$\epsilon_t(i) \equiv \left(\frac{p_t(i)}{P_t}\right)^{1-\theta} M_t^{-1} = \frac{P_t(i) Y_t(i)}{P_t Y_t} \tag{A.7}$$

is firm i's market share.

A.2 Product scope

This Appendix derives the firms' optimal product scope assuming increasing marginal costs, $\eta < 1$. Since the firm will charge the same price for all of its varieties, it will produce the

[14] Marginal cost depends on the level of production if $\eta \neq 1$ but note that each variety faces the same demand curve.

same quantity of each variety. Hence, the costs of production are

$$\begin{aligned}\int_0^{N_t(i)} w_t h_t(i,j) + r_t k_t(i,j) dj &= \eta N_t(i) mc_t(i) z_t k_t(i)^{\alpha\eta} h_t(i)^{(1-\alpha)\eta} \\ &= \eta N_t(i) mc_t(i) y_t(i) + N_t(i)\phi + \phi_f\end{aligned}$$

Profits can then be written as

$$\pi_t(i) = \left(\frac{p_t(i) - mc_t(i)\eta}{p_t(i)}\right) P_t Y_t \epsilon_t(i) - mc_t(i)\eta[N_t(i)\phi + \phi_f]. \tag{A.8}$$

Firm i takes the number of firms and their product scopes as given and maximizes its profits with respect to $N_t(i)$ by taking account the effect of its product scope decision on its own and all other producers' prices and marginal costs.[15] After some algebra, the first-order condition is

$$\begin{aligned}\frac{\partial \pi_t(i)}{\partial N_t(i)} &= \left(1 - \eta + \eta\theta \left(\frac{p_t(i) - mc_t(i)}{p_t(i)}\right)^2\right) P_t Y_t \frac{\partial \epsilon_t(i)}{\partial N_t(i)} - mc_t(i)\eta\phi \qquad \text{(A.9)} \\ &\quad + Y_t \epsilon_t(i) \left(\frac{p_t(i) - mc_t(i)\eta}{p_t(i)}\right) \frac{\partial P_t}{\partial N_t(i)} - \eta[N_t(i)\phi + \phi_f] \frac{\partial mc_t(i)}{\partial N_t(i)} = 0.\end{aligned}$$

Now to derive $\partial\epsilon_t(i)/\partial N_t(i)$, $\partial P_t/\partial N_t(i)$, $\partial mc_t(i)/\partial N_t(i)$, then substitute in (A.9) to obtain firm i's product scope. From (A.7):

$$\frac{\partial \epsilon_t(i)}{\partial N_t(i)} = (1-\theta)\frac{\epsilon_t(i)}{p_t(i)} \frac{\partial p_t(i)}{\partial N_t(i)} + (\theta - 1)\frac{\epsilon_t(i)}{P_t} \frac{\partial P_t}{\partial N_t(i)}. \tag{A.10}$$

Note that the second term on the right hand side of (A.10) would not be present in the case of monopolistic competition. As will be shown later, $\partial p_t(i)/\partial N_t(i)$ and $\partial P_t/\partial N_t(i)$ are both negative. From (A.6):

$$\frac{\partial p_t(i)}{\partial N_t(i)} = \frac{\mu_t(i)[\mu_t(i) - 1]mc_t(i)}{1 - \epsilon_t(i)} \frac{\partial \epsilon_t(i)}{\partial N_t(i)} + \mu_t(i) \frac{\partial mc_t(i)}{\partial N_t(i)}. \tag{A.11}$$

Since $p_t(i) = P_t(i)$, the aggregate price index can be written as

$$P_t = M_t^{\frac{1}{\theta-1}} \left(\sum_{k=1}^{M_t} p_t(k)^{1-\theta}\right)^{\frac{1}{1-\theta}}$$

and

$$\frac{\partial P_t}{\partial N_t(i)} = P_t^{\theta} M_t^{-1} \left[\sum_{k=1}^{M_t} p_t(k)^{-\theta} \frac{\partial p_t(k)}{\partial N_t(i)}\right]. \tag{A.12}$$

[15] Note that from (A.5) if $\eta \neq 1$ then the firm internalises the effect of the product scope on its marginal costs.

Under symmetry where all firms start off identical with $p_t(i) = p_t(k) = p_t$, this is equal to

$$\frac{\partial P_t}{\partial N_t(i)} = \left(\frac{p_t}{P_t}\right)^{-\theta} M_t^{-1} \left[(M_t - 1)\frac{\partial p_t(k)}{\partial N_t(i)} + \frac{\partial p_t(i)}{\partial N_t(i)}\right] \tag{A.13}$$

and using (A.11) can be written as[16]

$$\frac{\partial P_t}{\partial N_t(i)} = \left(\frac{p_t}{P_t}\right)^{-\theta} \frac{\mu_t}{M_t} \left[(M_t - 1)\frac{\partial mc_t(k)}{\partial N_t(i)} + \frac{\partial mc_t(i)}{\partial N_t(i)}\right]. \tag{A.14}$$

From (A.5)[17]

$$\begin{aligned}
\frac{\partial mc_t(i)}{\partial N_t(i)} &= \frac{1-\eta}{\eta} mc_t(i) \left(y_t(i) + \phi + \frac{\phi_f}{N_t(i)}\right)^{-1} \times \\
&\left[-\frac{y_t(i)}{N_t(i)} - \theta\frac{y_t(i)}{p_t(i)}\frac{\partial p_t(i)}{\partial N_t(i)} + \theta\frac{y_t(i)}{P_t}\frac{\partial P_t}{\partial N_t(i)} - \frac{\phi_f}{N_t(i)^2}\right]
\end{aligned}$$

$$\begin{aligned}
\frac{\partial mc_t(k)}{\partial N_t(i)} &= \frac{1-\eta}{\eta} mc_t(k) \left(y_t(k) + \phi + \frac{\phi_f}{N_t(k)}\right)^{-1} \times \\
&\left[-\theta\frac{y_t(k)}{p_t(k)}\frac{\partial p_t(k)}{\partial N_t(i)} + \theta\frac{y_t(k)}{P_t}\frac{\partial P_t}{\partial N_t(i)}\right].
\end{aligned}$$

Now assuming symmetry, setting the price index as the numeraire $P_t = p_t = 1$, and using (A.13):

$$\begin{aligned}
\frac{\partial mc_t(i)}{\partial N_t(i)} &= \frac{1-\eta}{\eta} mc_t \left(y_t + \phi + \frac{\phi_f}{N_t}\right)^{-1} \times \\
&\left[-\frac{y_t}{N_t} - \frac{\phi_f}{N_t^2} + \theta y_t \left[\frac{M_t - 1}{M_t}\frac{\partial p_t(k)}{\partial N_t(i)} + \frac{1 - M_t}{M_t}\frac{\partial p_t(i)}{\partial N_t(i)}\right]\right]
\end{aligned}$$

$$\frac{\partial mc_t(k)}{\partial N_t(i)} = \frac{1-\eta}{\eta} mc_t \left(y_t + \phi + \frac{\phi_f}{N_t}\right)^{-1} \theta y_t \left[-\frac{1}{M_t}\frac{\partial p_t(k)}{\partial N_t(i)} + \frac{1}{M_t}\frac{\partial p_t(i)}{\partial N_t(i)}\right].$$

Now use these in (A.14) to get

$$\frac{\partial P_t}{\partial N_t(i)} = \frac{1}{M_t}\frac{\eta - 1}{\eta} \left(y_t + \phi + \frac{\phi_f}{N_t}\right)^{-1} \left(\frac{y_t}{N_t} + \frac{\phi_f}{N_t^2}\right).$$

Manipulating the symmetric equilibrium version of the zero profit condition (A.8) gives $\frac{\phi M_t N_t + \phi_f M_t}{Y_t} = \frac{\mu_t}{\eta} - 1$. Then, noting that $y_t = \frac{Y_t}{M_t N_t}$ the above simplifies to

$$\frac{\partial P_t}{\partial N_t(i)} = \frac{(\eta - 1)(1 + \Phi_t^f)}{\mu_t M_t N_t} < 0 \tag{A.15}$$

[16] Note that $\sum_{k=1}^{M_t} \epsilon_t(k) = 1$. Then $\sum_{k=1}^{M_t} \frac{\partial \epsilon_t(k)}{\partial N_t(i)} = 0$, which under symmetry is $(M_t - 1)\frac{\partial \epsilon_t(k)}{\partial N_t(i)} + \frac{\partial \epsilon_t(i)}{\partial N_t(i)} = 0$.

[17] Since the marginal cost and price is the same for all firm i's varieties, $y_t(i,j) = y_t(i,k) = y_t(i)$.

where $\Phi_t^f \equiv \frac{\phi_f M_t}{Y_t}$ is the share of firm-level fixed costs in final output. Similar to the models with the love of variety, an expansion of the product scope reduces the aggregate price index. $\frac{\partial mc_t(i)}{\partial N_t(i)}$ can be rearranged to

$$\frac{\partial mc_t(i)}{\partial N_t(i)} = \frac{\eta - 1}{\mu_t^2} \left[\frac{1}{N_t} + \theta \frac{\partial p_t(i)}{\partial N_t(i)} - \theta \frac{\partial P_t}{\partial N_t(i)} + \Phi_t^f \frac{1}{N_t} \right]. \tag{A.16}$$

The next step is to find $\frac{\partial p_t(i)}{\partial N_t(i)}$. Combining (A.10) and (A.11):

$$\frac{\partial p_t(i)}{\partial N_t(i)} = \frac{\mu_t(i)[\mu_t(i) - 1]mc_t(i)}{1 - \epsilon_t(i)} \left((1-\theta) \frac{\epsilon_t(i)}{p_t(i)} \frac{\partial p_t(i)}{\partial N_t(i)} + (\theta - 1) \frac{\epsilon_t(i)}{P_t} \frac{\partial P_t}{\partial N_t(i)} \right) + \mu_t(i) \frac{\partial mc_t(i)}{\partial N_t(i)}$$

Applying symmetry with $\epsilon_t(i) = 1/M_t$ and some algebra gives

$$\frac{\partial p_t(i)}{\partial N_t(i)} = \frac{M_t + \frac{(\mu_t - 1)(\theta - 1)}{M_t - 1} + \frac{(1-\eta)\theta}{\mu_t}}{1 + \frac{(\mu_t - 1)(\theta - 1)}{M_t - 1} + \frac{(1-\eta)\theta}{\mu_t}} \frac{\partial P_t}{\partial N_t(i)} < 0. \tag{A.17}$$

Hence, $\frac{\partial p_t(i)}{\partial N_t(i)} < \frac{\partial P_t}{\partial N_t(i)} < 0$. From (A.10) and (A.11) it is now clear that $\frac{\partial \epsilon_t(i)}{\partial N_t(i)} > 0$ and $\frac{\partial mc_t(i)}{\partial N_t(i)} < 0$. An expansion of the product scope reduces the prices of the firm's varieties and increases its market share. This stands in contrast to Minniti and Turino (2013) where due to the love of variety, the firm would increase its prices. Using (A.10), under symmetry (A.9) can be written as

$$\begin{aligned} \frac{\partial \pi_t(i)}{\partial N_t(i)} &= \left(1 - \eta + \eta\theta \left(1 - \frac{1}{\mu_t}\right)^2\right) \left((1-\theta)\frac{\partial p_t(i)}{\partial N_t(i)} + (\theta - 1)\frac{\partial P_t}{\partial N_t(i)}\right) \\ &\quad + \left(1 - \frac{\eta}{\mu_t}\right) \frac{\partial P_t}{\partial N_t(i)} - (\mu_t - \eta)\frac{\partial mc_t(i)}{\partial N_t(i)} - \eta\phi \frac{1}{\mu_t} \frac{M_t}{Y_t} = 0. \end{aligned}$$

Finally, (A.15), (A.16), (A.17), and (10) are used in the above to solve for the product scope:

$$N_t = \frac{1-\eta}{\eta} \frac{\mu_t}{\phi} \frac{Y_t}{M_t} \left[\frac{1 + \Phi_t^f}{\mu_t} \left(1 - \frac{1}{M_t} - \frac{(M_t - 1)[\theta(M_t - 1) - M_t]}{M_t([\theta(M_t - 1) - M_t][\theta(M_t - 1)(\eta - 1) - \eta M_t] - \theta)} \right) \right] \tag{A.18}$$

Reminiscent of Minniti and Turino (2013), the big term in square brackets is less than one and is increasing in M_t (converging to unity as the number of firms becomes very large). An increase in the firm's product scope reduces its marginal costs and prices. Other firms respond by reducing their prices and to lower this price competition firms under-expand their product scopes relative to the case of monopolistic competition where such strategic interactions are absent. This strategic effect diminishes as the number of firms increases and this gives an incentive to introduce new varieties. Recall the share of fixed costs in final output:

$$\Phi_t \equiv \frac{\phi M_t N_t + \phi_f M_t}{Y_t} = \frac{\mu_t}{\eta} - 1.$$

Then, $\Phi_t = \Phi_t^f + \Phi_t^v$ together with (A.18) solves for $\Phi_t^f \equiv \phi_f M_t/Y_t$ and $\Phi_t^v \equiv \phi M_t N_t/Y_t$. As the number of firms becomes very large, these cost shares approach the levels in the monopolistic competition version of the model (see Appendix A.3). As the markup is countercyclical, it is clear that $\partial\Phi/\partial M < 0$. It can also be shown that $\partial\Phi^f/\partial M < 0$ and $\partial\Phi^v/\partial M > 0$. Firm entry leads to an expansion of product scopes and increases the variety-level fixed costs as a fraction of total output.

A.3 Monopolistic competition

This Appendix shows that under monopolistic competition, markups and the product scope are constant over the business cycle. When firms are too small to influence the aggregate price index, P_t, the last term in (A.1) is absent and the markup is constant at $\mu = \theta/(\theta - 1)$. Profits can be written as

$$\pi_t(i) = \frac{\mu - \eta}{\mu} P_t Y_t \epsilon_t(i) - mc_t(i)\eta(N_t(i)\phi + \phi_f)$$

where the market share is

$$\epsilon_t(i) = \left(\frac{mc_t(i)\mu}{P_t}\right)^{1-\theta} M_t^{-1}$$

The first-order condition is

$$\frac{\partial \pi_t(i)}{\partial N_t(i)} = -\left[\theta \frac{\mu - \eta}{\mu} P_t Y_t \frac{\epsilon_t(i)}{p_t(i)} + \eta(N_t(i)\phi + \phi_f)\right] \frac{\partial mc_t(i)}{\partial N_t(i)} - mc_t(i)\eta\phi = 0$$

and from (A.5)

$$\frac{\partial mc_t(i)}{\partial N_t(i)} = \frac{-\left(\frac{y_t(i)}{N_t(i)} + \frac{\phi_f}{N_t(i)^2}\right) mc_t(i)}{\frac{\eta}{1-\eta}\left(y_t(i) + \phi + \frac{\phi_f}{N_t(i)}\right) + \theta y_t(i)}.$$

Clearly, if $\eta < 1$, then $\frac{\partial p_t(i)}{\partial N_t(i)} = \mu \frac{\partial mc_t(i)}{\partial N_t(i)} < 0$. As in the previous section, $\frac{\phi M_t N_t + \phi_f M_t}{Y_t} = \frac{\mu}{\eta} - 1$ is obtained from the zero profit condition. Then, putting these together under symmetry and some algebra gives

$$\Phi_t^v = \frac{\theta\left(\frac{\mu}{\eta} - 1\right)\left(1 + \Phi_t^f\right)}{\frac{\mu}{1-\eta} + \theta} \tag{A.19}$$

where once again $\Phi_t^f \equiv \frac{\phi_f M_t}{Y_t}$ and $\Phi_t^v \equiv \frac{\phi M_t N_t}{Y_t}$. With constant markups and zero profits each period, these cost shares are constant each period. Using (A.19) and $\Phi_t^f + \Phi_t^v = \frac{\mu}{\eta} - 1$ then gives $\Phi_t^f = \mu - 1$ and $\Phi_t^v = \frac{\mu}{\eta} - \mu$. Since Φ_t^f is constant, the number of firms is proportional to final output and the product scope is constant:

$$N = \frac{1-\eta}{\eta}\frac{\mu}{\phi}\frac{Y_t}{M_t} = \frac{1-\eta}{\eta}\frac{\phi_f}{\phi}\theta.$$

Parameter η has no effect on local dynamics as output per firm and output per variety are constant. The dynamics of the model are identical to the constant markup mono-product model in Pavlov and Weder (2012) without the love of variety effects. Hence, indeterminacy cannot arise in this version of the model.

A.4 Data sources

This Appendix details the source and construction of the U.S. data used in Section 5. All data is quarterly and for the period 1955:I-2007:IV.

1. Gross Domestic Product. Seasonally adjusted at annual rates, billions of chained (2009) dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.6.

2. Gross Domestic Product. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

3. Personal Consumption Expenditures, Nondurable Goods. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

4. Personal Consumption Expenditures, Services. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

5. Gross Private Domestic Investment, Fixed Investment, Residential. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

6. Gross Private Domestic Investment, Fixed Investment, Nonresidential. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

7. Government Consumption Expenditures. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 3.9.5.

8. Government Gross Investment. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 3.9.5.

9. Nonfarm Business Hours. Index 2009=100, seasonally adjusted. Source: Bureau of Labor Statistics, Series Id: PRS85006033.

10. Civilian Noninstitutional Population. 16 years and over, thousands. Source: Bureau of Labor Statistics, Series Id: LNU00000000Q.

11. GDP Deflator = (2)/(1).

12. Real Per Capita Output, $Y_t = (1)/(10)$.

13. Real Per Capita Consumption, $C_t = [(3) + (4)]/(11)/(10)$.

14. Real Per Capita Investment, $X_t = [(5) + (6)]/(11)/(10)$.

15. Real Per Capita Government Expenditures, $G_t = [(7) + (8)]/(11)/(10)$.

16. Per Capita Hours Worked, $H_t = (9)/(10)$.

A.5 Alternative markup calibrations

Table A1 presents estimation results for two alternative markup calibrations: $\mu = 1.2$ and $\mu = 1.4$. Second moments and variance decompositions are virtually identical to Tables 2 and 3, and are not presented to conserve space. The most noticeable change is the higher θ and lower η when the calibrated μ is lower. This is due to parameters μ and θ jointly determining the elasticity of the markup (recall Section 3). A lower μ weakens the amplification mechanism from markup fluctuations and data then favors stronger amplification from product scope variations via lower variety-level returns to scale.

Table A1: Posterior distributions for alternative markup calibrations

	$\mu = 1.2$		$\mu = 1.4$	
Name	Mean	90% Interval	Mean	90% Interval
η	0.880	[0.870,0.890]	0.966	[0.948,0.985]
θ	22.492	[22.200,22.723]	13.391	[11.333,15.393]
ψ_A	0.003	[0.000,0.005]	0.016	[0.004,0.027]
ψ_Δ	0.989	[0.984,0.993]	0.989	[0.984,0.993]
ψ_g	0.991	[0.987,0.995]	0.991	[0.987,0.995]
ψ_{ag}	0.980	[0.969,0.993]	0.974	[0.959,0.990]
σ_s	0.540	[0.515,0.566]	0.526	[0.497,0.554]
σ_A	0.792	[0.749,0.836]	0.705	[0.658,0.751]
σ_Δ	0.461	[0.444,0.479]	0.456	[0.438,0.473]
σ_g	1.104	[1.036,1.168]	1.114	[1.047,1.179]
$\sigma^{m.e.}$	0.280	[0.279,0.280]	0.280	[0.279,0.280]
Ω_A	-0.529	[-0.595,-0.465]	-0.662	[-0.748,-0.577]
Ω_Δ	0.781	[0.667,0.885]	0.757	[0.637,0.880]
Ω_g	0.316	[0.270,0.362]	0.342	[0.292,0.391]

Prior distributions are identical to those from Table 1.