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Examining Quality Distortion

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EXAMINING SKEWED QUALITY *

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ABSTRACT

This paper defines quality as being skewed when the marginal rate of substitution (MRS) between quantity and quality differs from the marginal rate of transformation (MRT). Two classic analyses in the economics literature are reassessed in the light of this definition: (i) the model of monopoly with uniform pricing presented by Spence (1975) and Sheshinski (1976) (ii) the model of nonlinear price discrimination (equivalently vertical differentiation) of Maskin and Riley (1984). Conditions are found for the existence and direction of the skeweness in quality in the former model and in an extended version of the latter model.

JEL Classification: L11

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EXAMINING SKEWED QUALITY

One often hears people make statements such as “I am a person who prefers quality over quantity”. The implications of such comments are, of course, that such people recognize there is a tradeoff between these attributes. Unlike this popular saying, people do not have a preference for *either* quantity *or* quality but, rather, have a rate of substitution between the two. Such preferences will be apparent in the individual’s, and therefore market, demand. These preferences, together with market structure, will therefore influence equilibrium quality levels. This paper considers how to characterize the influence of market structure on the equilibrium quality level in the presence of this trade-off.

Pioneering papers modeling the determination of goods’ quality, such as Spence (1975) and Sheshinski (1976), implicitly incorporated the tradeoff between quantity and quality in a general way. These papers were concerned with relating the level of quality with its efficient (or competitive) level. Unambiguous results from these analyses were not forthcoming. The subsequent literature largely followed Mussa and Rosen (1978) in adopting a unit demand model. In these models each customer has a unit demand, and increases in quality linearly increase the willingness to pay. This approach provides great analytical convenience, and it has often proved possible to provide unambiguous results that relate quality to its efficient level. However the unit demand model does not allow substitution possibilities between quantity and quality. It thus abstracts from one of the fundamental features of markets with endogenous quality.

This paper proposes a definition of “skewed quality”. Quality is said to be skewed if the marginal rate of substitution (MRS) between quantity and quality differs from the marginal rate of transformation (MRT). The aim of introducing this definition is to provide a method to characterize the influence different market structures have on equilibrium quality levels. This method differs from traditional practice in that (i) it looks at the balance between quantity and quality rather than looking at the impact of marginal increments in quantity and quality individually (ii) this imbalance in the relative levels of quantity and quality is assessed locally rather than by relating the equilibrium quality level to the (globally) efficient quality level

The application and usefulness of this definition is tested by re-visiting two classic analyses in the economics literature. The first is the model presented in Spence (1975) and Sheshinski (1976). These authors considered a monopoly which set a uniform price. The representative consumer has an arbitrary set of preference over quality and quality. An alternative characterization of equilibrium quality than that provided by Spence and Sheshinski is provided. The skewness of quality in equilibrium is related directly to consumer preferences. In particular, it is shown that the manner in which the monopolist skews quality is determined by the elasticity of the representative consumer's indifference curve (showing the trade-off between quality and quantity). A number of useful special cases of preferences are discussed.

The second model that is reconsidered is the one introduced by Maskin and Riley (1984). Maskin and Riley consider how a monopolist can use nonlinear pricing to conduct price discrimination. In equilibrium the monopolist bundles output: low valuation customers purchase bundles with an inefficiently low quantity, whereas the highest valuation customer purchases a bundle with an efficient quantity. Maskin and Riley note that their model can be recast as the problem of vertical differentiation studied by Mussa and Rosen (1978). In doing so they adopt, as did Mussa and Rosen, the unit demand model. The results of this exercise parallel both their analysis of nonlinear pricing and Mussa and Rosen's results. Specifically they show, assuming a restriction on consumer preferences known as the "single crossing property", that the quality level supplied to low valuation customers is below the efficient level, whereas the quality level supplied to the highest valuation customer is efficient.

This conclusion is often taken to be a general characterization of vertical differentiation. In this paper the Maskin and Riley model is extended to allow both quantity and quality to be simultaneously endogenous. In this case it is shown, assuming a natural generalization of the single crossing property, that it is possible that low valuation customers purchase goods with upwardly skewed quality. Equilibrium when consumer preferences do not satisfy this generalization of the single crossing property is also considered. Under one restriction on preferences, referred to in this paper is as diverse ordering, vertical differentiation does not yield skewed quality.

Section 1 states the formal definition of skewed quality and discusses its implications. Section 2 reexamines the model the Spence model in light of the discussion in section. Section 3 extends the Maskin and Riley model and provides the conditions under which skewed quality occurs. Section 4 concludes the paper.

1. Skewed quality: A definition

Suppose a good has n varieties. The quantity produced of variety i is X^i, i=1,…,n., and each unit has quality level Z^i bundled with it. The total consumer benefit (utility) available from the production of these varieties is $V(X^i,Z^i,X^{-i},Z^{-i})$. The surplus from production of all varieties is therefore:

$$S(X^i,Z^i,X^{-i},Z^{-i}) = V(X^i,Z^i,X^{-i},Z^{-i}) - C(X^i,Z^i,X^{-i},Z^{-i}) \tag{1}$$

where $C(X^i,Z^i,X^{-i},Z^{-i})$ is total cost. The definition of skewed quality used in this paper is:

Definition 1: In a market equilibrium $\{X^i,Z^i,\ i=1,..n\}$, the quality of variety i is said to be downwardly (un-,upwardly) skewed if:

$$\frac{V_1(X^i,Z^i,X^{-i},Z^{-i})}{V_2(X^i,Z^i,X^{-i},Z^{-i})} < (=, >) \frac{C_1(X^i,Z^i,X^{-i},Z^{-i})}{C_2(X^i,Z^i,X^{-i},Z^{-i})} \tag{2}$$

where the RHS is the marginal rate of substitution (MRS) and the LHS is the marginal rate of transformation (MRT) of variety i.

The motivation for this definition is the observation that the unskewed combination of quantity and quality maximizes the surplus for a given level of resources devoted to production. Specifically the unskewed combination $\{\tilde{X}^i(X^{-i},Z^{-i}),\tilde{Z}^i(X^{-i},Z^{-i})\}$ satisfies:

$$\{\tilde{X}^i(X^{-i},Z^{-i},\bar{C}),\tilde{Z}^i(X^{-i},Z^{-i},\bar{C})\} = \underset{X^i\,Z^i}{\operatorname{argmax}}\ S(X^i,Z^i,X^{-i},Z^{-i}) \text{ subject to } C(X^i,Z^i,X^{-i},Z^{-i}) \le \bar{C} \tag{3}$$

The unskewed combination has optimal balance of quantity and quality. A variety has downwardly skewed quality if, by substituting quality for quantity, a higher surplus is obtainable for a given level of resources devoted to production. This is depicted graphically in figure 1 for the case when only one variety is produced. (A superscript on quality and quantity is thus unnecessary.) Quality is unskewed at the point $\tilde{\Pi}$ where the indifference curve $\tilde{V}$ is tangent to $\bar{C}$, the iso-cost curve. That is, quality is unskewed when the quantity-quality combination is $\{\tilde{X}, \tilde{Z}\}$. Note that at the point Π^* quality is downwardly skewed, as the MRS<MRT.

Note that when quality is unskewed, the implicit function theorem can be used to rewrite (2) as $\tilde{Z}^i = \tilde{z}^i(X^i, X^{-i}, Z^{-i})$. The function $\tilde{z}^i$ is called the contract curve for variety i, as it represents the locus of points of tangency between the indifference curves and the iso-cost curves. Quality skewness occurs when an outcome does not lie on the contract curve.

It is necessary to state the following companion to definition 1 that describes quantity distortion

> Definition 2: In a market outcome $\{X^i,Z^i, i=i,..n\}$, the quantity of variety i is said to be downwardly (un-,upwardly) distorted if:
>
> $$V_1(X^i,Z^i,X^{-i},Z^{-i}) \;>\; (=, <)\; C_1(X^i,Z^i,X^{-i},Z^{-i}) \qquad (4)$$
>
> and the quality of variety i is said to be downwardly (un-,upwardly) distorted if:
>
> $$V_2(X^i,Z^i,X^{-i},Z^{-i}) \;>\; (=, <)\; C_2(X^i,Z^i,X^{-i},Z^{-i}), \qquad (5)$$

The motivation for definition 2 is very familiar. It is, of course, the parallel of the relationship between price and marginal cost in textbook analysis. Note that the definitions summarized by (2) and (4) are stated given (X^{-i},Z^{-i}). In definition 2 there is no requirement that (X^{-i},Z^{-i}) be undistorted. In this sense both definitions 1 and 2 are 'local': it is possible to consider each variety one at a time. Similarly, in assessing quality distortion using definition 1 it is not necessary that X^i be undistorted. It is possible that quality be undistorted and X^i be downwardly distorted. The interpretation of this example is that the balance of quality and quantity is optimal, even though total output is sub-optimal. Note that if all varieties are undistorted, then the market outcome is efficient, i.e. the surplus (1) is maximized.

Condition (5) is referred to in this paper as the marginal rule for quality distortion. The marginal rule, usually with the additional requirement that (X^i,X^{-i},Z^{-i}) also be undistorted, is widely adopted as the definition of quality distortion, including in the seminal work of Spence (1975) and Sheshinski (1976). The marginal rule has proved a difficult analytical tool to use, (see, for example, Spence and Sheshinski). In addition, it does not capture the idea of there being an optimal trade-off quantity and quality at different output levels

Using definition 1 can be combined with (4) to provide a meaningful qualitative characterization of equilibrium quality. In some case this provides the same information as using the marginal rules in definition 2. Suppose in equilibrium that $V_1 > (<) C_1$ and $V_2 < (>) C_2$. In this case it is clear that the quantity produced is downwardly (upwardly) distorted. Under the marginal rule quality is upwardly (downwardly) distorted and by definition 1 quality is upwardly (downwardly) skewed. Now consider an outcome $V_1 > (<) C_1$ and $V_2 > (<) C_2$. Again the quantity produced is downwardly (upwardly) distorted. In addition quality is downwardly (upwardly) distorted. Both measures suggest that production is “insufficient” (excessive). In this sense the two rules composing definition 2, conditions (4) and (5), yield the same information. However by using definition 1 as a substitute (or in addition to) the marginal rule (5), additional information on the balance of quantity and quality is provided.

The locus of unskewed quality levels is also generated from the following optimization problem:

$$\{\tilde{X}^i(X^{-i},Z^{-i},\bar{C}),\tilde{Z}^i(X^{-i},Z^{-i},\bar{C})\} = \underset{X^i\, Z^i}{\text{argmin}}\; C(X^i,Z^i,X^{-i},Z^{-i}) \text{ subject to } V^i(X^i,Z^i,X^{-i},Z^{-i}) \geq \bar{V} \tag{6}$$

Thus $\tilde{Z}^i$ could thus be described as the cost minimizing quality level. This is the description used by Levhari and Peles (1973) and Kihlstrom and Levhari (1977) for the case of a monopoly where (implicitly) the representative customer utility was given by V=XZ. The adoption of definition 1 might therefore be seen as a generalization of the approach of Levhari and Peles (1973).

2. Monopoly with uniform pricing

In this section the problem posed by Spence (1975) and Sheshinski (1976) is reconsidered in light of definition 1. A monopolist sets a uniform price per unit and cannot segment the market according to customers' willingness to pay. There is a single representative consumer with benefit (utility) V(X,Z). The inverse demand function is $P(X,Z)=V_1(X,Z)$. Firm profit, Π, is given by:

$$\Pi = R(X,Z) - C(X,Z) \qquad (7)$$

where $R(X,Z) \equiv P(X,Z)X$ is revenue. The first order condition for profit maximization gives:

$$\frac{R_1}{R_2} = \frac{C_1}{C_2} \qquad (8)$$

The monopolist chooses quantity and quality so that the slope of the iso revenue curve equals the marginal rate of transformation.

To study the nature of quality skewness the following definition is useful:

Definition 3: The elasticity of indifference, E, is:

$$E \equiv \frac{XV_1}{ZV_2} \qquad (9)$$

E represents the elasticity of the indifference curve V(X,Z). Spence (1975) noted that quality is downwardly distorted according to the marginal rule if $V_2(X,Z)<P_2(X,Z)X$, that is, the average benefit of an increase in quality is less than the marginal benefit. Observe that this condition is equivalent to $E > E_{ZP}$, where $E_{ZP} \equiv P(X,Z) /ZP_2(X,Z)$ is the elasticity of quality with respect to price.

The following proposition shows that the elasticity of indifference is useful in identifying the nature of quality skewness as stated in definition 1.

Proposition 1: Quality is upwardly (un- downwardly) skewed by the monopolist if the elasticity of the indifference curve is increasing in quantity, $\partial E/\partial X >(=, <) 0$.

Proof: Let X, Z represent the profit maximizing quantity and quality. Then quality is downwardly (un-,upwardly) skewed if:

$$\frac{V_1(X,Z)}{V_2(X,Z)} < (=, >) \frac{V_1(X,Z) + XV_{11}(X,Z)}{XV_{12}(X,Z)} \tag{10}$$

or, equivalently:

$$V_1(X,Z)V_2(X,Z) + XV_{11}(X,Z)V_2(X,Z) - XV_{12}(X,Z)V_1(X,Z) > (=, <) \ 0 \tag{11}$$

Now:

$$\frac{\partial E}{\partial X} = \frac{V_1(X,Z)V_2(X,Z) + XV_{11}(X,Z)V_2(X,Z) - XV_{12}(X,Z)V_1(X,Z)}{ZV_2(X,Z)^2} \tag{12}$$

$\partial E/\partial X$ >(=, <) 0 if (11) holds. ||

Proposition 1 is illustrated in figure 1. When $\partial E/\partial X>0$, the iso-revenue curve is steeper than the indifference curve. The profit maximizing quantity and quality is shown as X* and Z*. The (profit maximizing) iso-revenue curve, R*, is tangent to the iso-cost curve at the point Π*. Note that a higher surplus could be achieved ($\tilde{V}$ rather than V*), for a given cost, if the firm produced quantity and quality $\tilde{X}$ and $\tilde{Z}$. However such an action is obviously not profit maximizing, as at the point $\tilde{\Pi}$ firm revenue is $\tilde{R}$< R*. Thus when $\partial E/\partial X > 0$ the firm downwardly skews quality.

The above analysis suggests an alternative way to view the classic question of how the efficient and profit maximizing quality levels are related. The monopolist's production decision can be conceptually decomposed into two steps: (i) the monopolist restricts "output" (which could be taken as either X or V) to increase profits in the manner described in textbooks, and (ii) the monopolist chooses the balance of output and quality to maximize profits. The former action is represented by movements along the contract curve, while the latter is shown as movements away from the contract curve. The impact of the former action on quality is ambiguous: it depends on the slope of the contract curve.

Two cases are shown in figure 1: in case A the contract curve, $\tilde{z}^A(X)$, is upward and in case B the contract curve, $\tilde{z}^B(X)$, is downward sloping. The efficient quantity is $\hat{X}$ and efficient quality is $\hat{Z}^A$ in case A and $\hat{Z}^B$ in case B. In case A, restriction (distortion) of quantity by the monopolist along the contract curve causes quality to fall from $\hat{Z}^A$ to $\tilde{Z}$. In addition, because the iso-revenue curves are steeper than the indifference curves, the monopolist downwardly skews quality from $\tilde{Z}$ to Z*. Overall, therefore, in case A profit maximizing quality is below the efficient level. In contrast, if the contract curve is upward sloping, as in case B, the monopolists two decision work in opposite directions, i.e. $Z^*<\tilde{Z}$ and $\tilde{Z}> \hat{Z}^B$. The net result on quality of these two actions is thus ambiguous. A similar analysis applies if, in contrast to the assumption in figure 1, the iso-revenue curves are flatter than the indifference curves and there is thus upwardly skewed quality. The profit maximising quality is unambiguously above the efficient quality if the contract curve is downward sloping. However if the contract curve is upward sloping, the firms two actions operate in different directions, and the relationship between the efficient and profit maximizing quality levels is ambiguous.

The application of Proposition 1 is considered in the following three examples.

Example 1: Multiplicative separable utility. Let $V(X,Z) = v(\xi(X)\zeta(Z))$, where $v'(.)>0$, $v''(.)<0$, $\xi'(X)>0$ and $\zeta'(Z)>0$. In this case:

$$E = \frac{X\xi'(X)\zeta(Z)}{y\zeta'(Z)\xi(X)} = \frac{\varepsilon_{\xi X}}{\varepsilon_{\zeta Z}} \qquad (13)$$

where $\varepsilon_{\xi X} \equiv X\xi'(X)/\xi(X)$ and $\varepsilon_{\zeta Z} \equiv y\zeta'(Z)/\zeta(Z)$. Thus quality is downwardly (un-, upwardly) skewed if $d\varepsilon_{\xi X}/dX < (=, >)\ 0$.

In this example, the term $\zeta(Z)$ can be interpreted as the experience of consuming a unit of the good with quality level Z. Call $\xi(X)\zeta(Z)$ the gratification from consuming X units of the good which, in this interpretation, represents the “total quality” from consumption. On average, a unit of the good provides gratification $\xi(X)\zeta(Z)/X$, while the marginal unit provides gratification $\xi'(X)\zeta(Z)$. In many instances, the quality of the marginal unit will be perceived by the consumer as providing the same quality as the average unit. If the marginal gratification is equal to the average gratification, then $\xi'(X)=\xi(X)/X$, hence $\xi(X) = AX$. In this event, $\varepsilon_{\xi X}=1$, and thus quality is unskewed.

Example 2: Iso-elastic $\xi(X)$. Following on from example 1, suppose $\xi(X)$ takes the isoelastic form $\xi(X) = AX^{\alpha}$, $A>0$, $\alpha>0$. Then $\varepsilon_{\xi X}=\alpha$, and thus quality is unskewed. In this case the relationship between the profit maximizing and efficient quality levels depends entirely on the slope of the contract curve, which in turn depends on the functional form of the cost function. The contract curve can be written:

$$\frac{XC_1}{ZC_2} = \frac{\alpha}{\varepsilon_{\zeta Z}} \qquad (14)$$

The LHS is the elasticity of the iso-cost curve. When cost is isoelastic, i.e. $C(X,Z)=X^{\chi}\psi(Z)$ $\chi>0$, $\psi'(Z)>0$ then the LHS is independent of X. In this case (14) shows that quality choice is independent of quantity choice. The contract curve is horizontal, and the monopolist chooses the efficient level of quality. That is, Swan invariance (Swan, 1970) holds.[1]

Example 3: Quantity dependent gratification. Let $V(X,Z) = \mathcal{V}(X\psi(Z),X)$.[2] The first argument of the utility function represents the gratification from consumption, as described in example 1. However there may be an "externality" to this consumption. For instance, increased consumption of Cola leads to increased tooth decay. This effect is captured by the second argument in the utility function. In this case the indifference elasticity is:

$$E = \frac{1+\varepsilon_{\mathcal{V}}}{\varepsilon_{\zeta Z}} \qquad (15)$$

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[1] Kihlstrom and Levhari (1977) show this result for the case in which $\alpha=1$.

[2] In theory any utility function, $V(X,Z)$, could be written in this form by adopting the transformation $\mathcal{V}(XZ,X) \equiv V(X,(XZ)/X)$. However the merit of applying this transformation will depend on whether the application under consideration suggests it has any intuitive justification.

where $\varepsilon_{\upsilon} \equiv \dfrac{X\upsilon_2}{X\psi(Z)\upsilon_1}$ is the elasticity of the indifference curve between gratification and quantity. Quality is upwardly (un- downwardly) skewed ed if $\partial\varepsilon_{\upsilon}/\partial X >(=, <)\ 0$.

The analysis presented in this section does not remove the ambiguity (concerning the relative size of the monopoly and efficient quality) that led Spence to describe his results as "somewhat discouraging" (p.428). However it does provide a way to decompose the influence of the monopolist’s market power into two effects: (i) the restriction of output along the contract curve and (ii) the distortion of balance of quantity and quality. The examples in this section provide a tractable approach to model the monopolist’s choice quality.

3. Vertical differentiation with nonlinear pricing

This section uses definition 1 to extend the analysis of Maskin and Riley (1984). Maskin and Riley's primarily focus is not on the determination of goods' quality, but rather on how output (quantity) might be bundled to conduct price discrimination. Specifically they show that an appropriately designed nonlinear pricing schedule can separate customer according to their willingness to pay. Their most prominent result is that, under the assumption that preferences satisfy a "single crossing property" low valuation customers obtain a low consumer surplus (with the lowest valuation customer obtaining a zero surplus) and inefficiently low consumption, while high valuation customers obtain a relatively high consumer surplus and consume efficiently.

Maskin and Riley, however, demonstrate (in their section 6) that there is a direct parallel between vertical differentiation in a unit demand model and the optimal quantity bundling discussed above. Specifically, again under the assumption that preferences satisfy a "single crossing property", customers with a low valuation of quality face low consumer surplus and inefficiently low quality, while consumers with a high valuation of quality have a relatively high consumer surplus and receive an efficient quality level.

This section revisits the Maskin and Riley model, but allows for more general preferences. While the Maskin and Riley analysis is one dimensional (either quantity or quality), this section will allow for general set of preference over quality and quality. Furthermore, a more general form of cost function is allowed for. The analysis is formally conducted in the context of a generalized "single crossing property". However the implications of the property not holding are also considered.

A monopoly has n customer types. The firm conducts non-linear pricing: it offers bundles with quantity X of quality Z for the (lump sum) fee T. The consumer surplus of type i customers from this offer is:

$$U^i(X,Z,T) = \begin{cases} V^i(X,Z) - T & \text{if } V^i(X,Z) \geq T \\ 0 & \text{if } V^i(X,Z) < T \end{cases} \qquad (16)$$

where it is assumed that customers do not purchase the bundle, and thus receive zero utility, if purchasing the bundle would yield negative consumer surplus. The firm knows the distribution of customer types, but cannot identify specific individuals as belonging to a customer type.

The firm offers a set of schedules $<X^i,Z^i,T^i>$, i=1,..n, with the aim that type i customers purchase the bundle for fee T^i. For this to occur the schedules must satisfy the selection constraints:

$$V^i(X^i,Z^i) - T^i \geq V^i(X^j,Z^j) - T^j \quad \text{for all } j \neq i \qquad (17)$$

Consumer i will only purchase bundle $<X^i,Z^i>$ if it provides non-negative consumer surplus. Consumer i's participation constraint is:

$$V^i(X^i,Z^i) \geq T^i \qquad (18)$$

If these constraints are satisfied, firm profit from type i customers is

$$\Pi^i = T^i - C(X^i,Z^i) \qquad (19)$$

The firm chooses X^i,Z^i,T^i, i=1,..,n, to maximize total profits, Π, i.e:

$$\max_{X^i Z^i T^i} \Pi = \max_{X^i Z^i T^i} \sum_{i=1}^{n} \Pi^i \text{ subject to (17) and (18).} \qquad (20)$$

The optimization problem (20) is formally identical to the one facing a monopolist that utilizes non-linear pricing to sell bundles consisting of two goods. Identifying the optimal nonlinear prices for a multiproduct monopolist has proved difficult for an arbitrary distribution of consumer preferences. Most approaches restrict consumer preferences by proposing some generalization of the single crossing condition adopted by Maskin and Riley (1984) (see Armstrong, 1996 McAfee and McMillan, 1988, and Sibley and Srinagesh, 1997). The following restriction on preferences, which can be thought of as a straightforward generalization of the one dimensional single crossing property of Maskin and Riley, is used below:

Definition 4: Preferences exhibit uniform ordering if for all i=2,..n: (i) $V^i(X,Z) > V^{i-1}(X,Z)$, (ii) $V^i_1(X,Z) > V^{i-1}_1(X,Z)$ and, (iii) $V^i_2(X,Z) > V^{i-1}_2(X,Z)$.

This definition of uniform ordering is adapted from the definition of Sibley and Srinagesh (1997). Sibley and Srinagesh (p.699) view uniform ordering as "unappealing" because it so restrictive on allowable preferences. However it ensures that preferences can be ranked as in an analogous way to the one dimensional case studied by Maskin and Riley. As the purpose of this analysis is to consider the implications of applying definition 1 to describe quality skewness, it is important to examine the case of uniform ordering in detail. The implications of relaxing the restrictions imposed by uniform ordering are considered below.

It is shown in the appendix that, under uniform ordering, only the downward-adjacent incentive compatibility constraints are binding, and that only type 1's participation constraint is binding. (This is the same result as the one dimensional case studied by Maskin and Riley.) The Lagrangian for the optimization problem (20) is:

$$L = \sum_{i=2}^{n}[T^i - C(X^i,Z^i) + \lambda^i\{V^i(X^i,Z^i) - V^i(X^{i-1},Z^{i-1}) - T^i + T^{i-1}\}]$$

$$+ T^1 - C(X^1,Z^1) + \mu(V^1(X^1,Z^1) - T^1)) \qquad (21)$$

where $\lambda^i \geq 0$ and $\mu \geq 0$ are the Langrange multipliers. Hence:

Proposition 2: Under uniform ordering variety i is downwardly (un-, upwardly) skewed if:

$$\frac{V_1^{i+1}(X^i,Z^i)}{V_2^{i+1}(X^i,Z^i)} < (=, >) \quad \frac{V_1^{i}(X^i,Z^i)}{V_2^{i}(X^i,Z^i)} \qquad (22)$$

Proof: The first order conditions of (21) are:

$$\partial L \partial T^1 = 1 + \lambda^2 - \mu = 0 \qquad (23)$$

$$\partial L \partial T^i = 1 + \lambda^{i-1} - \lambda^i = 0, \text{ for } i = 2,...,n-1. \qquad (24)$$

$$\partial L \partial T^n = 1 - \lambda^n = 0 \qquad (25)$$

Hence $\lambda^i = n-i+1$, for i=2,...n and $\mu = n$. Note that these multipliers are positive, so all of their concomitant constraints are binding.

$$\partial L \partial X^1 = - C_1(X^1,Z^1) + \mu V_1^i(X^1,Z^1) - \lambda^2 V_1^2(X^1,Z^1) = 0 \tag{26}$$

$$\partial L \partial X^i = - C_1(X^i,Z^i) + \lambda^i V_1^i(X^i,Z^i) - \lambda^{i+1} V_1^{i+1}(X^i,Z^i) = 0 \text{ for } i=2,...n-1 \tag{27}$$

$$\partial L \partial X^n = - C_1(X^n,Z^n) + \lambda^n V_1^n(X^n,Z^n) = 0 \tag{28}$$

$$\partial L \partial Z^1 = - C_2(X^1,Z^1) + \mu V_2^1(X^i,Z^i) - \lambda^2 V_2^2(X^1,Z^1) = 0 \tag{29}$$

$$\partial L \partial Z^i = - C_2(X^i,Z^i) + \lambda^i V_2^i(X^i,Z^i) - \lambda^{i+1} V_2^{i+1}(X^i,Z^i) = 0 \text{ for } i=2,...n-1 \tag{30}$$

$$\partial L \partial Z^n = - C_2(X^n,Z^n) + \lambda^n V_2^n(X^n,Z^n) = 0 \tag{31}$$

Dividing (28) by (31) gives:

$$\frac{C_1(X^n,Z^n)}{C_2(X^n,Z^n)} = \frac{V_1^n(X^n,Z^n)}{V_2^n(X^n,Z^n)} \tag{32}$$

Hence variety n exhibits unskewed quality. Dividing (27) by (30) and (28) by (31) gives:

$$\frac{C_1(X^i,Z^i)}{C_2(X^i,Z^i)} = \frac{(n-i+1)V_1^i(X^i,Z^i) - (n-i)V_1^{i+1}(X^i,Z^i)}{(n-i+1)V_2^i(X^i,Z^i) - (n-i)V_2^{i+1}(X^i,Z^i)} \qquad i=1,...,n-1 \tag{33}$$

Thus the quality of variety i, i=1,..,n-1, is downwardly (un-,upwardly) skewed if:

$$\frac{V_1^i(X^i,Z^i)}{V_2^i(X^i,Z^i)} < (=, >) \frac{(n-i+1)V_1^i(X^i,Z^i) - (n-i)V_1^{i+1}(X^i,Z^i)}{(n-i+1)V_2^i(X^i,Z^i) - (n-i)V_2^{i+1}(X^i,Z^i)} \tag{34}$$

(22) follows from (34). ||

Note that the requirement of uniform ordering does not impose a ranking in the magnitudes of types' MRS. The reason that quality is skewed when the MRS differs across types can be illustrated using figure 2. Assume there are two customer types with type 2 customers having a greater marginal rate of substitution:

$$\frac{V_1^1(X^i,Z^i)}{V_2^1(X^i,Z^i)} > \frac{V_1^2(X^i,Z^i)}{V_2^2(X^i,Z^i)} \qquad (35)$$

In this case, Proposition 2 indicates that quality is downwardly skewed. The explanation of this result begins by considering contract curve, $\tilde{z}^i(X)$, associated with each customer type i shown in figure 2(b). Suppose, for the moment, that the firm is constrained to produce bundles that lie on the contract curve. Then firm profit from type i customers, when these customers are guaranteed consumer surplus $\bar{U}^i$, is given by:

$$\Pi^i(X,\bar{U}^i) = N^i(X,\tilde{z}^i(X)) - \bar{U}^i \qquad (36)$$

Firm profit for each type, $\Pi^i(X,\bar{U}^i)$, is plotted in figure 2(a). This diagram mirrors figure 1 in Maskin and Riley's figure 1. However the curves $\Pi^i(X,\bar{U}^i)$ in figure 2(a) have been augmented to incorporate an endogenous level quality, so that production is constrained to lie on the contract curve $\tilde{z}^i(X)$. Maskin and Riley interpret the curves $\Pi^i(X,\bar{U}^i)$ as the customer type i's indifference curves across schedules (rather than bundles), i.e. they show lines of constant consumer surplus as a function of profits (or the fee) and quantity. If the firm could identify customer type, it would be profit maximizing to offer the schedule $<\hat{X}^i\ \hat{Z}^i\ \hat{T}^i>$, where $\hat{T}^i$ is equal to the consumer benefit of type i. Under these schedules consumers gain utility $\bar{U}_0^i$, which is the level of utility at which type i consumers are indifferent between purchasing the bundle $<\hat{X}^i\ \hat{Z}^i>$ or not. However, where the firm cannot identify customer type, type 2 customers would have an incentive to switch to type 1's bundle. Maskin and Riley show that, in order to optimally satisfy self selection, the firm must reduce the quantity to type 1 customer from $\hat{X}^1$ to $\tilde{X}^1$ and increase the level of utility to type 2 customers from $\bar{U}_0^2$ to $\bar{U}_1^2$ (by reducing the fee below the level of benefit). On the assumption that the firm is constrained to produce unskewed quality, quality will be $\tilde{Z}^1$. However observe that if the firm is not constrained to produce unskewed quality, it can increase profits by substituting quantity for quality in bundle 1. By doing so, type 1 customers continue to satisfy the participation constraint, and T^2 can be increased by $\tilde{V}^2$ - $\bar{V}^2$ while type 2 customers still satisfy self selection.

Following the analysis of Maskin and Riley it is assumed above that the firm supplies all customer types. However Armstrong (1996) points out, in the context of a multiproduct monopolist, that it may not be profit maximizing for the firm to supply the low valuation customers. In particular, suppose the valuation of, say, type 1 customers is very much lower than all other customers. If the firm is to supply these customers it must set a relatively low fee to all other customer to satisfy incentive compatibility. In this case it may be optimal for the firm to forgo the (small) profits available from supplying type 1 customers in order to raise the fee to all other customers. It is straightforward to show that the condition (22) identifies quality skewness in those varieties which are actually supplied. Note, however, when the firm drops the production of one or more varieties the optimal bundle for type i changes. In this event it is possible that the quality skewness of those varieties produced also changes.

It may be wondered whether, as with Maskin and Riley, the profit maximizing level of quality of all varieties but variety n is necessarily below their efficient level. The following example shows that it is not.

<u>Example 4</u>: Consider the case in which $V^i(X,Z) = A^i X Z^{\gamma_i}$ where A^i and γ_i, are positive parameters. Assume that $A^i<A^{i+1}$ and that $\gamma_i A^i<\gamma_{i+1}A^{i+1}$, in such a way that uniform ordering holds for relevant values of X and Z. In this case, by proposition 2, variety i is downwardly (un-,upwardly) skewed if γ_i < (=, >) γ_{i+1}. If, in addition, cost is given

by $C(X,Z) = \overline{C}X^{\chi}(\omega + \varpi Z^{\phi})$ where $\overline{C}$, ω, ϖ, ϕ, and χ are positive parameters, the contract curve is horizontal. The efficient quality of variety i, $\hat{Z}^i$, is given by:

$$\hat{Z}^i = \left(\frac{\gamma_i \chi \omega}{\varpi(\phi - \gamma_i \chi)}\right)^{\frac{1}{\phi}} \tag{37}$$

In this case Z^i < (=, >) $\hat{Z}^i$ if γ_i < (=, >) γ_{i+1}.

In this example the contract curves for each customer type are horizontal at $Z^i = \hat{Z}^i$. Thus differences in the MRS between type i and i+1 (i.e. differences between γ_i and γ_{i+1}) determine the relationship between quality and its efficient level. In contrast to the results of Mussa and Rosen and Maskin and Riley quality of variety i is above its efficient level if $\gamma_i > \gamma_{i+1}$. This suggests the results from unit demand models may be misleading with regard to the direction of quality distortion.

In the above analysis it is assumed that there are no cost spillovers in the production of different varieties. As pointed out by Kim and Kim (1996) many production processes, for example automobiles, are characterized by cost spillovers between varieties. For example, undertaking production of a small car (the low quality variety) will lower the cost of a mid-sized car (a higher quality variety). Definition 1 allows for the presence of cost spillovers. It is readily shown that Proposition 2 holds in the presence of cost spillover provided their presence does not alter which of the self selection and participation constraints are binding. However this may not be the case. For instance, it is possible that the presence of a spillover might substantially lower the marginal cost of producing variety 1 relative to other varieties. It may be optimal for the firm to seek to offer type 1 customers a bundle with higher quantity and quality than that of type 2 customers. In this case the incentive compatibility constraints would be upwardly binding and thus Proposition 2 would not hold.

Mussa and Rosen and Maskin and Riley’s results are also often interpreted as implying that quality distortion is ubiquitous when consumers are free from available bundles. However this aspect of Maskin and Riley’s results depends crucially on the assumption of the single crossing condition holding. The above results similarly depend on uniform ordering. This claim can be verified by considering the following alternative distribution of preferences:

Definition 5: Order consumers such that, i<j if $C(\hat{X}^i,\hat{Z}^i) < C(\hat{X}^{i+1},\hat{Z}^{i+1})$. Diverse ordering occurs if:

$$V^i(\hat{X}^i, \hat{Z}^i) > V^j(\hat{X}^j, \hat{Z}^j) \text{ for all } i \neq j. \tag{38}$$

Under diverse ordering the distribution of consumer utility is such that self selection holds if $T^i = V^i(\hat{X}^i, \hat{Z}^i)$. Thus $<\hat{X}^i\ \hat{Z}^i\ \hat{T}^i>$ is the incentive compatible profit maximising schedule. The firm produces the efficient quantity and quality is unskewed and undistorted.

Figure 3 is used to illustrate this outcome. Figure 3, like figure 2(a), is an augmented Maskin-Riley diagram. For simplicity, attention is restricted to two customer types in figure 3. Type 1 customer preferences are represented by the indifference curves labeled $\Pi^1(.)$, where $\Pi^1(X,\bar{U}_0^1)$ is the indifference curve along which the participation constraint is binding. Three types of preferences of type 2 customers are shown. A consumer preference satisfying uniform ordering (as discussed above), represented by the indifference curve $\Pi_U^2(X,\bar{U}_0^2)$, is included in figure 3 for comparison purposes. In contrast, the indifference curve $\Pi_D^2(X,\bar{U}_0^2)$ is an example of preferences which, when combined with those of type 1's preferences, exhibits diverse ordering. If the firm offers schedules $<\hat{X}^i\ \hat{Z}^i\ \hat{T}^i>$, i=1,2, type i's bundle is represented by the point M^i. It is apparent from figure 3 that type i would receive lower utility if they switched to the alternative bundle. Thus the schedule $<\hat{X}^i\ \hat{Z}^i\ \hat{T}^i>$, which includes the efficient bundle, is incentive compatible.

Using a unit demand model Donnenfeld and White (1988) and Srinagesh and Bradburd (1989) (DWSB) argue that, in contrast to the findings of Mussa and Rosen and Maskin and Riley, the high quality level may be upwardly distorted. Noting the direct parallel between quality choice with unit demand and quantity choice, the DWSB argument can be illustrated in figure 3. An example of the case considered by DWSB is represented by preferences $\Pi_T^2(X,\bar{U}_0^2)$. Observe that these preferences do not satisfy the single crossing condition – indeed as shown the indifference curves of different types may cross twice. DWSB do not provide a characterization of preferences under which their analysis holds. Rather they note that the slope of customer type 1's demand curve must be steeper than that of customer type 2's demand curve.[3] This is the case for output levels above $\hat{X}^1$.

In this case the schedule $<\hat{X}^i \hat{Z}^i \hat{T}^i>$ is inconsistent with incentive compatibility: type 1 customers (represented by the point M^1) receive a higher utility by consuming type 2's bundle (represented by the point $\hat{N}^2$). In order to satisfy incentive compatibility it is necessary to lower the fee to type 1 customers so they receive utility $\bar{U}_1^1$ (at point N^1). It is also necessary to increase the quantity available to type 2 customers from $\hat{X}^2$ to $\tilde{X}^2$, so that they move along their participation constraint from the point $\hat{N}^2$ to N^2. In this event the upward adjacent incentive compatibility constraints are binding.

However, as with the uniform ordering case, the point N^2 only represents an equilibrium for type 2 customers if quality is unskewed. The test for existence and direction of quality skewness in this case is found by reasoning analogous to the above analysis. It is readily shown that the condition for existence and direction of quality skewness is given by (22) with the direction of the inequality signs reversed.

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[3] DWSB adopt the unit demand model, thus consider the choice of quality assuming quantity is fixed. In their papers they refer to the customers' trade-off between price and quality as the marginal rate of substitution. However this trade-off is better described as the demand for quality.

4. Discussion

Since Mussa and Rosen (1978) the unit demand model has been the dominant methodology used to analyse the impact of market structure on the quality level of goods. The widespread use of the unit demand model is because of its tractability in relating equilibrium to the efficient level. However the unit demand model is highly restrictive on the set of preferences allowable. This paper has suggested a method to analyse more models with more general preference, and thereby bypass the restrictions imposed by the use of the unit demand model.

The definition of quality skewness proposed in this paper identifies the balance of quantity and quality in market equilibrium rather than, as is traditional, relate the equilibrium quality level to the efficient level. Specifically, quality skewness is defined on the basis of the relationship between equilibrium quality and the contact curve – the locus of tangencies between the consumer's indifference curves and the iso cost curve. If equilibrium quality is above (below) the contract curve then quality is upwardly (downwardly) skewed. The definition gives a useful economic insight into the nature of market equilibrium.

This usefulness of the approach is demonstrated by applying the definition to two classic studies. The model of Spence (1975) and Sheshinski (1976) is reconsidered. A condition, the rate of change of the indifference elasticity, is found to identify, from consumer preference, the conditions under which a profit maximising monopolists generates quality skewness. Quality is unskewed by the monopolist if preferences take the isoelastic form considered in example 2.

Maskin and Riley consider vertical differentiation with customer self selection, but only in the context of the unit demand model. Their results mirror Mussa and Rosen: that low quality is inefficiently low. This paper extends their model to allow for any preferences across quantity and quality that satisfy uniform ordering. Their conclusions do not always hold in the extended model. In particular under uniform ordering low valuation customers have upward skewed quality if the MRS of the low valuation customer is less than the MRS of the upwardly adjacent customer. Indeed the bundle consumed by low valuation customers might have quality that is above the efficient level.

Some cases in which uniform ordering does not hold are considered. Under diverse ordering there is no quality skewness or distortion. Indeed production is efficient, even with arbitrary preferences. Some one-dimensional cases in which the demand curve of the low valuation customers is flatter than that of the high valuation customers (and hence single crossing/uniform ordering do no hold) are considered by DWSB. DWSB's results indicate that the low valuation customers face inefficiently high quality. However, once the restriction of unit demand is removed, this result cannot be guaranteed. The low valuation customer will face downwardly skewed quality if the MRS of the low valuation customer is less than the MRS of the downwardly adjacent customer.

There is clearly scope for many other applications of the approach adopted in this paper. In these applications, it may sometimes be convenient to adopt the iso-elastic functional forms for utility and cost discussed in examples 2 and 4. With these restrictions on preferences and cost the contract curve is horizontal. In this case the unskewed and undistorted quality levels are both the same as the efficient quality level (for all levels of equilibrium quantity). This approach thus provides a significant degree of analytic simplification while, in contrast to the unit demand model, also allowing consumers to exhibit substitutability between quantity and quality.

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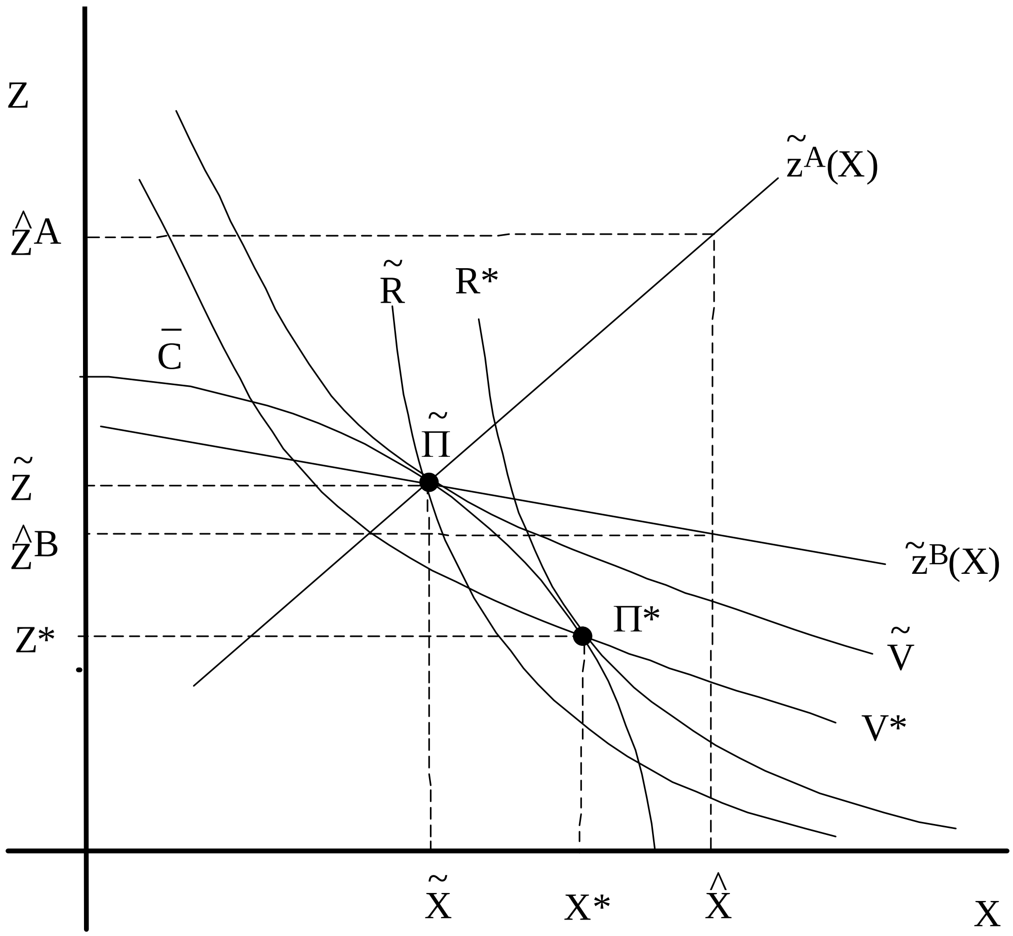

Figure 1: Quality skewness and monopoly

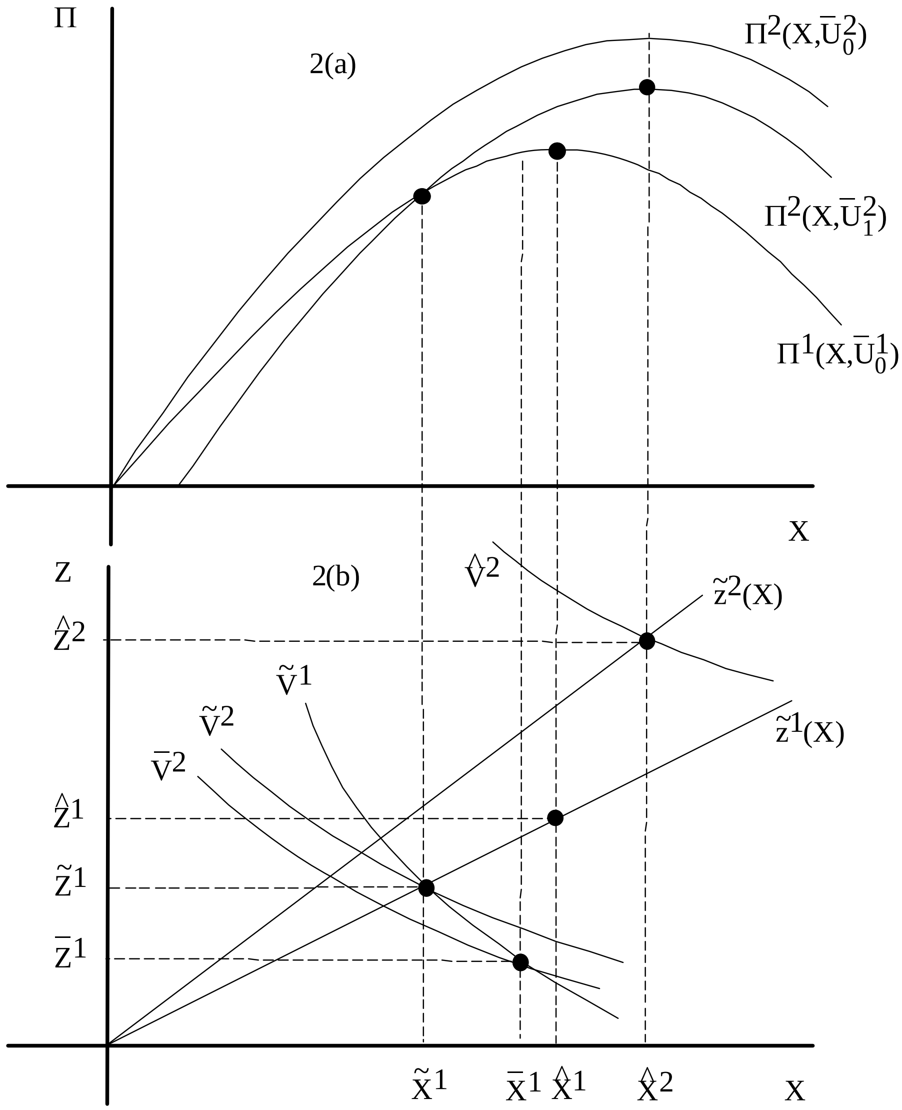

Figure 2: The augmented Maksin-Riley diagram
with tbe quantity-quality trade-off.

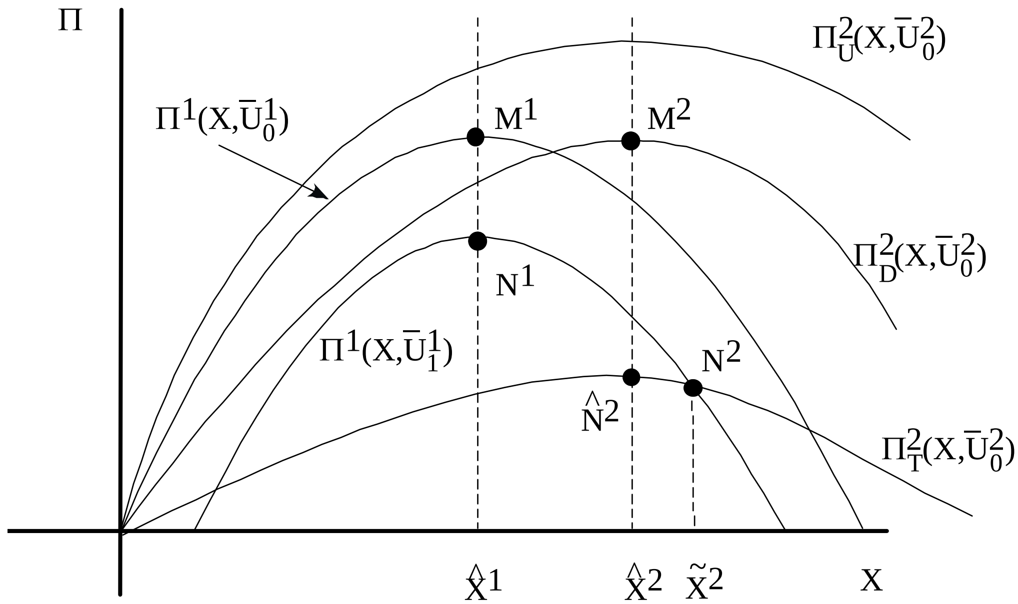

Figure 3: The augmented Maksin-Riley diagram
without uniform ordering

Appendix:

Lemma A1 shows that only the downward-adjacent incentive compatibility constraints are binding. Lemma A2 shows that only type 1 customer's participation constraint is binding.

Lemma A1: $\lambda^{ij} = 0$ for $j \neq i-1$.

Proof of Lemma A1. The proof proceeds in two steps. (i) Show that $X^i > X^{i-1}$ and $Z^i > Z^{i-1}$; and, (ii) $U^i(X^i,Z^i,T^i) > U^{i-j}(X^{i-j},Z^{i-j},T^{i-j})$ for j= 2,...,i-1.

(i) The firm's profit from selling a bundle <X,Z,T> to type i customers is:

$$R^i(X,Z,T) = N^i(X,Z) - U^i(X,Z,T)$$

where $N^i(X,Z) = V^i(X,Z) - C(X,Z)$ is the social surplus from selling to type i customers, which is assumed strictly concave. Let:

$$(\hat{X}^i,\hat{Z}^i) = \underset{X\ Z}{\operatorname{argmax}}\ N^i(X,Z)$$

Along type i's indifference curve, $\bar{U}^i = U^i(X,Z,T)$, profits from type i customers is given by:

$$\bar{R}^i(X,Z) = N^i(X,Z) - \bar{U}^i$$

(Note along the indifference curve T is adjusted to ensure keep U^i constant.) Observe that $\bar{R}^i_1 = N^i_1(X,Z)>0$ and $\bar{R}^i_2 = N^i_2(X,Z)>0$ for all $X<\hat{X}^i$ and $Z<\hat{Z}^i$.

Consider the bundle that lie along type i's indifference curve. These can be characterised in the following way:

$$\bar{U}^i = U^i(X,Z,T) = U^i(X+ \delta X,Z+ \delta Z,T+ \delta T)$$

For small variations in X, Z and T the following holds from Taylor's theorem:

$$U^i(X+\delta X, Z+\delta Z, T+\delta T) \approx U^i(X,Z,T) + \delta X V_1^i(X,Z) + \delta Z V_2^i(X,Z) - \delta T$$

Hence along an indifference curve:

$$\delta T = \delta X V_1^i(X,Z) + \delta Z V_2^i(X,Z)$$

Consider the preferences of type j>i customers for bundles along type i's indifference curves:

$$U^j(X+\delta X, Z+\delta Z, T+\delta T) \approx U^j(X,Z,T) + \delta X V_1^j(X,Z) + \delta Z V_2^j(X,Z) - \delta T$$

$$= U^j(X,Z,T) + \delta X V_1^j(X,Z) + \delta Z V_2^j(X,Z) - [\delta X V_1^i(X,Z) + \delta Z V_2^i(X,Z)]$$
$$> U^j(X,Z,T)$$

if $\delta X>0$ and $\delta Z>0$.

Thus if $U^i(X,Z,T) = U^i(X+\delta X, Z+\delta Z, T+\delta T)$ then $U^j(X+\delta X, Z+\delta Z, T+\delta T) > U^j(X,Z,T)$ for $\delta X>0$ and $\delta Z>0$. It may now be shown that a bundle involving $X^i \leq X^{i-1}$ or $Z^i \leq Z^{i-1}$ cannot be an equilibrium bundles. Suppose it was. Then it would satisfy the self selection constraints for both type i-1 and type i customers, i.e.:

$$U^i(X^i,Z^i,T^i) \geq U^i(X^{i-1},Z^{i-1},T^{i-1}) \text{ and } U^{i-1}(X^i,Z^i,T^i) \leq U^i(X^{i-1},Z^{i-1},T^{i-1})$$

Write $X^i=X^{i-1}+\delta X$, $Z^i=Z^{i-1}+\delta Z$ and $T^i=T^{i-1}+\delta T$. Then these constraints are satisfied if:

$$\delta X[V_1^i(X^{i-1},Z^{i-1}) - V_1^{i-1}(X^{i-1},Z^{i-1})] + \delta Z[V_2^i(X^{i-1},Z^{i-1}) - V_2^{i-1}(X^{i-1},Z^{i-1})] \geq 0 \qquad \text{(1A)}$$

This constraint could hold if $\delta X = \delta Z = 0$. However δX and δZ could be increased along type i's indifference curve without violating the incentive compatibility constraints. In this case profits will increase. Hence $\delta X = \delta Z = 0$ cannot be in the equilibrium bundle.

Similarly, under uniform ordering, (1A) could hold if $\delta X > 0$ and $\delta Z < 0$. However such a combination cannot be an equilibrium. Suppose that it were. Observe that, for any δX and δZ, it is profit maximizing to ensure that T^i is set so that type i's incentive compatibility constraint is binding, i.e. $U^i(X^i,Z^i,T^i) = U^i(X^{i-1},Z^{i-1},T^{i-1})$. Now if δZ was increased to zero (with δT adjusted to retain indifference), (1A) would continue to hold. However along the indifference curve profits increase. Hence $\delta X > 0$ and $\delta Z < 0$ cannot be an equilibrium combination. Similarly $\delta X < 0$ and $\delta Z > 0$ cannot be an equilibrium combination. Thus $X^i > X^{i-1}$ and $Z^i > Z^{i-1}$ in the equilibrium bundle.

(ii) Note that uniform ordering requires that $V^{i+1}(X^{i+1},Z^{i+1}) < V^i(X^i,Z^i)$ if $X^{i+1} < X^i$ and $Z^{i+1} < Z^i$. To show that $U^i(X^i,Z^i,T^i) > U^{i-j}(X^{i-j},Z^{i-j},T^{i-j})$ for j= 2,...,i-1 assume that (X^i,Z^i, T^i), i=1,…,n, represent the equilibrium bundles. In this case the following self selection constraints hold:

$$V^i(X^i,Z^i) - V^i(X^{i-1},Z^{i-1}) \geq T^i - T^{i-1}$$

and:

$$V^{i-1}(X^{i-1},Z^{i-1}) - V^{i-1}(X^{i-2},Z^{i-2}) \geq T^{i-1} - T^{i-2} \text{ for all } i>2.$$

Then, adding these constraints yields:

$$V^i(X^i,Z^i) - V^i(X^{i-2},Z^{i-2}) + \{V^i(X^{i-2},Z^{i-2}) - V^i(X^{i-1},Z^{i-1}) - [V^{i-1}(X^{i-2},Z^{i-2}) - V^{i-1}(X^{i-1},Z^{i-1})]\}$$
$$\geq T^i - T^{i-2}$$

Note that uniform ordering implies:

$$\{V^i(X^{i-2},Z^{i-2}) - V^i(X^{i-1},Z^{i-1}) - [V^{i-1}(X^{i-2},Z^{i-2}) - V^{i-1}(X^{i-1},Z^{i-1})]\} < 0,$$

Hence:

$$V^{i-1}(X^{i-1},Z^{i-1}) - V^{i-1}(X^{i-2},Z^{i-2}) > T^{i-1} - T^{i-2}$$

for all i>2. Similarly

$$V^{i}(X^{i+j},Z^{i+j}) - T^{i+j} < V^{i}(X^{i},Z^{i}) - T^{i} \text{ for all } j = 1,...n-i. \qquad ||$$

<u>Lemma A2</u>: $\mu^1 > 0$ and $\mu^i = 0$ for i = 2,...n.

Proof of lemma A2: Under uniform ordering, any schedule which gives type 1 customers with non-negative utility will provide all other customers with positive utility. Thus only the participation constraint of type 1 customers is binding. ||